CS109 Lecture 10 Worksheet: ANSWER KEY

Probabilistic Models

Summer 2026

Lecture 10 Worksheet: Answer Key

Instructor / TA copy. Problem-set solutions are hidden in the standard build; run `./convert-to-pdf.sh 10 solutions` *for the full instructor/TA edition.*

Problem 1 (Review): Normal

$X \sim N(100, 225)$, so $\sigma = 15$.

(a) $\sigma = \sqrt{225} = 15$.

(b) $z = \dfrac{130 - 100}{15} = 2$, so $P(X > 130) = 1 - \Phi(2) = 1 - 0.9772 = 0.0228$.

Problem 2: A Joint PMF

(a) All six entries are in $[0, 1]$ and they sum to $0.18 + 0.12 + 0.10 + 0.12 + 0.18 + 0.30 = 1.00$, so it is a valid joint PMF.

(b) $P(\mathrm{R}, \mathrm{Jr}) = 0.30$ and $P(\mathrm{S}, \mathrm{Fr}) = 0.18$ (read directly from the table).

(c) $P(Y = \mathrm{Jr}) = P(\mathrm{S}, \mathrm{Jr}) + P(\mathrm{R}, \mathrm{Jr}) = 0.10 + 0.30 = 0.40$.

Problem 3: Marginal Distributions

(a) $P(X = \mathrm{S}) = 0.18 + 0.12 + 0.10 = 0.40$; $\quad P(X = \mathrm{R}) = 0.12 + 0.18 + 0.30 = 0.60$.

(b) $P(Y = \mathrm{Fr}) = 0.18 + 0.12 = 0.30$; $\quad P(Y = \mathrm{So}) = 0.12 + 0.18 = 0.30$; $\quad P(Y = \mathrm{Jr}) = 0.10 + 0.30 = 0.40$.

(c) Summing a row fixes X and adds over all mutually exclusive values of Y (the law of total probability), which by definition gives the marginal probability of that value of X.

Problem 4: Conditioning and Independence

(a) $P(X = \mathrm{R} \mid Y = \mathrm{Jr}) = \dfrac{P(\mathrm{R}, \mathrm{Jr})}{P(Y = \mathrm{Jr})} = \dfrac{0.30}{0.40} = 0.75$.

(b) $P(X = \mathrm{S}, Y = \mathrm{Fr}) = 0.18$, but $P(X = \mathrm{S})\, P(Y = \mathrm{Fr}) = 0.40 \times 0.30 = 0.12$. Since $0.18 \neq 0.12$, X and Y are **not independent**.

Problem 5: Bayes with Random Variables (Baby Hearing Test)

(a)

$$P(Y = 1 \mid X = 14) = \frac{f(14 \mid Y = 1)\, P(Y = 1)}{f(14 \mid Y = 1)\, P(Y = 1) + f(14 \mid Y = 0)\, P(Y = 0)}.$$

(b) The shared constant cancels, leaving the exponential parts:

$$f(14 \mid Y = 1) \propto e^{-(14-15)^2/50} = e^{-0.02} \approx 0.9802, \qquad f(14 \mid Y = 0) \propto e^{-(14-8)^2/50} = e^{-0.72} \approx 0.4868.$$

$$P(Y = 1 \mid X = 14) = \frac{0.75(0.9802)}{0.75(0.9802) + 0.25(0.4868)} = \frac{0.7352}{0.7352 + 0.1217} \approx 0.858.$$

The observation 14 is closer to 15 than to 8, so the posterior belief in hearing rises slightly above the prior 0.75.

Problem 6: The Size of a Joint (pset4: joint_size)

(a) Each of the 20 variables independently indexes one of 5 values, so the number of joint assignments is

$$5^{20} = 95{,}367{,}431{,}640{,}625 \approx 9.5 \times 10^{13} \text{ cells.}$$

(b) A table this size is impossible to store or estimate directly, which is exactly why we use structured models with independence assumptions (Bayesian networks) that factor the joint into small conditional pieces.

Challenge Problem: Tired Baby (pset4: tired_baby)

(a)

$$P(\text{Tired} \mid t = 2) = \frac{f(2 \mid \text{Tired})\, P(\text{Tired})}{f(2 \mid \text{Tired})\, P(\text{Tired}) + f(2 \mid \text{Not})\, P(\text{Not})},$$

with $f(2 \mid \text{Tired}) = 3e^{-3\cdot 2} = 3e^{-6}$ and $f(2 \mid \text{Not}) = 1 \cdot e^{-1\cdot 2} = e^{-2}$.

(b)

$$P(\text{Tired} \mid t = 2) = \frac{\frac{3}{4}(3e^{-6})}{\frac{3}{4}(3e^{-6}) + \frac{1}{4}(e^{-2})} = \frac{0.00558}{0.00558 + 0.03384} \approx 0.142.$$

The baby is now **less** likely to be tired than the prior 0.75. A tired baby (Exp(3), mean $\frac{1}{3}$ min) would almost always rub its eyes well before 2 minutes; waiting a full 2 minutes is much more consistent with not being tired (Exp(1), mean 1 min).