CS109 Lecture 28 Worksheet: Final Review

A Slide-by-Slide Index into Ten Past Final Exams

Summer 2026

Lecture 28 Worksheet: Final Review

Topics: every concept on the Lecture 28 review deck, cross-referenced against every final exam posted on the CS109 final exam page.

How to use this sheet. This worksheet is not a problem set; it is a *map*. Walk through the review slides in order. Whenever a slide covers something you are shaky on, come here, find the slide number, and go work the listed exam problems. Each entry names the exam, the problem number and title, and the specific parts that exercise that slide's idea. The abbreviations (Spr26, Win26, F24, ...) are expanded with download links in the **Exam Index** at the end of this document.

A note on coverage. A concept that appears on nine of the ten finals is a concept you will see again. Rows below with long lists are the highest-yield material on the sheet. Rows with one or two entries are still fair game, but budget your time accordingly.

Block 1: Core Probability (slides 4–15)

Slides	Concept	Final exam problems
4–8	Axioms, $P(E \cup F)$, inclusion–exclusion, complement, De Morgan's laws, chain rule	F22 #1 Short Answer (c); F24 #2 Airline Flights (a, b); Win26 #1 Let's Get This Party Started (d); Spr17 #2 Letters of Recommendation (a, b); W22 #1 Short Answer (d)
9	Law of Total Probability	Win25 #1 Short Answer (c); Win26 #2 Powering a Town (a); Win25 #3 Bag of Coins (b); F23 #3 Popcorn (c); Spr17 #2 Letters of Recommendation (c)
10–11	Conditional probability; Bayes' Theorem	**Spr26 #2 Fraud Detection (a, b)**; Win25 #1 Short Answer (b); Win26 #2 Powering a Town (b); F24 #2 Airline Flights (c); W22 #2 Quantitative Art History (b); F17 #1 Missing Not at Random (b, c); Spr17 #2 Letters of Recommendation (d)
12–14	Independence; independence $\neq$ mutual exclusion	**F23 #1 Short Answer (c)** (how to *test* for each from probabilities alone); F21 #1 True or False (a); Win26 #1 Let's Get This Party Started (c); F22 #1 Short Answer (c); F17 #1 Missing Not at Random (e)
15	Conditional independence	**Spr26 #2 Fraud Detection (b)**; Win26 #4 Are We Cooked? (a); F22 #2 Machine Learning (b, Naive Bayes); Spr17 #9 Multiclass Bayes

Block 2: Random Variables (slides 17–37)

Slides	Concept	Final exam problems
17–21	PMF vs. PDF; expectation and variance; distributions sum/integrate to 1	Win26 #6 That's Private (a); F21 #6 P-Hacking (b); F23 #4 Delicate Polling (b); Spr17 #7 Differential Privacy (a, b); W22 #1 Short Answer (a)
22–24	Computing $P(X = x)$: discrete vs. continuous	Win26 #3 Let's Tree Max (a); Win25 #2 In-Vitro Fertilization (b); F24 #3 Board Games (a, b); Spr26 #3 Support Chat Requests (b)
25–30	Computing $P(X \leq x)$: summing a PMF vs. using a CDF	Win25 #1 Short Answer (a); F23 #1 Short Answer (a); F24 #1 Short Answer (a); W22 #1 Short Answer (b)
31–37	Computing $P(X \geq x)$; continuous CDFs and tail probabilities	**Spr26 #1 Short Answer (a)**; F23 #3 Popcorn (a) (Normal CDF, conditional); F23 #5 Water Levels (a); F21 #2 Algorithmic Art (a); F17 #2 Occupy Mt Gox (a)

Block 3: Common Distributions (slides 39–55)

