CS109 Lecture 3 Worksheet: Independence and Inclusion-Exclusion

Probability of OR, Probability of AND, and Independence

Summer 2026

Lecture 3 Worksheet: Independence and Inclusion-Exclusion

Topics: the probability of "OR" via inclusion-exclusion $P(E \cup F) = P(E) + P(F) - P(E \cap F)$ (and its three-set version), independence $P(E \cap F) = P(E)P(F)$, the independence of complements, and reliability of series/parallel systems.

How to use this sheet. Work in small groups. A TA and the instructor will circulate to help. We will go over selected solutions on the board. You do not need to finish every part; aim for understanding over speed. No calculators or code required, though a couple of problems suggest a quick check you could run later.

Problem 1: Inclusion-Exclusion with Two Events

In a certain class, 60% of students are enrolled in a CS course, 40% are enrolled in a math course, and 25% are enrolled in both.

(a) What fraction of students are enrolled in CS **or** math? State the inclusion-exclusion formula you use.

(b) What fraction are enrolled in **neither**?

(c) Why would simply adding $0.60 + 0.40$ give the wrong answer for part (a)?

Problem 2: Weather Prediction (Midterm Fall 2022)

We are building a weather prediction service for Cloud City. We have access to the full history of weather stored in a list called `history`. `history[i]` is `True` if it rained on day `i`. For example the history list for the last week at Stanford would look like: `history = [False, False, False, False, False, False, True]` if the only time it rained was the last day of the week. The `history` list for Cloud City has over 10,000 elements and represents the daily weather for the last 30 years.

a) Write pseudo-code to estimate the probability of rain on a random day.

b) Write pseudo-code to estimate the probability of rain tomorrow given that it was sunny today.

c) How would you test if rain on a given day is independent of rain on the previous day, using the values calculated in part a and b?

Problem 3a: Scheduling (a) (pset1, problem 5a)

You are trying to find a time when two people can meet. Assume a day is composed of 8 one hour blocks. Each block has $p = 0.7$ probability of being busy for each person. We assume that whether each block is busy for a person is independent of whether other blocks are busy for them, and also independent of whether that block is busy for other people.

Question: Two people are trying to schedule a meeting. What is the probability that there is at least one block of time when they are both free on a given day? Provide an answer to 3 decimal places.

Problem 3b: Scheduling (b) (pset1, problem 5b)

You are trying to find a time when two people can meet.

Question: A person in India and a person in the UK are trying to schedule a one-hour meeting. In India, time zones are offset by half an hour relative to time zones in the UK. Here are their overlapping blocks:

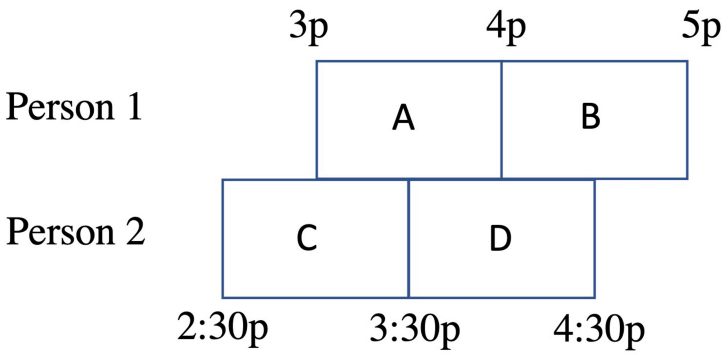

Figure 1: Overlapping schedule blocks

All times in the figure are in India Standard Time. Notice that Person 2 is in the UK and can't meet before 2:30p because it would be too early in their local time.

The two people can only meet if blocks A, C, and D are all free, or if blocks A, B, and D are all free. Each block still has $p = 0.7$ probability of being busy for each person. We still assume that whether each block is busy for a person is independent of whether other blocks are busy for them, and also independent of whether that block is busy for other people. What is the probability that the two people will be able to meet? Provide an answer to three decimal places.