Global Illumination


rspective
Lit
Show
TING NEEDS TO BE REBUILT (2,086 unbuilt objects)
ECTION CAPTURES NEED TO BE REBUILT (1 unbuilt)
10
10°
0.25
4
Level: NewMap_01 (Persistent)

Idea 1: Following Photons

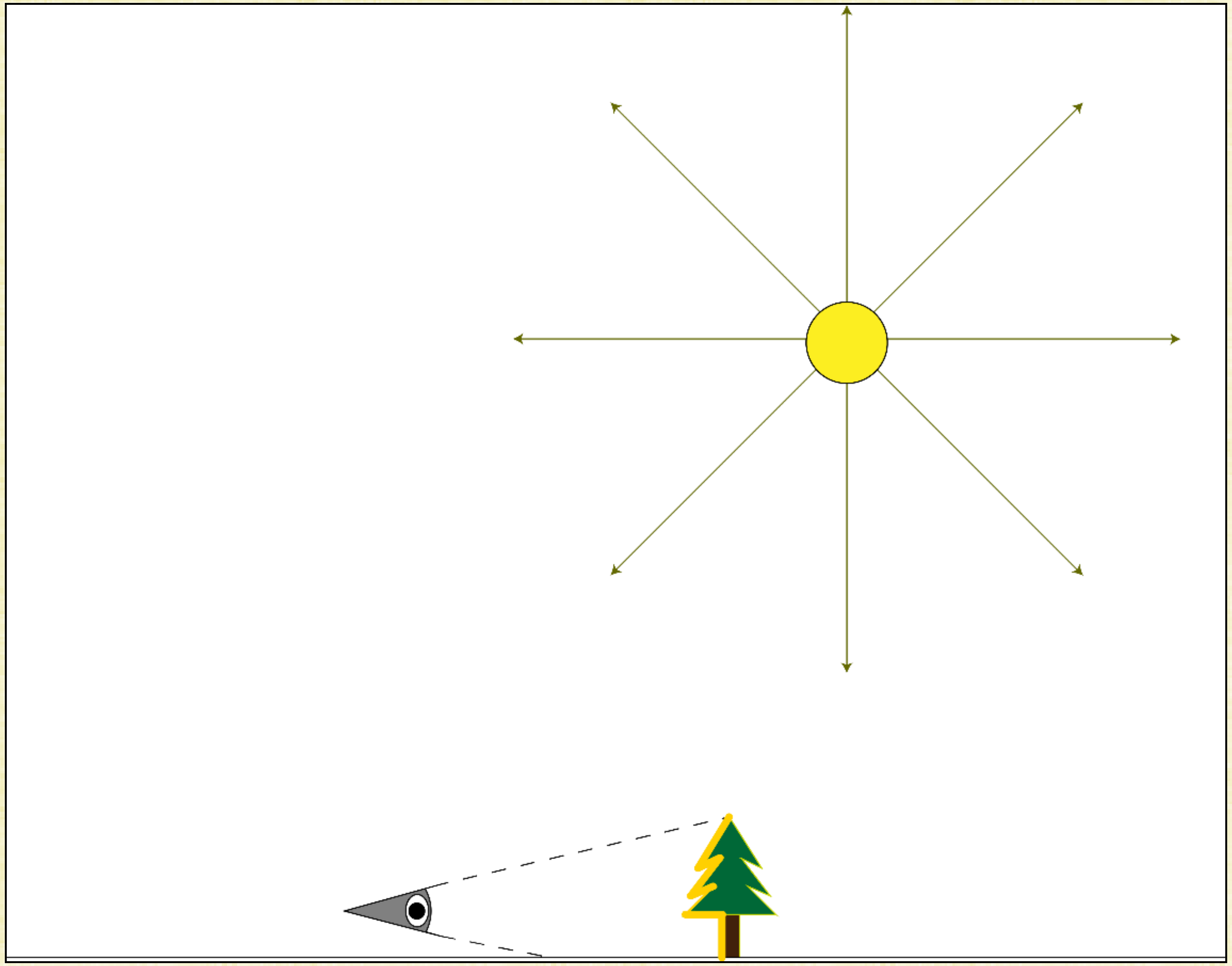
- For each light, choose a number of outgoing directions (on the hemisphere or sphere):
  - Emit a photon in each direction

- Each photon travels in a straight line, until it intersects an object
  - If Absorbed: Terminate the photon
  - If Reflected/Transmitted/Scattered: The photon goes off in a new direction, until it intersects an object

