# CS109 Lecture 1: LLM Learning Guide

Foundations of Probability

Summer 2026

## Lecture 1: LLM Learning Guide

**Before lecture:** read the Lecture 1 slides (through the "Sum of Two Dice" sample-space discussion). Then open your favorite LLM and work through the concepts below **in order**. Each concept has a *Learn* prompt to get an explanation and a *Test me* prompt to check that it stuck. Paste the prompts in, but do not just read the replies: try to answer the "Test me" questions yourself first, then ask the LLM to grade you.

**How to get the most out of this:**

- Tell the LLM your level: "I'm a student in Stanford's CS109 (probability for computer scientists). Explain at that level."
- When it gives an answer, ask "why?" at least once. Push for the reasoning, not just the result.
- If an explanation uses a term you do not know, ask it to define the term before moving on.
- Always attempt the practice yourself before revealing the answer. Then ask the LLM: "Here is my reasoning: _____. Where exactly am I wrong, if anywhere?"

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### Concept 1: Sample spaces and events

**Learn:** > "Explain the difference between a *sample space* $S$ and an *event* $E$ in probability, and why an event is defined as a subset of the sample space ($E \subseteq S$). Give three concrete examples (a coin flipped twice, a single die roll, and the number of emails I get in a day), writing out $S$ and one example event $E$ for each."

**Test me:** > "Quiz me: give me a random real-world scenario and ask me to (1) write the sample space and (2) describe one event as a subset. After I answer, tell me whether my sample space was complete and whether my event was a valid subset, and fix any mistakes."

### Concept 2: What a probability actually is

**Learn:** > "Explain what it means to say the probability of an event is a number between 0 and 1. Cover the long-run frequency (frequentist) interpretation: probability as the fraction of times an event happens over many repeated trials. Then explain how we can also read a probability off a dataset. Use a dart-throwing or weather example."

**Test me:** > "I throw darts and hit the target 11 times out of 24 throws. Ask me to estimate the probability and to explain in one sentence what that number means in terms of repeated trials. Then critique my explanation."

## Concept 3: Equally likely outcomes and $P(E) = |E|/|S|$

**Learn:** > "Explain the equally-likely-outcomes model and the formula $P(E) = |E|/|S|$. Crucially, explain when this formula is valid and when it fails. Use the classic example of summing two dice: contrast the correct 36-outcome sample space with the tempting-but-wrong idea that the 11 possible sums (2 through 12) are equally likely."

**Test me:** > "Give me a problem about rolling two dice (for example, the probability the sum is 5, or the probability the two dice match). Make me solve it by counting equally likely outcomes. Then check whether I used the correct 36-outcome sample space, and call out the 'sums are equally likely' trap if I fell into it."

## Concept 4: The Kolmogorov axioms

**Learn:** > "State the three Kolmogorov axioms of probability: (1) $0 \leq P(E) \leq 1$, (2) $P(S) = 1$, and (3) for mutually exclusive events, $P(E \cup F) = P(E) + P(F)$. For each axiom, explain in plain English why it is reasonable, and give one example of a claim that violates it."

**Test me:** > "Give me 4 short true/false statements about probabilities (for example, 'an event can have probability 1.2', or 'two events can each have probability 0.7'). Ask me to say true or false and name which axiom is involved. Grade my answers and correct my reasoning."

## Concept 5: Mutually exclusive events (and when you may add)

**Learn:** > "Explain what it means for two events to be *mutually exclusive* ($E \cap F = \varnothing$), and why Axiom 3 lets us add probabilities only in that case. Show a worked example where adding is valid and a second example where naively adding double-counts a shared outcome. I have not yet learned the general inclusion-exclusion formula, so keep the second example to direct counting of the union."

**Test me:** > "Using a single draw from a 52-card deck, ask me whether each of these pairs is mutually exclusive and whether I can add their probabilities: (a) heart vs. ace, (b) ace vs. face card. Make me compute $P(E \cup F)$ for each, then check my work and point out any double-counting."

## Concept 6: The complement rule

**Learn:** > "Explain the complement rule $P(E^C) = 1 - P(E)$, derive it from the axioms (using that $E$ and $E^C$ are mutually exclusive and their union is $S$), and explain why computing the complement is often easier than computing the event directly. Give an example such as 'the probability of at least one' versus 'the probability of none.' "

**Test me:** > "Pose a problem where the complement is clearly the easier route (for example, rolling a die several times and asking for 'at least one six' — but only set it up conceptually since I have not learned independence yet). Have me identify the complement, then walk me through whether my setup is correct."

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## Wrap-up prompt (after all six concepts)

> "Give me a single multi-part problem that combines sample spaces, equally likely outcomes, mutually exclusive events, and the complement rule (the kind I'd see early in CS109). Let me attempt the whole thing. Then grade each part, tell me which concept each part tested, and tell me which one concept I should review most before the next lecture."

*Bring any sticking points to lecture; the TAs and instructor will help you work through them on the worksheet.*