CS109 Lecture 5: LLM Learning Guide

The Binomial

Summer 2026

Lecture 5: LLM Learning Guide

Before lecture: read the Lecture 5 slides (random variables, probability mass functions, and the binomial distribution). Then open your favorite LLM and work through the concepts below **in order**. Each concept has a *Learn* prompt and a *Test me* prompt. Attempt the "Test me" questions yourself before asking the LLM to grade you.

How to get the most out of this:

- Tell the LLM your level: "I'm a student in Stanford's CS109 (probability for computer scientists). Explain at that level."
- After any answer, ask "why?" at least once and ask it to show each step.
- When it states a formula, ask it to define every symbol before continuing.
- Attempt practice yourself first, then say: "Here is my reasoning: _____. Where exactly am I wrong, if anywhere?"

Concept 1: Random variables

Learn: > "Explain what a random variable is, intuitively and formally: a variable whose value is uncertain, that maps random outcomes to numbers. Contrast a random variable with an event. Use examples like 'let X be the roll of a die' and 'let Y be the number of heads in 2 coin flips,' and show how 'X takes on a value' is itself an event."

Test me: > "Give me a short scenario and ask me to define an appropriate random variable for it, state the values it can take, and say which of my statements are events versus the random variable itself. Then check my answer."

Concept 2: The probability mass function (PMF)

Learn: > "Explain the probability mass function of a discrete random variable: $p_X(k) = P(X = k)$. Explain the two properties every PMF must satisfy ($0 \leq p_X(k) \leq 1$ and the values sum to 1). Show how to build the PMF of 'number of heads in 2 coin flips' by listing outcomes, and mention that a PMF can be given as a table, a formula, or even code."

Test me: > "Give me a small random variable (like the sum of two dice, or heads in 3 flips). Make me write its full PMF and verify it sums to 1. Then check my table and point out any value I got wrong."

Concept 3: Using and normalizing a given PMF

Learn: > "Show how to work with a PMF someone hands you: plugging in a value to get a probability, summing over a range to get things like $P(X \leq 2)$, and finding a normalizing constant c so that the probabilities sum to 1. Use an example with a formula PMF, such as $P(X = k) = c \cdot 2^{-k}$ for $k \geq 1$."

Test me: > "Give me a PMF with an unknown constant c. Make me solve for c, then compute a specific probability and a cumulative probability like $P(X \leq 2)$. Check each step."

Concept 4: Recognizing a binomial scenario

Learn: > "Explain the three conditions that define a binomial random variable: (1) a fixed number n of independent trials, (2) the same success probability p on every trial, and (3) we count the number of successes k. Give several scenarios that fit (bit strings, ad clicks, votes, working servers) and at least one that does NOT (e.g. dealing cards without replacement) and explain which condition fails."

Test me: > "Give me four short scenarios and make me decide which are binomial. For each binomial one, make me state n and p and write $X \sim \text{Bin}(n, p)$. For each non-binomial one, make me name the assumption that fails. Then grade me."

Concept 5: The binomial PMF

Learn: > "Derive and explain the binomial PMF $P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$. Explain each piece: why $p^k(1-p)^{n-k}$ is the probability of one specific sequence with k successes, and why $\binom{n}{k}$ counts how many such sequences there are. Connect it to the 'many coin flips' story. Work a numeric example like $P(X = 3)$ for $X \sim \text{Bin}(8, 0.5)$."

Test me: > "Give me a binomial with specific n, p, and k and make me compute $P(X = k)$ by hand, writing out the $\binom{n}{k}$, the p^k, and the $(1-p)^{n-k}$ separately. Check my arithmetic and my setup."

Concept 6: Cumulative binomial probabilities ("at least" / "fewer than")

Learn: > "Explain how to compute probabilities like $P(X \geq k)$, $P(X < k)$, and $P(a \leq X \leq b)$ for a binomial by summing the PMF over the right values, and when it is easier to use the complement (e.g. $P(X \geq 2) = 1 - P(X = 0) - P(X = 1)$). Then explain a common bug: why you canNOT compute 'at least k successes' as $\binom{n}{k} p^k$ — the chosen-slot events are not mutually exclusive (it double-counts), and a quick check at $p = 1$ exposes the error."

Test me: > "Give me a binomial 'at least k' or 'fewer than k' problem (servers, a best-of-7 series, defective items). Make me set it up as a correct sum over the right values, and make me explain in one sentence why the $\binom{n}{k} p^k$ shortcut is wrong. Check both."

Wrap-up prompt (after all six concepts)

> "Give me one multi-part CS109-style problem built around a binomial random variable: have me (1) argue that the scenario is binomial and state n and p, (2) write the PMF, (3) compute the probability of an exact count k, and (4) compute an 'at least k' probability correctly. Let me solve every part, then grade each part, tell me which concept it tested, and tell me the single concept I should review most before the next lecture."

Note: expectation and variance of the binomial (its mean np and variance $np(1-p)$) come in the next lecture on moments, so don't worry if the LLM brings them up early; focus here on the PMF and computing *probabilities.*

Bring any sticking points to lecture; the TAs and instructor will help you work through them on the worksheet.