CS109 Lecture 4 Worksheet: Counting

The Product Rule, Permutations, Combinations, and Counting for Probability

Summer 2026

Lecture 4 Worksheet: Counting

Topics: counting with steps (the product rule), permutations of distinct and semi-distinct objects, combinations $\binom{n}{k}$, and using counting to compute probabilities of equally likely outcomes (poker hands, coin flips).

How to use this sheet. Work in small groups. A TA and the instructor will circulate to help. We will go over selected solutions on the board. You do not need to finish every part; aim for understanding over speed. No calculators or code required, though a couple of problems suggest a quick check you could run later.

Problem 1 (Review of Lecture 3): Independence and "At Least One"

A web service depends on 3 independent components, each of which is up with probability 0.95.

(a) Find the probability that **all three** are up (service fully healthy).

(b) Find the probability that **at least one** component is down. (Hint: complement.)

Problem 2: Counting with Steps (the Product Rule)

A phone unlock code is a sequence of 4 digits, each from 0 to 9.

(a) How many distinct codes are possible?

(b) How many codes use **no repeated digit**?

(c) A 6-digit code is known to use 6 **distinct** digits, and the screen shows exactly 6 smudges (one per digit used), but not their order. How many orderings of those 6 known digits are possible?

Problem 3: Fantasy Sports Draft (Spring 2026 Midterm Question)

A fantasy sports league holds a draft. There are 20 available players in the player pool: 8 forwards, 7 midfielders, and 5 goalkeepers. A team selects 6 players from the pool in a ranked sequence (the 1^{st} pick is ranked most valuable, the 2^{nd} pick second most, and so on). No player may be selected more than once.

a) How many ranked sequences of 6 players contain *exactly 2 goalkeepers*?

b) Suppose that exactly 4 of the 6 drafted players are forwards. After the draft, the team's coach randomly selects 3 of the 6 players to start the next match (the order of the starters does not matter). What is the probability that all 3 starters are forwards?

Problem 4: Permutations of Letters (Semi-Distinct Objects)

Recall from lecture that the letters of **BOBA** can be arranged in $\frac{4!}{2!} = 12$ ways (the two B's are interchangeable).

(a) How many distinct arrangements are there of the letters in **BANANA**?

(b) How many distinct arrangements are there of the letters in **MISSISSIPPI**?

(c) State the general rule for counting arrangements of n objects when there are repeated groups of sizes $n_1, n_2, \ldots, n_k$.

Problem 5: Combinations

(a) How many different 5-card hands can be dealt from a standard 52-card deck (order does not matter)?

(b) A committee of 3 is chosen from 10 people. How many committees are possible?

(c) How many bit strings of length 10 contain exactly three 1's? Explain why this is a combination.

Problem 6: Counting to Find Probabilities (Poker Hands)

A 5-card hand is dealt uniformly at random from a 52-card deck, so every one of the $\binom{52}{5}$ hands is equally likely.

(a) Find the probability of a **flush** (all 5 cards the same suit). Count the favorable hands as (number of suits) $\times$ (ways to choose 5 cards of that suit).

(b) Find the probability of **four of a kind** (four cards of one rank plus any fifth card).

Problem 7: Counting and Coin Flips

A fair coin is flipped 10 times and the sequence of heads and tails is recorded.

(a) How many equally likely outcomes (sequences) are there?

(b) Find the probability of getting **exactly 4 heads**. Explain why the count of favorable outcomes is $\binom{10}{4}$.

(c) Find the probability of getting **at least 8 heads**.

Challenge Problem (optional): Exactly Two Aces

A 5-card hand is dealt from a 52-card deck.

(a) Count the hands that contain **exactly two aces**, as (ways to choose 2 of the 4 aces) $\times$ (ways to choose the other 3 cards from the 48 non-aces).

(b) Convert this count to a probability.