CS109 Lecture 9: LLM Learning Guide

The Normal Distribution

Summer 2026

Lecture 9: LLM Learning Guide

Before lecture: read the Lecture 9 slides (the Normal distribution, standardization, Φ, and the normal approximation to the binomial). Then open your favorite LLM and work through the concepts below **in order**. Each concept has a *Learn* prompt and a *Test me* prompt. Attempt the "Test me" questions yourself before asking the LLM to grade you.

How to get the most out of this:

- Tell the LLM your level: "I'm a student in Stanford's CS109 (probability for computer scientists). Explain at that level."
- After any answer, ask "why?" at least once and ask it to show each step.
- When it states a formula, ask it to define every symbol before continuing.
- Attempt practice yourself first, then say: "Here is my reasoning: _____. Where exactly am I wrong, if anywhere?"

Concept 1: The Normal random variable

Learn: > "Introduce the Normal (Gaussian) random variable $X \sim N(\mu, \sigma^2)$: its bell-shaped PDF, the roles of μ (center) and σ^2 (spread), and why it appears so often (sums of many small effects, the 68-95-99.7 rule). Emphasize that the parameter is the variance σ^2, but code libraries usually take the standard deviation σ."

Test me: > "Give me a normal random variable with specific μ and σ^2 and ask me to state its mean, variance, and standard deviation, and sketch where most of its mass lies. Check my answer, especially variance vs standard deviation."

Concept 2: Standardizing and the standard normal Φ

Learn: > "Explain how to standardize: $Z = \frac{X-\mu}{\sigma} \sim N(0,1)$, and how the standard-normal CDF $\Phi(z) = P(Z \leq z)$ lets you compute normal probabilities. Show how to compute $P(X < x)$ by converting to a z-score and looking up (or computing) $\Phi(z)$."

Test me: > "Give me a normal random variable and a threshold, and make me compute a z-score and then $P(X < x)$ using Φ. Check both the standardization and the lookup."

Concept 3: Symmetry and interval probabilities

Learn: > "Explain the symmetry property $\Phi(-a) = 1 - \Phi(a)$ and how to compute interval probabilities $P(a \leq X \leq b) = \Phi(z_b) - \Phi(z_a)$. Derive the 68% rule ($P(\mu - \sigma \leq X \leq \mu + \sigma) \approx 0.68$)."

Test me: > "Give me a two-sided probability like $P(|X-\mu| \le k\sigma)$ or a tail like $P(X > x)$ and make me use symmetry and Φ correctly. Check my use of $\Phi(-a) = 1 - \Phi(a)$."

Concept 4: Linear transforms and sums of normals

Learn: > "Explain that a linear transform of a normal is normal: if $X \sim N(\mu, \sigma^2)$ then $aX + b \sim N(a\mu + b,\ a^2\sigma^2)$. Also explain that a sum of independent normals is normal, with means adding and variances adding. Work an example like converting Celsius to Fahrenheit, or $2X + Y$."

Test me: > "Give me one or two independent normals and a linear combination, and make me find the resulting distribution's mean and variance, then a probability. Check that I squared the coefficients on the variances."

Concept 5: Normal approximation to the binomial

Learn: > "Explain when a binomial $\text{Bin}(n, p)$ can be approximated by a normal (large n, moderate p, i.e. variance $np(1-p)$ comfortably above ~10), using $\mu = np$ and $\sigma^2 = np(1-p)$. Contrast with the Poisson approximation (large n, small p). Work an example where the binomial is too big to compute directly."

Test me: > "Give me a large-n binomial and make me decide between the normal and Poisson approximations, justify it, and set up the approximate probability. Check my choice and my μ and σ."

Concept 6: The continuity correction

Learn: > "Explain the continuity correction used when approximating a discrete binomial with a continuous normal: to approximate $P(X \le k)$ use $\Phi\big(\frac{k+0.5-\mu}{\sigma}\big)$, and for $P(X \ge k)$ use $1 - \Phi\big(\frac{k-0.5-\mu}{\sigma}\big)$. Explain why the ± 0.5 matters, with a small worked comparison."

Test me: > "Give me a binomial probability like $P(X \ge k)$ or $P(X = k)$ and make me write the normal approximation WITH the continuity correction. Check that I shifted by the correct ± 0.5 in the right direction."

Wrap-up prompt (after all six concepts)

> "Give me one multi-part CS109-style problem that requires me to (1) compute a normal probability by standardizing and using Φ, (2) use symmetry for a two-sided probability, and (3) approximate a large binomial with a normal using the continuity correction. Let me solve every part, then grade each part, tell me which concept it tested, and tell me the single concept I should review most before the next lecture."

Note: next lecture we start combining multiple random variables (joint distributions) and introduce Bayes' theorem with random variables, the gateway to inference.

Bring any sticking points to lecture; the TAs and instructor will help you work through them on the worksheet.