CS109 Lecture 8 Worksheet: Continuous Random Variables

PDFs, CDFs, the Uniform, and the Exponential

Summer 2026

Lecture 8 Worksheet: Continuous Random Variables

Topics: probability density functions (PDFs), the fact that $P(X = x) = 0$ for a continuous random variable, finding a normalizing constant, the cumulative distribution function (CDF), the Uniform random variable, and the Exponential random variable (PDF $\lambda e^{-\lambda x}$, CDF $1 - e^{-\lambda x}$, mean $1/\lambda$, memorylessness).

How to use this sheet. Work in small groups. A TA and the instructor will circulate to help. We will go over selected solutions on the board. You do not need to finish every part; aim for understanding over speed. You may leave answers as exact expressions (e.g. $1 - e^{-0.06}$) unless a decimal is requested.

Problem 1 (Review of Lecture 7): Poisson

A primatologist collects usable biological samples at an average rate of 15 per day. Let X be the number collected in a day, modeled as $X \sim \text{Poi}(15)$.

(a) Write an expression for the probability of collecting **exactly 10** samples, $P(X = 10)$.

(b) State $E[X]$ and $\text{Var}(X)$.

Problem 2: The Uniform Random Variable

(a) The function `random()` returns $X \sim \text{Uniform}(0, 1)$. For $0 \le a \le b \le 1$, what is $P(a \le X \le b)$? What is $P(X = 0.5)$ exactly, and why?

(b) Now let $X \sim \text{Uniform}(5, 7)$. Write its PDF $f(x)$ (give its constant height and where it is nonzero), and find $P(5.5 \le X \le 6)$.

(c) **Curse of dimensionality.** Let $X_1, \ldots, X_{100}$ be independent $\text{Uniform}(0, 1)$ values forming a point in 100 dimensions. Call a coordinate "near an edge" if it is below 0.01 or above 0.99. Find the probability a single coordinate is near an edge, then the probability the 100-dimensional point has **at least one** coordinate near an edge.

Problem 3: Building a PDF from Scratch (pset3: continuous_practice)

Let X be a continuous random variable with density

$$f(x) = \begin{cases} c\,(2 - 2x^2) & -1 < x < 1 \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find the constant c that makes f a valid PDF (it must integrate to 1 over $-1 < x < 1$).

(b) Find the CDF $F(a) = P(X \leq a)$ for $-1 < a < 1$.

(c) Evaluate $F(0.5)$.

Problem 4: PDF vs PMF Conceptual Check

(a) True or false: a PDF $f(x)$ can take values greater than 1. Justify with a one-line example.

(b) For a continuous random variable, explain why $P(X = c) = 0$ for any single point c, and why this means $P(a \leq X \leq b) = P(a < X < b)$.

(c) What do you get when you integrate a PDF over its entire support, and why?

Problem 5: The Exponential Random Variable (Earthquakes)

Major (magnitude 8.0+) earthquakes occur at a rate of $\lambda = 0.002$ per year. Let Y be the number of years until the next one, modeled as $Y \sim \text{Exp}(0.002)$, with CDF $F(y) = 1 - e^{-\lambda y}$ for $y \geq 0$.

(a) Write the PDF $f(y)$.

(b) Find the probability the next major earthquake is within the **next 30 years**, $P(Y \leq 30)$. Leave it as an expression and also give a decimal.

(c) Find $E[Y]$ and $\text{SD}(Y)$ (both equal $1/\lambda$ for an exponential).

Problem 6: Using the CDF to Avoid Integrals

Let $X \sim \text{Exp}(1)$, so $F(x) = 1 - e^{-x}$ for $x \geq 0$.

(a) Find $P(X < 2)$.

(b) Find $P(X > 1)$.

(c) Find $P(1 < X < 2)$ using $F(2) - F(1)$.

Problem 7: Minimum of Two Exponentials (pset3: exponentials)

Let X and Y be independent, each $\text{Exp}(1)$. Define $L = \min(X, Y)$.

(a) Express the event $\{L > 2\}$ in terms of X and Y, and use independence to find $P(L > 2)$.

(b) Find $P(L \leq 2)$. Give a decimal to four places (a quick check: about 0.9817).

Challenge Problem (optional): The Exponential is Memoryless

Let $X \sim \text{Exp}(\lambda)$, so $P(X > t) = e^{-\lambda t}$ for $t \geq 0$.

(a) Show that $P(X > s + t \mid X > s) = P(X > t)$ for any $s, t \geq 0$. (Use the definition of conditional probability and $\{X > s + t\} \subseteq \{X > s\}$.)

(b) Interpret this in words: if you have already waited s years for an earthquake, what does the result say about how much longer you should expect to wait?