- When a photon goes through the camera aperture and hits a pixel on the film, it contributes to the color of that pixel

Idea 1: Following Photons

- Most of the photons never hit the film (far too inefficient, impractical)

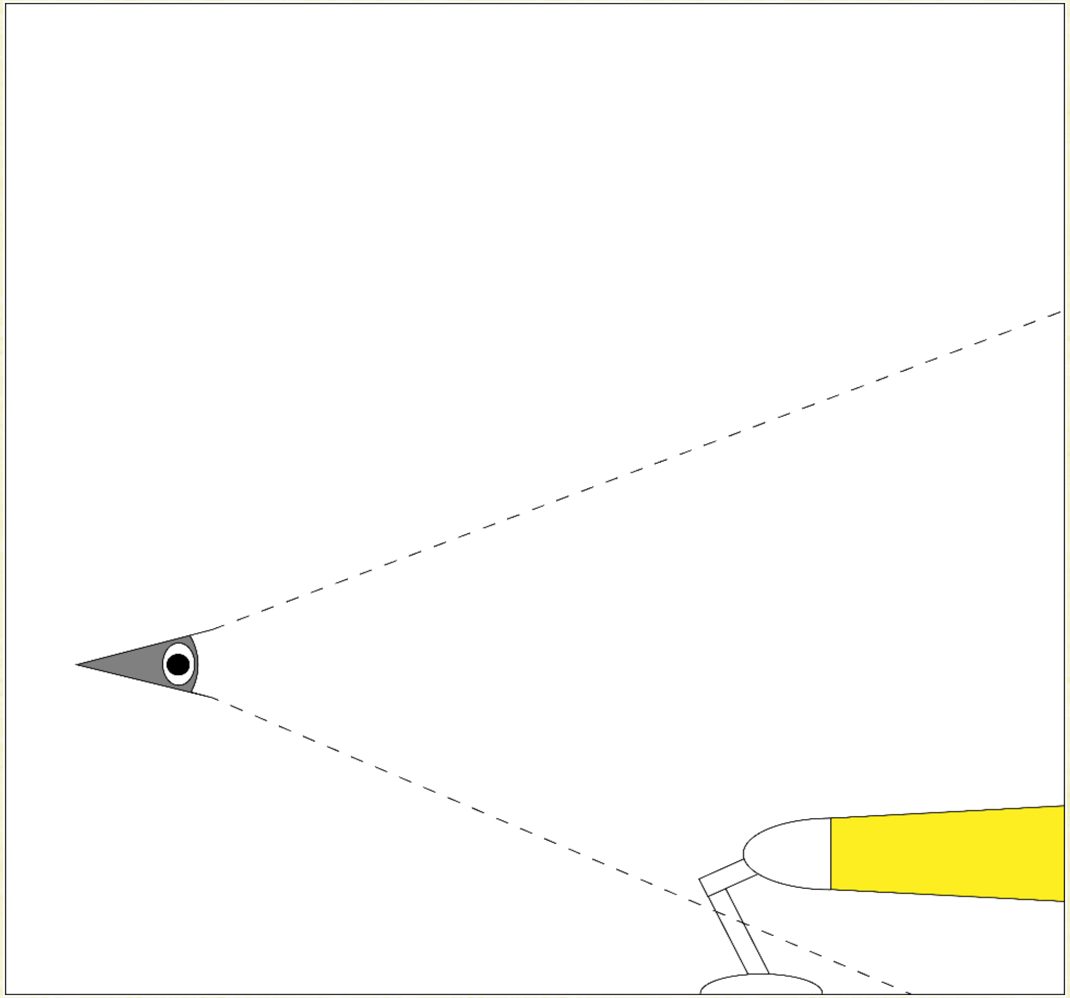


Idea 2: Backward Tracing of Photons

- For each pixel, send a ray through the aperture to backward trace a photon that would have hit the pixel (same as in ray tracing)
- If the ray hits an object, cast rays in all directions of the hemisphere in order to backwards trace incoming photons
  - Every new ray that hits another surface spawns an entire hemisphere of rays of its own (exponential growth makes this impractical)
- Follow all rays until they hit a light source
- Once a ray hits a light source, a path for photons (from the light source to the pixel) has been created
  - Emit photons along this path, bounce them off all the objects along the path, check to see if absorbed (if not absorbed, the photon continues on towards the pixel)
  - The absorption of photons results in a specific color/brightness of light hitting the pixel

Idea 2: Backward Tracing of Photons

- Most paths take too long to find their way back to the light source (inefficient)



