Chris Gregg
CS 109

Tutorial #5
November 3, 2026

Tutorial 5

1 Bayesian RNA Quantification

Let N_A and N_B be the count of type-A RNA and type-B RNA in a person's body. Both are Poisson distributed:

$$N_A \sim \text{Poi}(12 \cdot X) \qquad N_B \sim \text{Poi}(12 \cdot (1 - X)).$$

N_A and N_B have rates that come from an unknown continuous variable X. X takes on values between 0 and 1 and is indicates a persons overall health. N_A and N_B are independent, conditioned on knowing X.

a. For part (a) only, assume $X = 1/4$. What is $P(N_A > 10)$?

b. Our prior belief is that $X \sim \text{Beta}(a = 2, b = 2)$. An experiment observes $N_A = 12$ and $N_B = 16$. Based on these observations, what is your updated distribution for X? In other words, what is $f(X = x \mid N_A = 12, N_B = 16)$? You can leave your answer in terms of a normalization constant.

c. What is the variance of your updated belief distribution for X, found in part (b)?

2. Rejection Sampling

Explain how the steps of rejection sampling are analogous to applying the definition of conditional probability.

3. Hockey Fights

You meet 10 die-hard Canadian ice hockey fans and politely interrupt their discussion about who the best team in the National Hockey League is. Each fan independently roots for exactly one of the following three teams: Toronto Maple Leafs, Edmonton Oilers, Montreal Canadiens. A fan is equally likely to root for any of these teams.

(a) What is the probability 5 of them are Leafs fans, 3 are Oilers fans and 2 are Canadiens fans?

(b) What is the probability each team has at least one fan?

(c) What is the expected number of distinct teams being rooted for among the 10 fans?

(d) (Optional) What is the probability one team has strictly more fans than the other two teams combined?