CS109 Lecture 6: LLM Learning Guide

Moments (Expectation)

Summer 2026

Lecture 6: LLM Learning Guide

Before lecture: read the Lecture 6 slides (the classic discrete random variables and expectation). Then open your favorite LLM and work through the concepts below **in order**. Each concept has a *Learn* prompt and a *Test me* prompt. Attempt the "Test me" questions yourself before asking the LLM to grade you.

How to get the most out of this:

- Tell the LLM your level: "I'm a student in Stanford's CS109 (probability for computer scientists). Explain at that level."
- After any answer, ask "why?" at least once and ask it to show each step.
- When it states a formula, ask it to define every symbol before continuing.
- Attempt practice yourself first, then say: "Here is my reasoning: _____. Where exactly am I wrong, if anywhere?"

Concept 1: The Bernoulli and Geometric random variables

Learn: > "Explain the Bernoulli random variable $\text{Bern}(p)$ as an indicator of a single success/failure trial, and the Geometric random variable $\text{Geo}(p)$ as the number of independent trials until the first success, with PMF $P(X = k) = (1-p)^{k-1}p$. Give a concrete example of each and explain when each applies."

Test me: > "Give me four one-sentence scenarios and make me classify each as Bernoulli or Geometric (or neither), stating the parameter. Then check my answers."

Concept 2: The Negative Binomial (and the family of classics)

Learn: > "Explain the Negative Binomial random variable $\text{NegBin}(r, p)$ as the number of independent trials until the r-th success, and how it relates to the Geometric (the $r = 1$ case) and to the Binomial. Summarize the family Bernoulli / Binomial / Geometric / Negative Binomial in a small table: what each counts and its parameters."

Test me: > "Quiz me: for each of Bernoulli, Binomial, Geometric, and Negative Binomial, make me state in words what it counts and give an example. Then give me a tricky scenario and ask which one fits."

Concept 3: Expectation (expected value)

Learn: > "Define the expected value of a discrete random variable, $E[X] = \sum_x x\, P(X = x)$. Explain the interpretations (mean, weighted average, center of mass) and emphasize that $E[X]$ need not be a value X can actually take. Work the expectation of a fair die roll step by step."

Test me: > "Give me a small PMF (as a table) and make me compute $E[X]$ from the definition. Check my arithmetic and ask me whether $E[X]$ is an attainable value."

Concept 4: Linearity of expectation

Learn: > "Explain linearity of expectation: $E[aX + b] = aE[X] + b$ and $E[X + Y] = E[X] + E[Y]$, and stress that the second holds **even when X and Y are not independent**. Show a worked example, like the expected number of cards left in their original position after a shuffle, using indicator random variables."

Test me: > "Give me a problem where I should sum indicator random variables and use linearity (e.g. expected number of fixed points in a permutation, or expected matches). Make me set up the indicators and apply linearity, then check whether I wrongly assumed independence anywhere."

Concept 5: Law of the Unconscious Statistician (LOTUS)

Learn: > "Explain LOTUS: to get the expectation of a function of a random variable you do NOT need its new distribution; you compute $E[g(X)] = \sum_x g(x)P(X = x)$. Work $E[X^2]$ for a fair die, and point out that $E[X^2] \neq (E[X])^2$ in general."

Test me: > "Give me a random variable and a function g (like $g(X) = X^2$ or $g(X) = 2^X$). Make me compute $E[g(X)]$ via LOTUS, and make me compare $E[X^2]$ with $(E[X])^2$. Check my work."

Concept 6: Expectations of the classic random variables

Learn: > "State and give intuition for the expectations of the classics: $E[\text{Bern}(p)] = p$, $E[\text{Bin}(n, p)] = np$, $E[\text{Geo}(p)] = 1/p$, $E[\text{NegBin}(r, p)] = r/p$. Show how the binomial result np follows from writing the binomial as a sum of n Bernoulli indicators plus linearity."

Test me: > "Give me word problems (free throws over a season, dates until a match, trials until the 3rd success) and make me identify the classic random variable and compute its expectation using the right formula. Check both the identification and the arithmetic."

Wrap-up prompt (after all six concepts)

> "Give me one multi-part CS109-style problem about a real scenario that requires me to (1) identify a classic discrete random variable and its parameters, (2) compute an expectation using the right formula, (3) apply linearity of expectation with indicator variables, and (4) use LOTUS for $E[g(X)]$. Let me solve every part, then grade each part, tell me which concept it tested, and tell me the single concept I should review most before the next lecture."

Note: variance and standard deviation (the spread of a random variable) come next lecture, building directly on $E[X^2]$ from LOTUS, so keep that quantity in mind.

Bring any sticking points to lecture; the TAs and instructor will help you work through them on the worksheet.