Ray Tracing (as an efficient Backward Tracing)

- Ignore most incoming directions on the hemisphere, only keeping the most important ones:
  - Rays incoming directly from the light source have a lot of photons
    - A Shadow Ray is used to account for this incoming light
    - This is called direct illumination, since the light is coming directly from a light source
  - Reflective objects bounce a lot of photons in the mirror reflection direction
    - This incoming light is accounted for with a Reflected Ray
  - Transparent objects transmit a lot of photons along the transmitted ray direction
    - This incoming light is accounted for with a Transmitted Ray

- Downside: ray tracing ignores a lot of the incoming light, and thus cannot reproduce many visual effects

Solution: Bidirectional Ray Tracing

- Combine forward and backward tracing:
  - Step 1: Emit photons from the light sources, bathe objects in those photons, and record the result in a light map
    - Photons bounce around illuminating shadowed regions, bleeding color, etc.
    - Note: light maps don't change when the camera moves (so they can be precomputed)
  - Step 2: Ray trace the scene, using the light map to estimate indirect lighting (from the ignored directions of the hemisphere)

- IMPORTANT: Still treat the most important directions on the hemisphere with ray tracing, for increased accuracy
  - Shadow Rays for direct illumination
  - Reflected Rays
  - Transmitted Rays

Light Maps

- Light maps work great for soft shadows, color bleeding, etc.
- They can also generate many other interesting effects:

Recall: Lighting Equation

- Multiplying the BRDF by an incoming irradiance gives the outgoing radiance

$$dL_{o\ due\ to\ i}(\omega_i, \omega_o) = BRDF(\omega_i, \omega_o) dE_i(\omega_i)$$

- For even more realistic lighting, we'll bounce light all around the scene
- It's tedious to convert between E and L, so use $L = \frac{dE}{d\omega \cos\theta}$ to obtain:

$$dL_{o\ due\ to\ i}(\omega_i, \omega_o) = BRDF(\omega_i, \omega_o)\, L_i \cos\theta_i \, d\omega_i$$

- Then,

$$L_o(\omega_o) = \int_{i \in hemi} BRDF(\omega_i, \omega_o) L_i \cos\theta_i \, d\omega_i$$

Lighting Equation

- Explicitly add the dependencies on surface location x and incoming angle ω_i
- Add an emission term L_e, so x can be a location on the surface of an actual light too

$$L_o(x, \omega_o) = L_e(x, \omega_o) + \int_{i \in hemi} BRDF(x, \omega_i, \omega_o) L_i(x, \omega_i) \cos\theta_i \, d\omega_i$$

- Incoming light from direction ω_i left some other surface point x' going in direction $-\omega_i$
- So, replace $L_i(x, \omega_i)$ with $L_o(x', -\omega_i)$

$$L_o(x, \omega_o) = L_e(x, \omega_o) + \int_{i \in hemi} BRDF(x, \omega_i, \omega_o) L_o(x', -\omega_i) \cos\theta_i \, d\omega_i$$

An Implicit Equation

- Computing the outgoing radiance $L_o(x, \omega_o)$ on a particular surface requires knowing the outgoing radiance $L_o(x', -\omega_i)$ on all the other (relevant) surfaces
- But the outgoing radiance from those other surfaces (typically) depends on the outgoing radiance from the surface under consideration (circular dependencies)

$$L_o(x, \omega_o) = L_e(x, \omega_o) + \int_{i \in hemi} L_o(x', -\omega_i) \boxed{BRDF(x, \omega_i, \omega_o) \cos\theta_i \, d\omega_i}$$

Reflected Light	Emission	Reflected Light	BRDF	incident angle
UNKNOWN	KNOWN	UNKNOWN	KNOWN	KNOWN

