CS109 Lecture 3: LLM Learning Guide

Independence and Inclusion-Exclusion

Summer 2026

Lecture 3: LLM Learning Guide

Before lecture: read the Lecture 3 slides (inclusion-exclusion through independence and the network-reliability examples). Then open your favorite LLM and work through the concepts below **in order**. Each concept has a *Learn* prompt and a *Test me* prompt. Attempt the "Test me" questions yourself before asking the LLM to grade you.

How to get the most out of this:

- Tell the LLM your level: "I'm a student in Stanford's CS109 (probability for computer scientists). Explain at that level."
- After any answer, ask "why?" at least once and ask it to show each step.
- When it states a formula, ask it to define every symbol before continuing.
- Attempt practice yourself first, then say: "Here is my reasoning: _____. Where exactly am I wrong, if anywhere?"

Concept 1: Inclusion-exclusion for two events

Learn: > "Explain why $P(E \cup F) = P(E) + P(F) - P(E \cap F)$, and why we must subtract the intersection. Connect it back to Axiom 3 (which only adds probabilities for mutually exclusive events). Use a Venn diagram description and a concrete example such as students enrolled in CS, math, or both."

Test me: > "Give me a two-event 'or' problem with an overlap. Make me apply inclusion-exclusion and also find the probability of 'neither.' Check whether I subtracted the overlap correctly."

Concept 2: Inclusion-exclusion for three events

Learn: > "Extend inclusion-exclusion to three events: $P(E \cup F \cup G) = P(E) + P(F) + P(G) - P(EF) - P(EG) - P(FG) + P(EFG)$. Explain the alternating add/subtract pattern. Work the example of numbers from 1 to 100 divisible by 2, 3, or 5."

Test me: > "Pose a three-set counting problem (for example, integers divisible by some set of numbers, or people subscribing to three services). Make me apply the three-set formula term by term, and check that I included the triple-intersection with the correct sign."

Concept 3: Definition of independence

Learn: > "Define independence of two events: $P(E \cap F) = P(E)P(F)$. Emphasize that this is a numerical condition, not a statement about cause and effect. Give one example of two events that turn out to be

independent and one example of two events that seem related but are (or are not) independent, and show the check both times."

Test me: > "Using two dice, give me two events and make me determine whether they are independent by checking $P(E \cap F) = P(E)P(F)$. Include at least one case where the answer is counterintuitive. Grade my check."

Concept 4: Independence and complements; probability of AND

Learn: > "Explain why, if A and B are independent, then A and B^C are also independent. Then explain how independence makes 'AND' easy: for independent events you just multiply. Show how this lets you compute the probability that several independent things all happen."

Test me: > "Give me a problem with several independent events (for example, independent components each working with some probability). Make me compute the probability they all succeed by multiplying, and check my setup."

Concept 5: Series and parallel reliability ("at least one")

Learn: > "Explain how to compute the probability that 'at least one' of several independent events occurs, using the complement: $1 - P(\text{none occur})$. Apply it to a network with n independent routers in parallel, and contrast with a series system where all must work."

Test me: > "Pose an 'at least one' problem with independent trials (routers, bonds forming, servers staying up). Make me solve it with the complement method, then sanity-check that the answer increases toward 1 as the number of redundant components grows."

Concept 6: Conditional independence (a subtlety)

Learn: > "Explain the difference between independence and conditional independence, and give a clear example where two events are independent but become dependent once you condition on a third event (for example, two coin flips and the event that they match). Explain why conditioning can create or destroy independence."

Test me: > "Give me two events that are independent unconditionally, plus a third event to condition on. Make me check independence before and after conditioning, and explain in words why it changed."

Wrap-up prompt (after all six concepts)

> "Give me one multi-part CS109-style problem that combines inclusion-exclusion, independence, and an 'at least one' complement calculation. Let me solve every part, then grade each part, tell me which concept it tested, and tell me the single concept I should review most before the next lecture."

Bring any sticking points to lecture; the TAs and instructor will help you work through them on the worksheet.