CS109 Lecture 9 Worksheet: ANSWER KEY

The Normal Distribution

Summer 2026

Lecture 9 Worksheet: Answer Key

Instructor / TA copy. Problem-set solutions are hidden in the standard build; run `./convert-to-pdf.sh 9 solutions` *for the full instructor/TA edition.*

Problem 1 (Review): Exponential

$T \sim \text{Exp}(0.5)$.

(a) $P(T < 2) = 1 - e^{-0.5 \cdot 2} = 1 - e^{-1} \approx 0.632$.

(b) $E[T] = 1/\lambda = 1/0.5 = 2$ hours.

(c) By **memorylessness**, $P(T > 5 \mid T > 3) = P(T > 2) = e^{-0.5 \cdot 2} = e^{-1} \approx 0.368$.

Problem 2: Anatomy of the Normal

$X \sim N(70, 16)$.

(a) Mean $\mu = 70$; standard deviation $\sigma = \sqrt{16} = 4$.

(b) $z = \dfrac{74 - 70}{4} = 1$.

(c) $P(X < 74) = \Phi(1) \approx 0.841$.

Problem 3: Symmetry of Φ

(a) $\Phi(-a) = 1 - \Phi(a)$ (the standard normal is symmetric about 0).

(b) $P(-1 \le Z \le 1) = \Phi(1) - \Phi(-1) = \Phi(1) - (1 - \Phi(1)) = 2\Phi(1) - 1 = 2(0.8413) - 1 \approx 0.683$.

(c) $P(Z > 1.31) = 1 - \Phi(1.31) = 1 - 0.9049 = 0.0951$.

Problem 4: Submarine Manufacturing

$X \sim N(500, 36)$, so $\sigma = 6$.

(a) $z_{490} = \dfrac{490 - 500}{6} = -\dfrac{10}{6} \approx -1.667$, and $z_{510} = +1.667$.

(b)

$$P(490 \le X \le 510) = \Phi(1.667) - \Phi(-1.667) = 2\Phi(1.667) - 1 = 2(0.9522) - 1 \approx 0.904.$$

About **90.4%** of panels meet standards.

Problem 5: Website Analytics (pset3: website_visitors)

(Problem set problem)

Problem 6: Normal Approximation to a Binomial (Midterm: Website Testing)

Under "no effect," $X \sim \text{Bin}(10^6, 0.5)$.

(a) $E[X] = np = 500{,}000$ and $\text{Var}(X) = np(1-p) = 250{,}000$, so the approximating normal has $\mu = 500{,}000$ and $\sigma = \sqrt{250{,}000} = 500$.

(b) With continuity correction, $P(X \ge 501{,}000) = P(X \ge 500{,}999.5)$ in the normal:

$$P(X \ge 501{,}000) \approx 1 - \Phi\left(\frac{500{,}999.5 - 500{,}000}{500}\right) = 1 - \Phi(1.999) \approx 0.0228.$$

(Matches the exact-ish target of 0.02270.) So even with no real effect, there is about a 2.3% chance the CEO endorses.

(c) The binomial has large n with **moderate** p (here $p = 0.5$), so $\text{Var}(X) = 250{,}000 \gg 10$: this is the **normal** regime. (The Poisson approximation is for large n with **small** p.)

Problem 7: Continuity Correction Concept

$X \approx N(50, 25)$, $\sigma = 5$.

(a) The integer 55 "spreads" over the interval $[54.5, 55.5]$ in the continuous approximation, so $P(X = 55) \approx \Phi\left(\frac{55.5-50}{5}\right) - \Phi\left(\frac{54.5-50}{5}\right)$. We widen by ± 0.5 because a discrete point has no width but the bar it represents does.

(b) $P(X \le 60) \approx \Phi\left(\dfrac{60.5 - 50}{5}\right) = \Phi(2.1)$.

Challenge Problem: Sum of Independent Normals (pset4: normal_transform)

(Problem set problem)