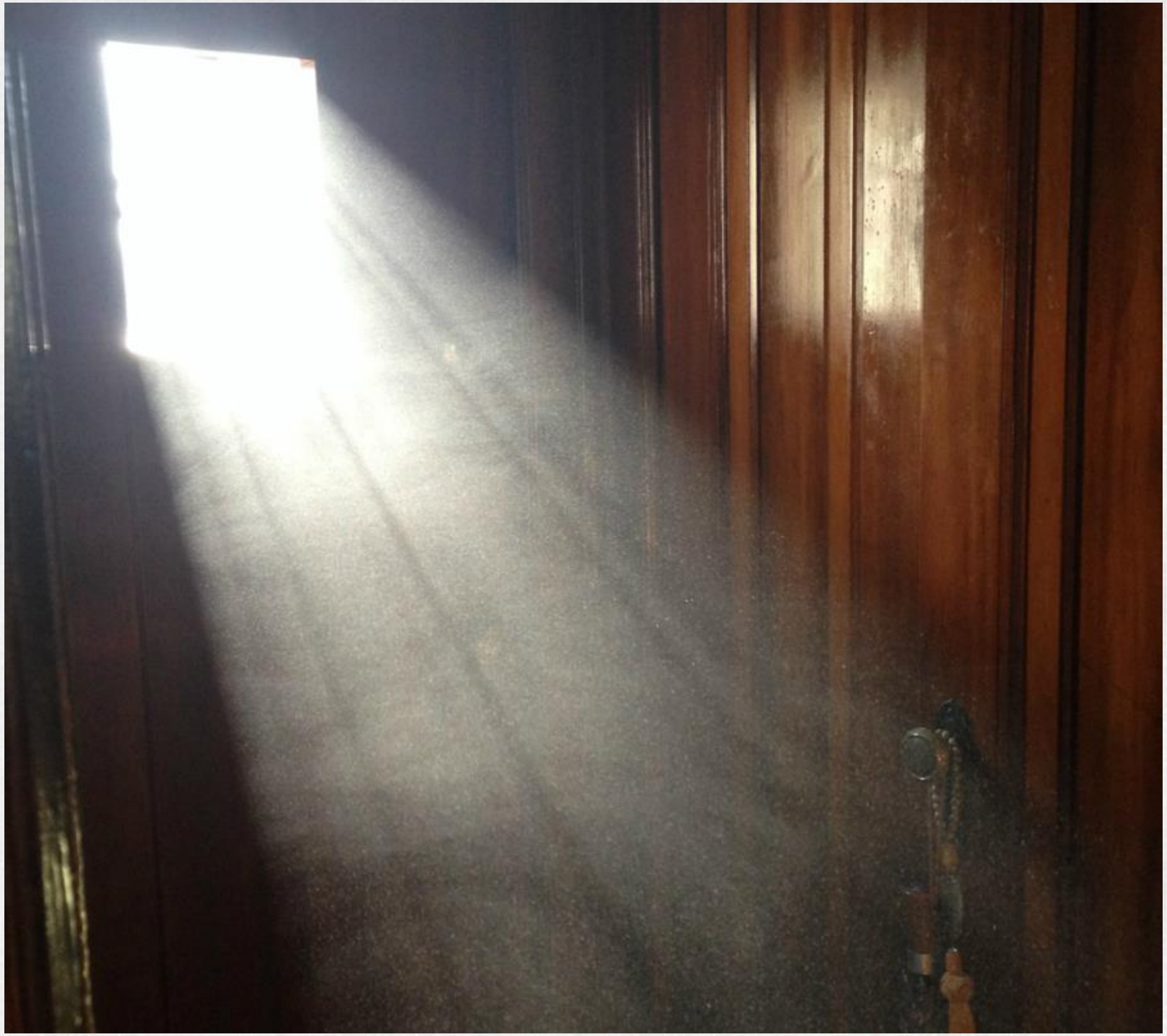
- Fredholm Integral Equation of the second kind (extensively studied) given in canonical form with kernel $k(u, v)$ by:

$$l(u) = e(u) + \int l(v) \boxed{k(u, v) dv}$$

Aside: Participating Media

- "Air" typically contains participating media (e.g. dust, droplets, smoke, etc.)
- L should actually be defined over all of 3D space (not just on 2D surfaces)
- The incoming light should be considered in a sphere centered around each point in 3D space

- Neglecting this assumes that "air" is a vacuum
- That restricts L to surfaces



Discretization of the Lighting Equation

- Choose p points, each representing a chunk of surface area
  - This is a 2D discretization (for participating media, volume chunks are 3D)
- For each of the p points: Choose q outgoing directions, each representing a chunk of solid angles of the hemisphere (or sphere)
  - This is a 2D discretization

- L_o and L_e then each have $p * q$ unknowns
  - This a 4D (or 5D) discretization