Slides	Concept	Final exam problems
39	Binomial: fixed n, independent trials, constant p	Win25 #2 In-Vitro Fertilization (b); Win25 #3 Bag of Coins (a, b); F23 #2 Don't Quit Until You Are Ahead (a); F24 #7 Sorted Random Values (a); Win26 #3 Let's Tree Max (a); W22 #4 Bayesian Carbon Dating (b)
40–45	Approximating a Binomial with a Poisson or a Normal	Win26 #3 Let's Tree Max (b); F23 #2 Don't Quit Until You Are Ahead (b); F21 #3 How Lucky! (b); Spr17 #3 Hindenbug (a, c)
46	Continuity correction	Spr17 #3 Hindenbug (c); F23 #2 Don't Quit Until You Are Ahead (b)
47–48, 54	Geometric and Negative Binomial: trials until success	**Win26 #1 Let's Get This Party Started (a)**; W22 #1 Short Answer (b); F24 #8 MLE of Negative Binomial (a); F21 #3 How Lucky! (a); F23 #2 Don't Quit Until You Are Ahead (c)
49–50	Choosing the right distribution from the problem's conditions; the Multinomial	**Win26 #1 Let's Get This Party Started (a–d)** (a different distribution in each part); Win25 #1 Short Answer (d, multinomial); W22 #6 Rank Ordering of Samples (a, b, multinomial); F23 #3 Popcorn (b) (Binomial of Normals); F24 #3 Board Games (a)
51–53	Poisson vs. Exponential: counts in an interval vs. time until next event	**Spr26 #3 Support Chat Requests (a, b, c)**; Win26 #1 Let's Get This Party Started (b); F23 #1 Short Answer (f); F21 #1 True or False (e); Spr26 #5 Hidden Chambers (a); F24 #6 Hidden Chambers (a); Spr17 #3 Hindenbug (a, b)
55	Uniform (discrete and continuous)	Win25 #4 Microcontroller Precision Error (a); Win25 #5 Exponential Backoff (a); F24 #7 Sorted Random Values (a, b); F17 #5 Expected Maximum Uniform

Block 4: Inference (slides 56–60)

Slides	Concept	Final exam problems
56–57	Recognizing an inference problem: prior, observation, likelihood, update	Win25 #3 Bag of Coins (c); F21 #5 Chess.com Puzzles; W22 #2 Quantitative Art History (b)
58–60	Bayesian updating over a discrete or continuous prior (the carbon-dating pattern)	**W22 #4 Bayesian Carbon Dating (c, d)** (this is the slides' worked example); **Spr26 #5 Hidden Chambers in the Pyramid of the Sun (b)**; F24 #6 Hidden Chambers in the Great Pyramid (b); Win26 #4 Are We Cooked? (b); F23 #7 Estimating Child Vocab Size (b); F17 #1 Missing Not at Random (c, d)

Block 5: Probabilistic Models and Joint Distributions (slides 61–70)

Slides	Concept	Final exam problems
61–65	Random variables varying together; conditional distributions	Win25 #6 Indus Valley Script (b); F21 #4 Probabilistic Supply Chains (b); F24 #5 What Name Doesn't Give Away Age?
66–70	Joint distributions; how to represent them (tables, functions, code)	**F21 #4 Probabilistic Supply Chains (a, b, c, d)**; Win26 #4 Are We Cooked? (a); Spr17 #5 Gaussian Blur (a, b)

Block 6: The Beta Distribution (slides 72–79)

Slides	Concept	Final exam problems
72–73	A distribution beats a point estimate; Laplace / prior counts	F22 #2 Machine Learning (b); F23 #1 Short Answer (e); F21 #7 Calibration of ML Algorithms (b)
74–77	Beta as a distribution over a probability; $E[X]$, $\mathrm{Var}(X)$	W22 #3 Beta ±; F21 #2 Algorithmic Art (c); F17 #4 Learning While Helping (a, b, d)
78–79	Inference with Beta: posterior = Beta(a+successes, b+fails)	**Spr26 #7 Testing a New Medicine (a, b)**; **Win26 #5 Two LLMs Walk Into A Bar (a, b)**; Win25 #8 Bayesian RNA Sequencing (b, c); F22 #5 Better; F24 #7 Sorted Random Values (b, c); F17 #4 Learning While Helping (a–d)

Block 7: Adding Random Variables and the CLT (slides 81–93)

Slides	Concept	Final exam problems
81–83	IID: independent and identically distributed	F21 #1 True or False (d); F23 #1 Short Answer (d); Win25 #4 Microcontroller Precision Error (b)
84–89	Sums of random variables; rules for common sums; convolution	F22 #3 Night Sight (b); Spr17 #4 NBA Finals Week (b); F17 #3 Timing Attack (b, c); F23 #1 Short Answer (b)

Slides	Concept	Final exam problems
90–93	Central Limit Theorem in practice	**Spr26 #4 Warehouse Shipping (a, b)**; **Win26 #2 Powering a Town (c, d)**; Win26 #6 That's Private (b); Win25 #4 Microcontroller Precision Error (b); F22 #1 Short Answer (a); F21 #6 P-Hacking (c, d); F17 #2 Occupy Mt Gox (b); Spr17 #3 Hindenbug (c)

Block 8: Sampling, Sample Mean, and Standard Error (slides 95–101)

