CS109 Lecture 7: LLM Learning Guide

Variance and the Poisson

Summer 2026

Lecture 7: LLM Learning Guide

Before lecture: read the Lecture 7 slides (variance and the Poisson distribution). Then open your favorite LLM and work through the concepts below **in order**. Each concept has a *Learn* prompt and a *Test me* prompt. Attempt the "Test me" questions yourself before asking the LLM to grade you.

How to get the most out of this:

- Tell the LLM your level: "I'm a student in Stanford's CS109 (probability for computer scientists). Explain at that level."
- After any answer, ask "why?" at least once and ask it to show each step.
- When it states a formula, ask it to define every symbol before continuing.
- Attempt practice yourself first, then say: "Here is my reasoning: _____. Where exactly am I wrong, if anywhere?"

Concept 1: Variance and standard deviation

Learn: > "Define variance $\text{Var}(X) = E[(X - \mu)^2]$ where $\mu = E[X]$, and explain why we square the deviations. Then explain standard deviation as $\sqrt{\text{Var}(X)}$ and why it is more interpretable (units). Use a small example of two distributions with the same mean but different spread."

Test me: > "Give me two PMFs with the same mean but different spreads and make me predict which has larger variance, then compute both. Check my reasoning about spread."

Concept 2: Computing variance with $E[X^2] - (E[X])^2$

Learn: > "Derive the computational formula $\text{Var}(X) = E[X^2] - (E[X])^2$ from the definition, and show how to get $E[X^2]$ using LOTUS. Work the variance of a fair die roll step by step."

Test me: > "Give me a random variable with a small PMF and make me compute its variance using $E[X^2] - (E[X])^2$. Check both $E[X]$ and $E[X^2]$, and make sure I didn't confuse $E[X^2]$ with $(E[X])^2$."

Concept 3: Variance of the classic random variables

Learn: > "State the variances of the classics: $\text{Var}(\text{Bern}(p)) = p(1 - p)$ and $\text{Var}(\text{Bin}(n, p)) = np(1 - p)$. Explain how the binomial variance follows from the sum of independent Bernoullis (variance of a sum of independent variables is the sum of variances). Explain why $p(1 - p)$ is maximized at $p = 1/2$."

Test me: > "Give me binomial scenarios and make me compute the variance and standard deviation. Then ask me which p maximizes spread for fixed n and why. Check my answers."

Concept 4: The Poisson distribution

Learn: > "Introduce the Poisson random variable: the number of events in a fixed interval when events happen at an average rate λ, with PMF $P(X = k) = \frac{e^{-\lambda}\lambda^k}{k!}$. Explain that $E[X] = \text{Var}(X) = \lambda$. Work an example like the probability of exactly 3 earthquakes in a year when $\lambda = 2.79$."

Test me: > "Give me a rate-based scenario (earthquakes, server hits, typos per page) and make me compute a Poisson probability such as $P(X = k)$ or $P(X \leq k)$. Check that I used the PMF correctly and matched the time unit to the rate."

Concept 5: Matching the rate to the interval

Learn: > "Explain why the Poisson rate λ must be scaled to the interval in the question: if events arrive at 2 per second, the rate over 5 seconds is $\lambda = 10$. Show a worked example where forgetting to rescale gives the wrong answer."

Test me: > "Give me a problem where the stated rate and the asked-about interval differ (e.g. rate per minute, question per hour). Make me rescale λ correctly before computing, and flag if I forget."

Concept 6: Poisson as an approximation to the Binomial

Learn: > "Explain when and why a Poisson approximates a Binomial: when n is large and p is small, with $\lambda = np$. Give the intuition (Poisson is the binomial in the limit as $n \to \infty$, $p \to 0$). Work an example like bit errors in a long string or strings hashed into a bucket, and note why the Poisson is easier to compute."

Test me: > "Give me a large-n, small-p binomial scenario and make me (1) justify the Poisson approximation, (2) find $\lambda = np$, and (3) compute a probability. Check my justification and arithmetic."

Wrap-up prompt (after all six concepts)

> "Give me one multi-part CS109-style problem that requires me to (1) compute a variance and standard deviation, (2) set up a Poisson probability from a rate while matching the time unit, and (3) recognize a large-n, small-p binomial and approximate it with a Poisson. Let me solve every part, then grade each part, tell me which concept it tested, and tell me the single concept I should review most before the next lecture."

Note: next lecture moves from discrete random variables to continuous ones, where probabilities come from integrating a density. The rate λ idea from the Poisson returns as the Exponential distribution (time until the next event).

Bring any sticking points to lecture; the TAs and instructor will help you work through them on the worksheet.