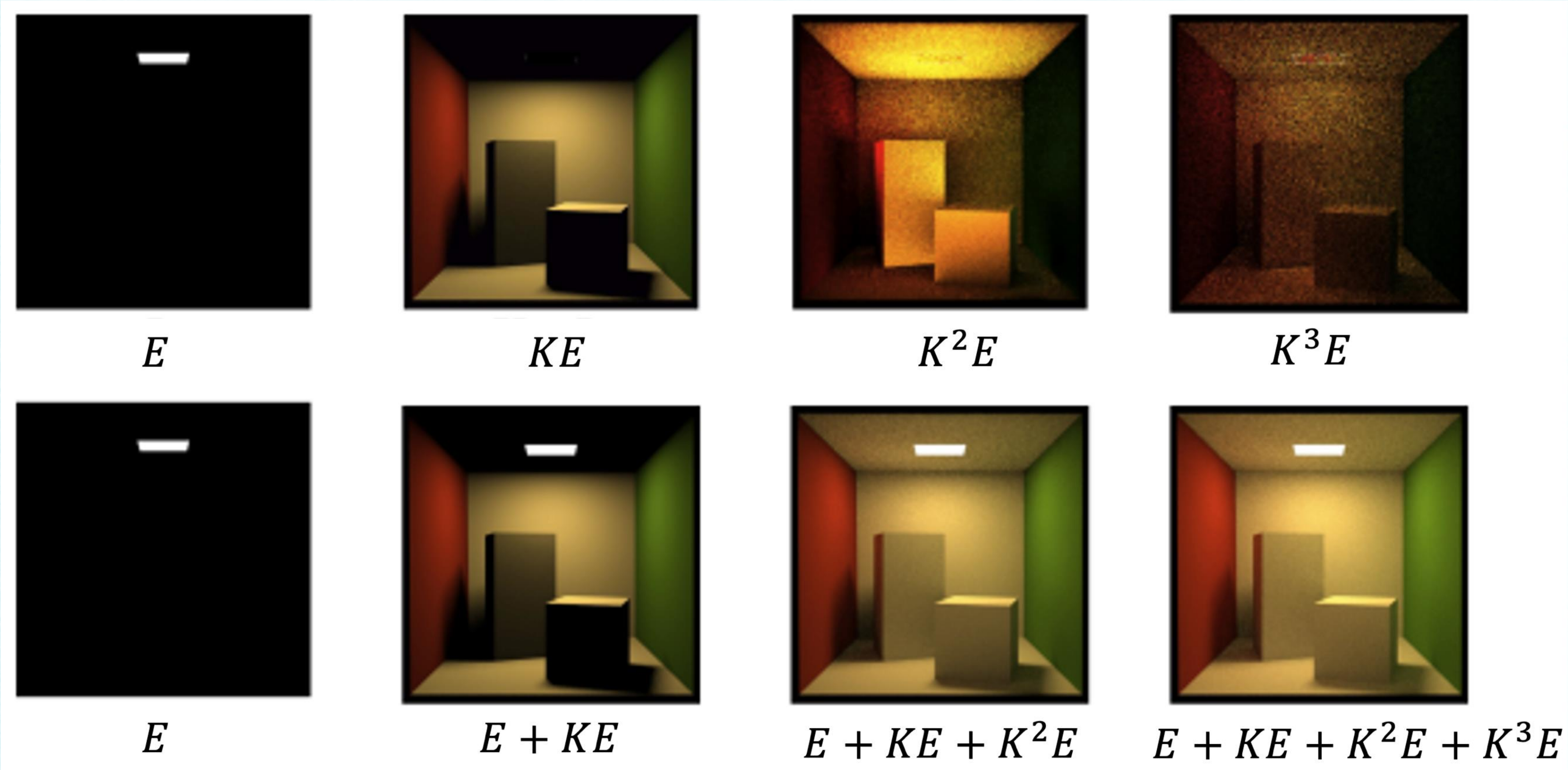
- The linear system of equations is: $L = E + KL$ or $(I - K)L = E$
  - L and E are length $p * q$, and the light transport "kernel" matrix K has size $p * q$ by $p * q$
- Solution: $L = (I - K)^{-1}E = (I + K + K^2 + \cdots)E$
  - Since K bounces only a fraction of the light (the rest is absorbed), higher powers are smaller (and the infinite series can be truncated)

Power Series

$$L = E + KE + K^2E + K^3E + \cdots$$

- E: Emission directly from Light Sources
- KE: Direct Illumination (light bounces only once)
- K^2E: Global Illumination (indirect lighting, two bounces)
- K^3E: Global Illumination (indirect lighting, three bounces)
- $\cdots$: Etc.

