CS109 Lecture 8: LLM Learning Guide

Continuous Random Variables

Summer 2026

Lecture 8: LLM Learning Guide

Before lecture: read the Lecture 8 slides (continuous random variables, PDFs, CDFs, the Uniform, and the Exponential). Then open your favorite LLM and work through the concepts below **in order**. Each concept has a *Learn* prompt and a *Test me* prompt. Attempt the "Test me" questions yourself before asking the LLM to grade you.

How to get the most out of this:

- Tell the LLM your level: "I'm a student in Stanford's CS109 (probability for computer scientists). Explain at that level."
- After any answer, ask "why?" at least once and ask it to show each step.
- When it states a formula, ask it to define every symbol before continuing.
- Attempt practice yourself first, then say: "Here is my reasoning: _____. Where exactly am I wrong, if anywhere?"

Concept 1: From PMF to PDF (continuous random variables)

Learn: > "Explain the jump from discrete to continuous random variables: why we can no longer assign positive probability to each exact value, and why $P(X = c) = 0$ for a continuous random variable. Introduce the probability density function (PDF) as 'probability per unit length' and explain why a PDF can exceed 1 while still integrating to 1."

Test me: > "Ask me whether each statement is true for a continuous random variable: '$P(X = 3) = 0$', 'a PDF must be ≤ 1', '$P(a \leq X \leq b) = P(a < X < b)$'. Make me justify each, then correct me."

Concept 2: Probabilities as areas, and normalizing a PDF

Learn: > "Explain that for a continuous random variable, $P(a \leq X \leq b) = \int_a^b f(x)\,dx$, and that $\int_{-\infty}^{\infty} f(x)\,dx = 1$. Show how to find an unknown constant c in a density by setting its integral to 1. Work an example like $f(x) = c(2 - 2x^2)$ on $(-1, 1)$."

Test me: > "Give me a density with an unknown constant on a finite interval. Make me (1) solve for the constant by integrating to 1, and (2) compute a probability over a sub-interval. Check each integral."

Concept 3: The cumulative distribution function (CDF)

Learn: > "Define the CDF $F(a) = P(X \leq a) = \int_{-\infty}^{a} f(x)\,dx$, and explain how it lets you avoid integrals: $P(a < X < b) = F(b) - F(a)$, and $f(x) = F'(x)$. Work an example deriving a CDF from a PDF and using it to answer a probability question."

Test me: > "Give me a PDF and make me derive its CDF, then use the CDF to compute $P(X < a)$, $P(X > a)$, and $P(a < X < b)$. Check my CDF and that I used it instead of re-integrating."

Concept 4: The Uniform random variable

Learn: > "Explain the continuous Uniform random variable Uniform(α, β): its constant density $\frac{1}{\beta-\alpha}$ on $[\alpha, \beta]$, and how to compute $P(\text{start} \leq X \leq \text{end}) = \frac{\text{end}-\text{start}}{\beta-\alpha}$. Connect it to `random()` returning Uniform$(0, 1)$."

Test me: > "Give me a Uniform random variable on some interval and make me write its PDF and compute a couple of interval probabilities. Check the density constant and the arithmetic."

Concept 5: The Exponential random variable

Learn: > "Introduce the Exponential random variable as the time until the next event in a Poisson process with rate λ: PDF $f(x) = \lambda e^{-\lambda x}$, CDF $F(x) = 1 - e^{-\lambda x}$, mean and standard deviation both $1/\lambda$. Connect it to the Poisson (counts vs. waiting times) and work an example like time until the next earthquake."

Test me: > "Give me a rate-based waiting-time scenario and make me (1) write the exponential PDF/CDF, (2) compute a probability like $P(X < t)$ using the CDF, and (3) state the mean. Check each step and that I matched the rate's time unit."

Concept 6: Memorylessness

Learn: > "Explain the memoryless property of the exponential: $P(X > s + t \mid X > s) = P(X > t)$. Derive it from the CDF, and explain in words why 'how long you've already waited' does not change the distribution of remaining wait. Mention that the exponential is the only continuous distribution with this property."

Test me: > "Pose a memorylessness question (e.g. given a device has already lasted s hours, the probability it lasts t more). Make me apply the property and justify it, then check my reasoning."

Wrap-up prompt (after all six concepts)

> "Give me one multi-part CS109-style problem on continuous random variables that requires me to (1) find a normalizing constant for a PDF, (2) derive and use a CDF, (3) compute a probability for a Uniform or Exponential random variable, and (4) use the memoryless property. Let me solve every part, then grade each part, tell me which concept it tested, and tell me the single concept I should review most before the next lecture."

Note: next we meet the most important continuous distribution of all, the Normal (Gaussian), and the Central Limit Theorem that explains why it appears everywhere.

Bring any sticking points to lecture; the TAs and instructor will help you work through them on the worksheet.