CS109 Lecture 4: LLM Learning Guide

Counting

Summer 2026

Lecture 4: LLM Learning Guide

Before lecture: read the Lecture 4 slides (counting with steps through permutations, combinations, and counting-based probabilities like poker hands and coin flips). Then open your favorite LLM and work through the concepts below **in order**. Each concept has a *Learn* prompt and a *Test me* prompt. Attempt the "Test me" questions yourself before asking the LLM to grade you.

How to get the most out of this:

- Tell the LLM your level: "I'm a student in Stanford's CS109 (probability for computer scientists). Explain at that level."
- After any answer, ask "why?" at least once and ask it to show each step.
- When it states a formula, ask it to define every symbol before continuing.
- Attempt practice yourself first, then say: "Here is my reasoning: _____. Where exactly am I wrong, if anywhere?"

Concept 1: Counting with steps (the product rule)

Learn: > "Explain the product rule (counting with steps): if a process has independent stages with a, then b, then c choices, the total is $a \cdot b \cdot c$. Use examples like the number of possible PINs or license plates, and contrast 'with repetition allowed' versus 'no repetition.' "

Test me: > "Give me a counting-with-steps problem (passwords, license plates, or seating). Ask me to solve it both with and without repetition allowed, and check whether I multiplied the right number of choices at each stage."

Concept 2: Permutations of distinct objects

Learn: > "Explain permutations: why there are $n!$ orderings of n distinct objects, and why the number of ordered arrangements of k chosen from n is $\frac{n!}{(n-k)!}$. Use a small concrete example and expand the factorials."

Test me: > "Pose a permutation problem (arranging books, ranking finishers). Make me decide whether order matters and apply the right formula, then check my work."

Concept 3: Permutations with repeated (semi-distinct) objects

Learn: > "Explain how to count arrangements when some objects are identical, using the formula $\frac{n!}{n_1!\,n_2!\cdots n_k!}$. Explain why we divide by the factorials of the repeat counts. Work the example of arranging the letters of a word like BANANA or MISSISSIPPI."

Test me: > "Give me a word with repeated letters and ask me to count its distinct arrangements. Check that I divided by the factorial of each repeated-letter count."

Concept 4: Combinations

Learn: > "Explain combinations $\binom{n}{k} = \frac{n!}{k!(n-k)!}$ and how they differ from permutations (order does not matter). Explain the connection to choosing a subset, and to counting bit strings with a fixed number of 1's. Work an example like choosing a committee or a poker hand."

Test me: > "Give me a selection problem and make me decide whether it is a permutation or a combination, then solve it. Include a bit-string example. Check my reasoning about whether order matters."

Concept 5: Counting to compute probabilities (equally likely outcomes)

Learn: > "Explain how counting feeds into probability: when all outcomes are equally likely, $P(E) = |E|/|S|$, and both the numerator and denominator are counting problems. Work a poker example, such as the probability of a flush or four of a kind, counting favorable hands over $\binom{52}{5}$ total hands."

Test me: > "Pose a poker-hand probability question (one pair, a flush, three of a kind). Make me count the favorable hands and divide by $\binom{52}{5}$. Check both the favorable count and the total."

Concept 6: Counting and coin flips (toward the binomial)

Learn: > "Explain how to count the number of ways to get exactly k heads in n fair coin flips, why that count is $\binom{n}{k}$, and why the probability is $\binom{n}{k}/2^n$. Mention that this idea generalizes to the binomial distribution we will see soon."

Test me: > "Give me a coin-flip problem (exactly k heads, or at least k heads, in n flips). Make me set up the count of favorable sequences over 2^n, and check that I summed the right combinations for 'at least.' "

Wrap-up prompt (after all six concepts)

> "Give me one multi-part CS109-style counting problem that mixes the product rule, permutations with repeats, combinations, and a counting-based probability (for example, a card or coin scenario). Let me solve every part, then grade each part, tell me which concept it tested, and tell me the single concept I should review most before the next lecture."

Bring any sticking points to lecture; the TAs and instructor will help you work through them on the worksheet.