

CS109 Lecture 10 Worksheet: ANSWER KEY

Probabilistic Models

Summer 2026

Lecture 10 Worksheet: Answer Key

Instructor / TA copy. Problem-set solutions are hidden in the standard build; run `./convert-to-pdf.sh 10 solutions` for the full instructor/TA edition.

Problem 1 (Review): Normal

$X \sim N(100, 225)$, so $\sigma = 15$.

(a) $\sigma = \sqrt{225} = 15$.

(b) $z = \frac{130 - 100}{15} = 2$, so $P(X > 130) = 1 - \Phi(2) = 1 - 0.9772 = 0.0228$.

Problem 2: A Joint PMF

- (a) All six entries are in $[0, 1]$ and they sum to $0.18 + 0.12 + 0.10 + 0.12 + 0.18 + 0.30 = 1.00$, so it is a valid joint PMF.
- (b) $P(R, Jr) = 0.30$ and $P(S, Fr) = 0.18$ (read directly from the table).
- (c) $P(Y = Jr) = P(S, Jr) + P(R, Jr) = 0.10 + 0.30 = 0.40$.

Problem 3: Marginal Distributions

- (a) $P(X = S) = 0.18 + 0.12 + 0.10 = 0.40$; $P(X = R) = 0.12 + 0.18 + 0.30 = 0.60$.
- (b) $P(Y = Fr) = 0.18 + 0.12 = 0.30$; $P(Y = So) = 0.12 + 0.18 = 0.30$; $P(Y = Jr) = 0.10 + 0.30 = 0.40$.
- (c) Summing a row fixes X and adds over all mutually exclusive values of Y (the law of total probability), which by definition gives the marginal probability of that value of X .

Problem 4: Conditioning and Independence

- (a) $P(X = R \mid Y = Jr) = \frac{P(R, Jr)}{P(Y = Jr)} = \frac{0.30}{0.40} = 0.75$.
- (b) $P(X = S, Y = Fr) = 0.18$, but $P(X = S)P(Y = Fr) = 0.40 \times 0.30 = 0.12$. Since $0.18 \neq 0.12$, X and Y are **not independent**.

Problem 5: Bayes with Random Variables (Baby Hearing Test)

(a)

$$P(Y = 1 \mid X = 14) = \frac{f(14 \mid Y = 1) P(Y = 1)}{f(14 \mid Y = 1) P(Y = 1) + f(14 \mid Y = 0) P(Y = 0)}.$$

(b) The shared constant cancels, leaving the exponential parts:

$$f(14 \mid Y = 1) \propto e^{-(14-15)^2/50} = e^{-0.02} \approx 0.9802, \quad f(14 \mid Y = 0) \propto e^{-(14-8)^2/50} = e^{-0.72} \approx 0.4868.$$

$$P(Y = 1 \mid X = 14) = \frac{0.75(0.9802)}{0.75(0.9802) + 0.25(0.4868)} = \frac{0.7352}{0.7352 + 0.1217} \approx 0.858.$$

The observation 14 is closer to 15 than to 8, so the posterior belief in hearing rises slightly above the prior 0.75.

Problem 6: The Size of a Joint (pset4: joint_size)

(Problem set problem)

Challenge Problem: Tired Baby (pset4: tired_baby)

(Problem set problem)