CS109 Lecture 8 Worksheet: ANSWER KEY

Continuous Random Variables

Summer 2026

Lecture 8 Worksheet: Answer Key

Instructor / TA copy. Full worked solutions follow.

Problem 1 (Review): Poisson

$X \sim \text{Poi}(15)$.

(a)

$$P(X = 10) = \frac{e^{-15}\,15^{10}}{10!} \approx 0.0486.$$

(b) For a Poisson, $E[X] = \text{Var}(X) = \lambda = 15$.

Problem 2: The Uniform Random Variable

(a) For $X \sim \text{Uniform}(0, 1)$, $P(a \le X \le b) = b - a$ (the length of the interval). $P(X = 0.5) = 0$: a single point has zero width, so it carries zero probability under a continuous distribution.

(b) $X \sim \text{Uniform}(5, 7)$ has constant density on its support:

$$f(x) = \begin{cases} \frac{1}{7-5} = \frac{1}{2} & 5 \le x \le 7 \\ 0 & \text{otherwise.} \end{cases}$$

$$P(5.5 \le X \le 6) = \frac{6 - 5.5}{7 - 5} = \frac{0.5}{2} = 0.25.$$

(c) One coordinate is near an edge with probability $P(X_i < 0.01) + P(X_i > 0.99) = 0.01 + 0.01 = 0.02$. The point is near an edge if **any** coordinate is, so by the complement (coordinates independent):

$$P(\text{at least one near edge}) = 1 - (1 - 0.02)^{100} = 1 - (0.98)^{100} \approx 0.867.$$

In high dimensions almost every random point is near a boundary, the “curse of dimensionality.”

Problem 3: Building a PDF from Scratch (pset3: continuous_practice)

(Problem set problem)

Problem 4: PDF vs PMF Conceptual Check

(a) **True.** A density is probability *per unit length*, not a probability, so it can exceed 1. Example: Uniform$(0, 0.5)$ has $f(x) = \frac{1}{0.5} = 2$ on its support, yet the total area is still 1.

(b) $P(X = c) = \int_c^c f(x)\,dx = 0$ (an integral over a zero-width interval). Because endpoints contribute zero probability, including or excluding them does not change the integral, so $P(a \le X \le b) = P(a < X < b)$.

(c) Integrating a PDF over its entire support gives 1, because the total probability of all outcomes must be 1 (the second axiom of probability).

Problem 5: The Exponential Random Variable (Earthquakes)

$Y \sim \text{Exp}(\lambda = 0.002)$.

(a) $f(y) = \lambda e^{-\lambda y} = 0.002\, e^{-0.002y}$ for $y \ge 0$ (and 0 for $y < 0$).

(b)

$$P(Y \le 30) = F(30) = 1 - e^{-0.002\cdot 30} = 1 - e^{-0.06} \approx 0.0582.$$

(c) $E[Y] = \dfrac{1}{\lambda} = \dfrac{1}{0.002} = 500$ years, and $\text{SD}(Y) = \dfrac{1}{\lambda} = 500$ years.

Problem 6: Using the CDF to Avoid Integrals

$X \sim \text{Exp}(1)$, $F(x) = 1 - e^{-x}$.

(a) $P(X < 2) = F(2) = 1 - e^{-2} \approx 0.865$.

(b) $P(X > 1) = 1 - F(1) = e^{-1} \approx 0.368$.

(c) $P(1 < X < 2) = F(2) - F(1) = (1 - e^{-2}) - (1 - e^{-1}) = e^{-1} - e^{-2} \approx 0.233$.

Problem 7: Minimum of Two Exponentials (pset3: exponentials)

$X, Y \overset{\text{iid}}{\sim} \text{Exp}(1)$, $L = \min(X, Y)$.

(a) $\min(X, Y) > 2 \iff X > 2$ **and** $Y > 2$. By independence,

$$P(L > 2) = P(X > 2)\, P(Y > 2) = e^{-2} \cdot e^{-2} = e^{-4}.$$

(b)

$$P(L \le 2) = 1 - P(L > 2) = 1 - e^{-4} \approx 0.9817.$$

(Note $L = \min(X, Y) \sim \text{Exp}(2)$, consistent with $1 - e^{-2\cdot 2} = 1 - e^{-4}$.)

Challenge Problem: The Exponential is Memoryless

(a) Since $\{X > s + t\} \subseteq \{X > s\}$, the intersection is just $\{X > s + t\}$, so

$$P(X > s + t \mid X > s) = \frac{P(X > s + t)}{P(X > s)} = \frac{e^{-\lambda(s+t)}}{e^{-\lambda s}} = e^{-\lambda t} = P(X > t).$$

(b) The waiting time "forgets" the past: having already waited s years with no earthquake tells you nothing new, the distribution of additional waiting time is exactly the original Exp(λ). You should still expect $1/\lambda = 500$ more years on average, no matter how long you have already waited.