Power Series



Tractability

- A typical scene might warrant thousands or tens of thousands of area chunks
  - So, p could be 1e3, 1e4, 1e5, 1e6, etc.
- Incoming light could vary significantly across the hemisphere
  - So, q might need to be 1e2, 1e3, 1e4, etc.
- L and E would then range in length from 1e5 to 1e10
- The matrix K would then range in size from 1e5 by 1e5 up to 1e10 by 1e10

- K would have between 1e10 and 1e20 entries!

- This tractability analysis is for the 4D problem (the 5D problem is even worse)
- The curse of dimensionality makes problems in 4D and 5D (and higher) hard to discretize with this numerical quadrature

In order to addressing tractability…

- Separate the diffuse and specular contributions, and treat them separately

Diffuse Approximation:

- Assume all materials are purely diffuse (i.e. with no specular contributions)
- Compute the view-independent global illumination for the entire scene
- This can be done in a pre-processing step

Specular Approximation:

- Compute (view-dependent) specular illumination on-the-fly as the camera moves
- Use the Phong Shading model (or any other model)

Radiosity & Albedo

- Radiosity: power per unit surface area leaving a surface (similar to irradiance, but outgoing instead of incoming):

$$B(x) = \frac{d\Phi}{dA} = \int_{hemi} L_o(x, \omega_o) \cos\theta_o \, d\omega_o$$

- When L_o is independent of ω_o (an approximation for purely diffuse surfaces):

$$B(x) = \frac{d\Phi}{dA} = L(x) \int_{hemi} \cos\theta_o \, d\omega_o = \pi L(x)$$

- Albedo: a “reflection coefficient” relating incoming light hitting a surface patch (irradiance E_i) to outgoing light emitted in all possible directions

$$\rho(x) = \int_{hemi} BRDF(x, \omega_o, \omega_i) \cos\theta_o \, d\omega_o$$

- When the BRDF is independent of ω_o and ω_i (an approximation for purely diffuse surfaces):

$$\rho(x) = BRDF(x) \int_{hemi} \cos\theta_o \, d\omega_o = \pi \, BRDF(x)$$

Purely Diffuse Surface Lighting Equation

- Multiply

$$L_o(x,\omega_o) = L_e(x,\omega_o) + \int_{i \in hemi} L_o(x',-\omega_i) BRDF(x,\omega_i,\omega_o) \cos\theta_i \, d\omega_i$$

through by $\cos\theta_o d\omega_o$ and integrate over the hemisphere (i.e. over $d\omega_o$):

$$B(x) = E(x) + \int_{i \in hemi} B(x') BRDF(x,\omega_i,\omega_o) \cos\theta_i \, d\omega_i$$

- B is a 2D function (of x), whereas L was a 4D function (of x and ω_o)

- In addition, assume that all surfaces have a diffuse BRDF independent of angle:

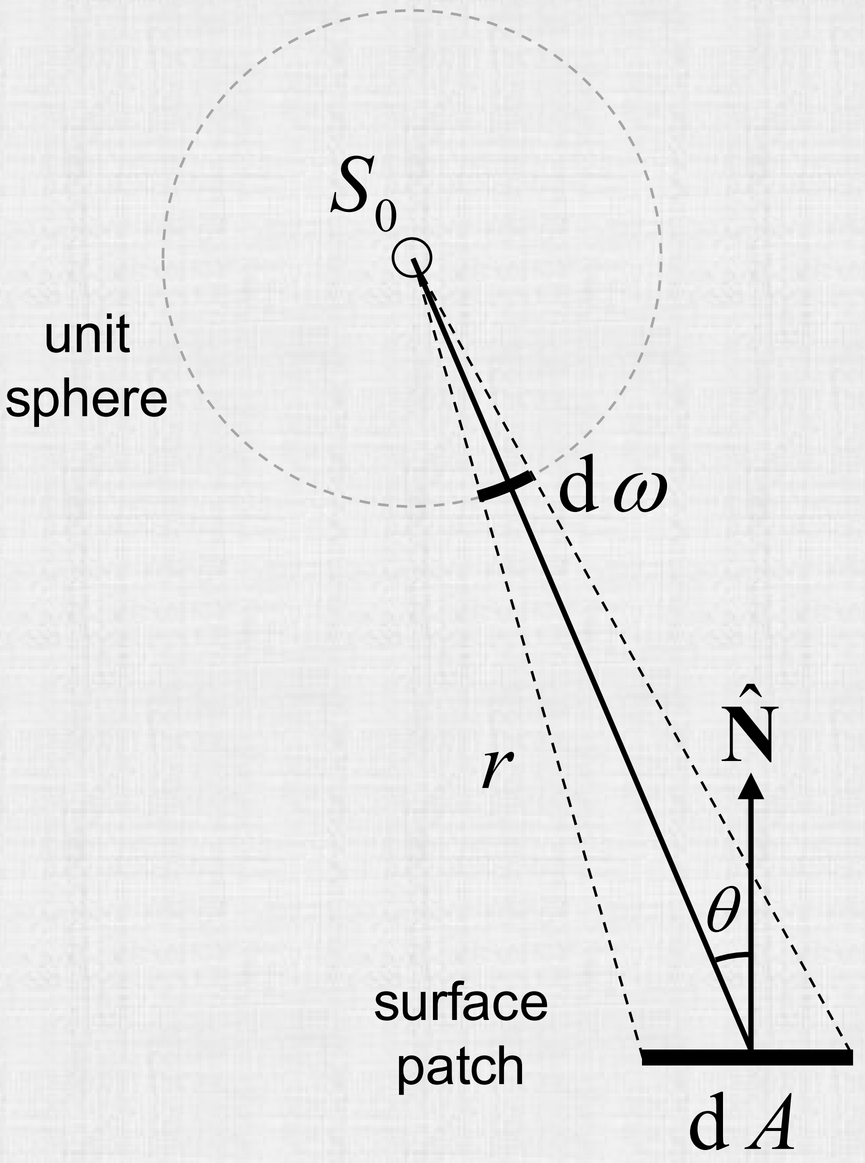
$$B(x) = E(x) + \frac{\rho(x)}{\pi} \int_{i \in hemi} B(x') \cos\theta_i \, d\omega_i$$

