

CS109 Lecture 3 Worksheet: ANSWER KEY

Independence and Inclusion-Exclusion

Summer 2026

Lecture 3 Worksheet: Answer Key

Instructor / TA copy. Full worked solutions follow.

Problem 1: Inclusion-Exclusion with Two Events

Let $C = \text{CS}$, $M = \text{math}$. $P(C) = 0.60$, $P(M) = 0.40$, $P(C \cap M) = 0.25$.

- (a) $P(C \cup M) = P(C) + P(M) - P(C \cap M) = 0.60 + 0.40 - 0.25 = 0.75$.
- (b) $P(\text{neither}) = 1 - P(C \cup M) = 1 - 0.75 = 0.25$.
- (c) Adding $0.60 + 0.40 = 1.00$ counts the 25% enrolled in both courses twice; inclusion-exclusion subtracts that overlap once to correct the double-count.

Problem 2: Weather Prediction (Midterm Fall 2022)

- a) Write pseudo-code to estimate the probability of rain on a random day.

```
rainy_days = 0
days = len(history)
for i in range(days):
    if history[i]:
        rainy_days += 1
return rainy_days / days
```

- b) Write pseudo-code to estimate the probability of rain tomorrow given that it was sunny today.

```
rain_tomorrow_sun_today = 0
sun_today = 0
days = len(history)
for i in range(days - 1):
    if !history[i]:
        sun_today += 1
        if history[i + 1]:
            rain_tomorrow_sun_today += 1
return rain_tomorrow_sun_today / sun_today
```

- c) How would you test if rain on a given day is independent of rain on the previous day, using the values calculated in part a and b?

In part a, we calculate $a = P(\text{rain})$, and in part b, we calculate $b = P(\text{rain}|\text{sun on the previous day})$. To check for independence between rain and whether it rained on the previous day, we can equivalently check for independence between rain and it not raining on the previous day. So, we can test

$$P(\text{rain}) \stackrel{?}{\approx} P(\text{rain} | (\text{rain on the previous day})^C) = P(\text{rain} | \text{sun on the previous day}).$$

In other words, we test $a \stackrel{?}{\approx} b$.

Problem 3a: Scheduling (a) (pset1, problem 5a)

(homework problem)

Problem 3b: Scheduling (b) (pset1, problem 5b)

(homework problem)