CS109 Lecture 9 Worksheet: The Normal Distribution

Gaussians, Standardization, and the Normal Approximation

Summer 2026

Lecture 9 Worksheet: The Normal Distribution

Topics: the Normal (Gaussian) random variable $X \sim N(\mu, \sigma^2)$, standardizing to the standard normal $Z = \frac{X-\mu}{\sigma}$, using Φ (the standard-normal CDF) and its symmetry, computing interval probabilities, linear transforms of a normal, and the normal approximation to the binomial (with continuity correction).

How to use this sheet. Work in small groups. A TA and the instructor will circulate to help. We will go over selected solutions on the board. You may use a Φ table or `scipy.stats.norm.cdf`. Remember `scipy` takes the **standard deviation**, not the variance. You do not need to finish every part; aim for understanding over speed.

Problem 1 (Review of Lecture 8): Exponential

The time (in hours) until a server's next request is $T \sim \text{Exp}(\lambda = 0.5)$, with CDF $F(t) = 1 - e^{-\lambda t}$.

(a) Find $P(T < 2)$.

(b) What is $E[T]$?

(c) Given that no request has arrived in the first 3 hours, what is the probability none arrives in the next 2 hours? (Name the property you are using.)

Problem 2: Anatomy of the Normal

Let $X \sim N(\mu = 70, \sigma^2 = 16)$ be an exam score.

(a) What are the mean and standard deviation of X?

(b) Standardize: write the Z-score of a raw score $x = 74$, i.e. $z = \frac{x-\mu}{\sigma}$.

(c) Find $P(X < 74)$ using $\Phi(z)$. (Check: $z = 1$, and $\Phi(1) \approx 0.841$.)

Problem 3: Symmetry of Φ

(a) Using symmetry of the standard normal, express $\Phi(-a)$ in terms of $\Phi(a)$.

(b) For $Z \sim N(0, 1)$, find $P(-1 \le Z \le 1)$ using $\Phi(1) \approx 0.8413$. (This is the "68% rule.")

(c) Find $P(Z > 1.31)$ given $\Phi(1.31) = 0.9049$.

Problem 4: Submarine Manufacturing

Your team produces submarine panels whose thickness is $X \sim N(\mu = 500, \sigma^2 = 36)$ microns. A panel meets standards if it is within 10 microns of 500 (i.e. $490 \leq X \leq 510$).

(a) Standardize the endpoints 490 and 510.

(b) Write $P(490 \leq X \leq 510)$ as $\Phi(\cdot) - \Phi(\cdot)$ and evaluate it. What fraction of panels meet standards?

Problem 5: Website Analytics (pset3: website_visitors)

Let X be the number of weekly visitors to a site, where $X \sim N(\mu = 2200, \sigma^2 = 52900)$.

(a) What is the standard deviation σ?

(b) Find the probability that the weekly number of visitors **exceeds 2000**, $P(X > 2000)$, to three decimal places.

Problem 6: Normal Approximation to a Binomial (Midterm: Website Testing)

A new website design is tested on 1,000,000 users. Let X be the number whose time-on-site increases. Under the assumption that the design has **no effect**, each user independently improves with probability 0.5, so $X \sim \text{Bin}(10^6, 0.5)$. The CEO endorses the design if $X \geq 501{,}000$.

(a) Find $E[X]$ and $\text{Var}(X)$, and hence the μ and σ of the approximating normal.

(b) Using the normal approximation **with continuity correction**, write $P(X \geq 501{,}000)$ as $1 - \Phi(\cdot)$ and evaluate it. (Check: about 0.0227.)

(c) Why do we use the normal (not Poisson) approximation here? State the rough rule of thumb.

Problem 7: Continuity Correction Concept

A fair coin is flipped 100 times; let X be the number of heads, approximated by $X \approx N(50, 25)$.

(a) To approximate $P(X = 55)$ exactly (a single integer), what interval of the continuous normal do you integrate over, and why?

(b) Write $P(X \leq 60)$ with the correct continuity correction as $\Phi(\cdot)$.

Challenge Problem (optional): Sum of Independent Normals (pset4: normal_transform)

Let $X \sim N(1, 2)$ and $Y \sim N(1, 2)$ be independent. Consider $W = 2X + Y$.

(a) A linear combination of independent normals is normal. Find the mean and variance of W. (Recall $\text{Var}(aX) = a^2\text{Var}(X)$ and variances of independent variables add.)

(b) Find $P(W < 5)$ to three decimal places.