Recall: Solid Angle vs. Cross-Sectional Area

- Definition of solid angle: $d\omega = \frac{dA_{sphere}}{r^2}$
- From the previous slide: the (orthogonal) cross-sectional area of the surface patch is $dA \cos\theta$
- So, given a sphere of radius r (in the figure):

$$d\omega = \frac{dA \cos\theta}{r^2}$$

- The solid angle decreases as the surface tilts away from the light (increasing θ, decreasing $\cos\theta$)
- The solid angle decreases as the surface is moved further from the light (increasing r)



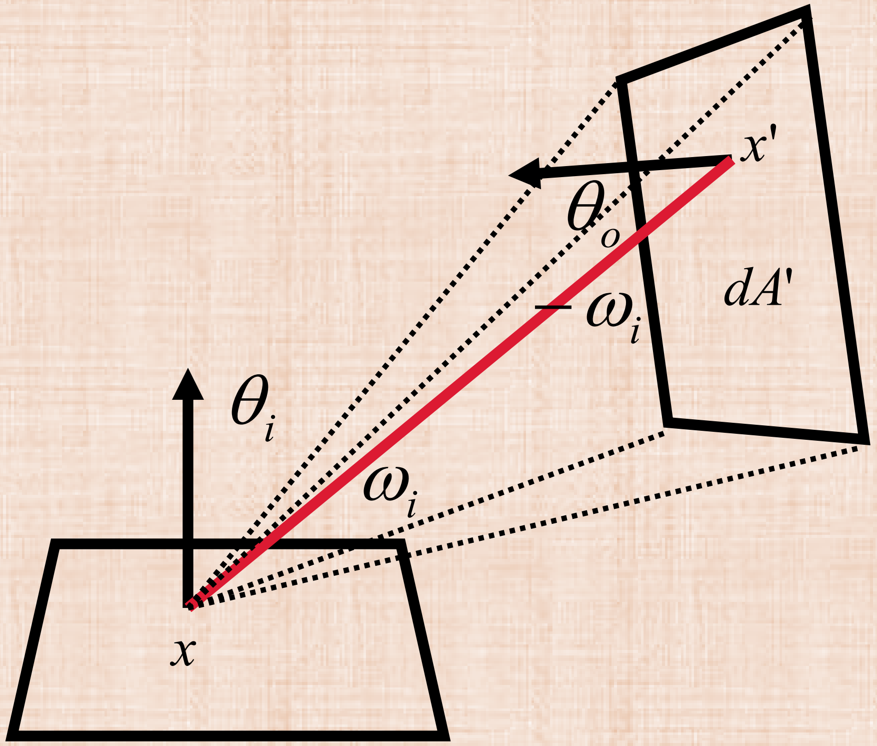
Replace Solid Angle with Surface Area

- Rewrite $d\omega = \frac{dA\cos\theta}{r^2}$ as $d\omega_i = \frac{dA'\cos\theta_o}{\|x-x'\|_2^2}$
- Substituting this into the equation from the last (orange) slide:

$$B(x) = E(x) + \rho(x)\int_{i\in hemi} B(x')\frac{\cos\theta_i\cos\theta_o}{\pi\|x-x'\|_2^2}dA'$$