Slides	Concept	Final exam problems
95–96	Why we sample; population vs. sample; with vs. without replacement	**Win26 #9 Genomics and Sampling (c)**; Win25 #7 Estimating Course Size (a); F23 #4 Delicate Polling (c)
97–99	Sample mean and standard error	**W22 #1 Short Answer (c)**; Win26 #6 That's Private (b); F22 #3 Night Sight (b); Win25 #7 Estimating Course Size (a); F21 #1 True or False (d)
100–101	Sample variance (the $n-1$ correction)	Win25 #7 Estimating Course Size (b); Spr17 #6 Improved Exam Grading (a)

Block 9: Bootstrapping, Hypothesis Testing, and p-values (slides 103–107)

Slides	Concept	Final exam problems
103–104	What bootstrapping assumes; bootstrapping as code (resample **with** replacement)	**Spr26 #8 Debugging a Bootstrap** (find the three bugs); Win25 #7 Estimating Course Size (c); Win26 #9 Genomics and Sampling (b, c); F21 #1 True or False (c); F22 #6 B-Reel (d); F23 #4 Delicate Polling (c)
105–107	Hypothesis testing and p-values under the null	**F24 #4 Song of the Quarter (c)**; F21 #6 P-Hacking (e, f, g); Spr17 #6 Improved Exam Grading (b); F17 #4 Learning While Helping (e)

Block 10: Algorithm Analysis and Expectation 2.0 (slides 109–114)

Slides	Concept	Final exam problems
109–111	Expectation of a sum = sum of expectations (indicator variables)	**Win26 #3 Let's Tree Max (c)** (expected empty zones); F22 #8 Approximate Counting Algorithm; F17 #5 Expected Maximum Uniform; Win25 #5 Exponential Backoff (b)
112	Conditional expectation $E[X \mid Y = y]$	F21 #4 Probabilistic Supply Chains (b); Win26 #10 The Doctor Will See You Now (a, b)
113–114	Law of Total Expectation	**Spr26 #1 Short Answer (c)** (drone deliveries by weather); **F24 #1 Short Answer (c)** (server crashes by age); F21 #4 Probabilistic Supply Chains (d)

Block 11: Information Theory (slides 116–119)

Slides	Concept	Final exam problems
116–117	Entropy as expected surprise; computing $H(X)$ in bits	**Spr26 #9 Decision Tree Entropy (a)**; Win25 #6 Indus Valley Script (a); F24 #5 What Name Doesn't Give Away Age?; Win26 #7 Playing it Safe (a)
118–119	Expected entropy after an observation; information gain	**Spr26 #9 Decision Tree Entropy (b, c)**; Win25 #6 Indus Valley Script (c); Win26 #7 Playing it Safe (a, b)

Block 12: Maximum Likelihood Estimation (slides 121–130)

Slides	Concept	Final exam problems
121–126	The modeling workflow: pick a model, fit parameters, predict	F22 #2 Machine Learning (a); Win25 #10 Logistic Regression Conceptual (b); Win26 #10 The Doctor Will See You Now (c)
127–128, 130	MLE: choose θ to maximize $P(\text{data} \mid \theta)$; log-likelihood; set derivative to 0	**Win25 #9 MLE of a Truncated Normal Distribution**; **F24 #8 MLE of Negative Binomial (b)**; F23 #5 Water Levels (b); F22 #7 Code Survival; W22 #5 Reliability Engineering; Spr17 #8 Windfarm Modeling; F17 #2 Occupy Mt Gox (c, d); F21 #1 True or False (b)
129	Gradient ascent (when there is no closed form)	**Win26 #8 It's Gamma Time**; F23 #6 Logistic Regression Priors (c); F21 #2 Algorithmic Art (d)

Block 13: Logistic Regression (slides 131–138)

Slides	Concept	Final exam problems
131	The logistic regression assumption $P(Y = 1 \mid \mathbf{x}) = \sigma(\theta^T \mathbf{x})$	Win25 #10 Logistic Regression Conceptual (a, b, c); F22 #2 Machine Learning (a, c); Win26 #9 Genomics and Sampling (a)
132–135	Training: log-likelihood, partial derivatives, gradient ascent update	**Spr26 #6 Logistic Regression (b)**; F23 #6 Logistic Regression Priors (b, c, MAP version); Win25 #10 Logistic Regression Conceptual (d); Spr17 #10 Logistic Vision Test; F17 #7 The Most Important Features (a, b, c); W22 #7 Machine Teaching (b, c)
136–138	Prediction and the decision boundary	**Spr26 #6 Logistic Regression (a)**; **Win26 #10 The Doctor Will See You Now (a, b, c)**; F17 #7 The Most Important Features (e); W22 #7 Machine Teaching (d)

