CS109 Lecture 2: LLM Learning Guide

Conditional Probability and Bayes

Summer 2026

Lecture 2: LLM Learning Guide

Before lecture: read the Lecture 2 slides (conditional probability through Bayes' theorem and the SARS/spam examples). Then open your favorite LLM and work through the concepts below **in order**. Each concept has a *Learn* prompt and a *Test me* prompt. Try the "Test me" questions yourself before asking the LLM to grade you.

How to get the most out of this:

- Tell the LLM your level: "I'm a student in Stanford's CS109 (probability for computer scientists). Explain at that level."
- After any answer, ask "why?" at least once and ask it to show each step.
- When it states a formula, ask it to define every symbol before continuing.
- Attempt practice yourself first, then say: "Here is my reasoning: _____. Where exactly am I wrong, if anywhere?"

Concept 1: Conditional probability

Learn: > "Explain conditional probability and the formula $P(E \mid F) = P(E \cap F)/P(F)$. Explain the intuition that conditioning on F shrinks the sample space to just the outcomes in F. Use rolling two dice as the example: compare $P(\text{sum} = 8)$ with $P(\text{sum} = 8 \mid \text{first die is 5})$."

Test me: > "Give me a two-dice conditional probability problem. Make me identify the conditioning event, recompute the restricted sample space, and apply the formula. Then check each step and correct me."

Concept 2: The chain rule (multiplication rule)

Learn: > "Explain the chain rule $P(E \cap F) = P(E)\,P(F \mid E)$ and its generalization to several events. Show why it is the natural tool for sequential experiments like drawing cards without replacement. Work the example of drawing two aces in a row from a 52-card deck."

Test me: > "Pose a 'without replacement' drawing problem (cards or colored balls). Make me apply the chain rule step by step, and check whether I correctly updated the conditional probabilities after each draw."

Concept 3: The law of total probability

Learn: > "Explain the law of total probability: how to compute $P(E)$ by splitting the sample space into mutually exclusive, exhaustive cases and summing $P(E \mid B_i)P(B_i)$. Use a spam-filter example where a word appears with different rates in spam versus legitimate email, and draw the probability tree."

Test me: > "Give me a scenario with two or three underlying categories and a feature that appears at different rates in each. Make me compute the overall probability of the feature using the law of total probability. Check my partition (mutually exclusive and exhaustive?) and my arithmetic."

Concept 4: Bayes' theorem

Learn: > "State and explain Bayes' theorem. Define prior, likelihood, and posterior, and explain how Bayes' theorem 'flips' a conditional probability from $P(\text{evidence} \mid \text{hypothesis})$ to $P(\text{hypothesis} \mid \text{evidence})$. Use the spam-filter example: given a message contains 'free', find the probability it is spam."

Test me: > "Give me a Bayes problem and make me label the prior, likelihood, and posterior explicitly, then solve it. Check that I used the law of total probability correctly in the denominator."

Concept 5: The base-rate effect (medical testing)

Learn: > "Using a medical test example (a rare disease, high sensitivity, small false-positive rate), explain why a positive test result can still mean the patient probably does NOT have the disease. Show the calculation two ways: with Bayes' theorem, and with natural frequencies on a population of 1000 people."

Test me: > "Make up a rare-disease testing scenario with your own numbers. Have me compute $P(\text{disease} \mid \text{positive})$ both with Bayes' theorem and with the natural-frequency method, and confirm the two answers agree. Then ask me to explain in plain English why the answer is lower than intuition suggests."

Concept 6: Monty Hall (putting conditioning to work)

Learn: > "Walk me through the Monty Hall problem. Explain carefully why switching wins with probability 2/3, by conditioning on where the prize is and using the fact that the host knowingly avoids opening the prize door. Address the common wrong intuition that it should be 50/50."

Test me: > "Quiz me: ask me to compute the probability of winning by switching versus staying, and to explain what information the host's choice reveals. Then point out any flaw in my reasoning."

Wrap-up prompt (after all six concepts)

> "Give me one multi-part problem in the style of CS109 that uses conditional probability, the law of total probability, and Bayes' theorem together (for example, a diagnostic-test or spam-classification scenario). Let me solve all parts, then grade each part, tell me which concept it tested, and tell me the single concept I should review most before the next lecture."

Bring any sticking points to lecture; the TAs and instructor will help you work through them on the worksheet.