CS109 Lecture 1 Worksheet: Foundations of Probability

Sample Spaces, Events, Equally Likely Outcomes, and the Axioms

Summer 2026

Lecture 1 Worksheet: Foundations of Probability

Topics: events and sample spaces, probability as long-run frequency, equally likely outcomes ($P(E) = |E|/|S|$), the Kolmogorov axioms, mutually exclusive events, and the complement rule.

How to use this sheet. Work in small groups. A TA and the instructor will circulate to help. We will go over selected solutions on the board. You do not need to finish every part; aim for understanding over speed. No calculators or code required, though a couple of problems suggest a quick check you could run later.

Problem 1: Sample Spaces and Events

You flip a fair coin **three times** and record the sequence of results (for example, `HTH`).

(a) Write out the sample space S. How many outcomes does it contain?

(b) Let E be the event "at least two heads." List every outcome in E, then give $|E|$.

(c) Let F be the event "the first flip is tails." List the outcomes in F.

(d) Are E and F **mutually exclusive**? Justify your answer by referring to $E \cap F$.

Problem 2: Two Dice and the Right Sample Space

You roll **two fair six-sided dice** and look at the results.

(a) Treat the dice as distinguishable (say, one red and one blue). Write down $|S|$, the number of equally likely outcomes.

(b) Let E be the event "the sum is 7." Find $P(E)$.

(c) Let G be the event "the sum is 2." Find $P(G)$.

(d) A classmate proposes a different sample space: "the sum can be any value in $\{2, 3, 4, \ldots, 12\}$, which is 11 outcomes, so $P(\text{sum} = 7) = \frac{1}{11}$." This answer is wrong. Explain precisely what is incorrect about this reasoning. (Hint: are those 11 outcomes *equally likely*?)

Problem 3: Drawing a Card, and When You May Add Probabilities

You draw a single card uniformly at random from a standard 52-card deck.

(a) Let H be the event "the card is a heart." Find $P(H)$.

(b) Let A be the event "the card is an ace." Find $P(A)$.

(c) Are H and A mutually exclusive? Can you compute $P(H \cup A)$ by simply adding $P(H) + P(A)$? Explain using **Axiom 3**, and then compute $P(H \cup A)$ correctly by counting the outcomes in $H \cup A$.

(d) Let F be the event "the card is a face card (J, Q, or K)." Are A and F mutually exclusive? Use this to find $P(A \cup F)$.

(e) Use the **complement rule** to find P(the card is **not** a spade).

Problem 4: The Complement Is Your Friend

A spinner is divided into 12 equal wedges numbered 1 through 12. You spin once; every wedge is equally likely.

(a) Let $T =$ "the number is a multiple of 3" and let $L =$ "the number is less than or equal to 4." Find $P(T)$ and $P(L)$.

(b) Are T and L mutually exclusive? Determine $P(T \cup L)$ by directly counting the outcomes in the union. Compare this to $P(T) + P(L)$ and explain any difference.

(c) Now consider the event "the number is **not** 7." Find its probability using the complement rule, and confirm the axioms are satisfied (i.e., the answer lies in $[0, 1]$).

Problem 5: What Probability Means, and a Check on the Axioms

(a) **Long-run frequency.** A weather dataset records that over the past 365 days, 219 were sunny. Treating this dataset as our source of probability, estimate P(sunny). In one sentence, state what this number means in terms of repeated trials.

(b) **Axiom check (true/false, with one line of justification each).**

i. A friend claims a certain event has probability 1.2. Is this possible? Which axiom does it violate?

ii. For two events E and F, your classmate writes $P(E) + P(F) = 1.3$ and concludes "that's impossible, probabilities can't sum above 1." Is the classmate right? Explain.

iii. If $P(E) = 0.4$, what is the largest that $P(E^C)$ could be, and what is it exactly? Which identity are you using?