Block 14: Beyond Binary Classification (slides 139–140)

Slides	Concept	Final exam problems
139–140	Regression, calibration, and other output types	**F24 #9 Recalibrating an Uncalibrated Model (a, b)**; **F21 #7 Calibration of ML Algorithms (a–d)**; Spr26 #1 Short Answer (b); F24 #1 Short Answer (b); F17 #6 Gaussian Naive Bayes; F17 #7 The Most Important Features (d, deep nets); Spr17 #9 Multiclass Bayes

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Exam Index

All exams and solutions are linked from the CS109 final exam page.

Code	Exam	Problems	Solutions
Spr26	Final Spring 2026	PDF	Soln
Win26	Final Winter 2026	PDF	Soln
Win25	Final Winter 2025	PDF	Soln
F24	Final Fall 2024	PDF	Soln
F23	Final Fall 2023	PDF	Soln
F22	Final Fall 2022	PDF	Soln
W22	Final Winter 2022	PDF	Soln
F21	Final Fall 2021	PDF	Soln
F17	Final Fall 2017	PDF	Soln
Spr17	Final Spring 2017	PDF	Soln

Problem Titles by Exam

A quick lookup so you can find a problem without opening every PDF.

Exam	Problems
Spr26	1 Short Answer; 2 Fraud Detection; 3 Support Chat Requests; 4 Warehouse Shipping; 5 Hidden Chambers in the Pyramid of the Sun; 6 Logistic Regression; 7 Testing a New Medicine; 8 Debugging a Bootstrap; 9 Decision Tree Entropy
Win26	1 Let's Get This Party Started; 2 Powering a Town; 3 Let's Tree Max; 4 Are We Cooked?; 5 Two LLMs Walk Into A Bar; 6 That's Private; 7 Playing it Safe; 8 It's Gamma Time; 9 Genomics and Sampling; 10 The Doctor Will See You Now
Win25	1 Short Answer; 2 In-Vitro Fertilization; 3 Bag of Coins; 4 Microcontroller Precision Error; 5 Exponential Backoff; 6 Indus Valley Script; 7 Estimating Course Size; 8 Bayesian RNA Sequencing; 9 MLE of a Truncated Normal Distribution; 10 Logistic Regression Conceptual
F24	1 Short Answer; 2 Airline Flights; 3 Board Games; 4 Song of the Quarter; 5 What Name Doesn't Give Away Age?; 6 Hidden Chambers in the Great Pyramid; 7 Sorted Random Values; 8 MLE of Negative Binomial; 9 Recalibrating an Uncalibrated Model
F23	1 Short Answer; 2 Don't Quit Until You Are Ahead; 3 Popcorn; 4 Delicate Polling; 5 Water Levels; 6 Logistic Regression Priors; 7 Estimating Child Vocab Size

Exam	Problems
F22	1 Short Answer; 2 Machine Learning; 3 Night Sight; 4 Penalty Shootout; 5 Better; 6 B-Reel; 7 Code Survival; 8 Approximate Counting Algorithm; 9 Ranking
W22	1 Short Answer; 2 Quantitative Art History; 3 Beta $\pm$; 4 Bayesian Carbon Dating; 5 Reliability Engineering; 6 Rank Ordering of Samples; 7 Machine Teaching
F21	1 True or False; 2 Algorithmic Art; 3 How Lucky!; 4 Probabilistic Supply Chains; 5 Chess.com Puzzles; 6 P-Hacking; 7 Calibration of ML Algorithms
F17	1 Missing Not at Random; 2 Occupy Mt Gox; 3 Timing Attack; 4 Learning While Helping; 5 Expected Maximum Uniform; 6 Gaussian Naive Bayes; 7 The Most Important Features
Spr17	1 Random Encounter Redux; 2 Letters of Recommendation; 3 Hindenbug; 4 NBA Finals Week; 5 Gaussian Blur; 6 Improved Exam Grading; 7 Differential Privacy; 8 Windfarm Modeling; 9 Multiclass Bayes; 10 Logistic Vision Test

Two closing observations.

1. The exam will concentrate on machine learning problems, though there will be some others that you will want to be able to solve (see the next bullet).
2. The four topics that appear on essentially *every* final are Bayes' Theorem, choosing between Poisson and Exponential, the Central Limit Theorem, and MLE. If you are triaging, start there.

Good luck. You have done the reading, the psets, and twenty+ worksheets. You are ready.