- Let $V(x,x') = 1$ when x and x' are mutually visible and $V(x,x') = 0$ otherwise, then:

$$B(x) = E(x) + \rho(x)\int_{all\ x'} B(x')V(x,x')\frac{\cos\theta_i\cos\theta_o}{\pi\|x-x'\|_2^2}dA'$$



A Tractable Discretization

- Choose p points, each representing a chunk of surface area (a 2D discretization)
- Then $B_i = E_i + \rho_i \sum_{j \neq i} B_j F_{ij}$ with a purely geometric $F_{ij} = V(x_i, x_j) \frac{\cos\theta_i \cos\theta_j}{\pi \|x_i - x_j\|_2^2} A_j$
- Rearrange to $B_i - \rho_i \sum_{j \neq i} B_j F_{ij} = E_i$ and put into matrix form:

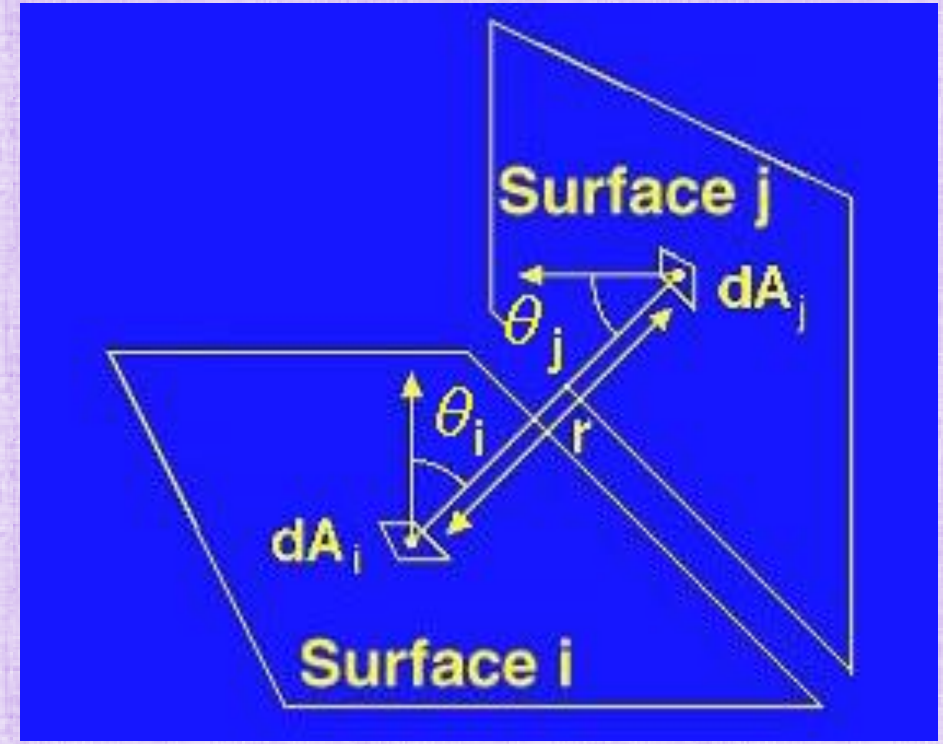
$$\begin{pmatrix} 1 & -\rho_1 F_{12} & \cdots & -\rho_1 F_{1p} \\ -\rho_2 F_{21} & 1 & \cdots & -\rho_2 F_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ -\rho_p F_{p1} & -\rho_p F_{p2} & \cdots & 1 \end{pmatrix} \begin{pmatrix} B_1 \\ B_2 \\ \vdots \\ B_p \end{pmatrix} = \begin{pmatrix} E_1 \\ E_2 \\ \vdots \\ E_p \end{pmatrix}$$

- For p ranging from 1e3 to 1e6: B and E have the same size, and the matrix has 1e6 to 1e12 entries (still large, but 1e4 to 1e8 times smaller than previously)

Form Factor

- Write $F_{ij} = V(x_i, x_j)\,\frac{\hat{F}_{ij}}{A_i}$ and $F_{ji} = V(x_i, x_j)\,\frac{\hat{F}_{ij}}{A_j}$ with the symmetric form factor:

$$\hat{F}_{ij} = \frac{\cos\theta_i \cos\theta_j}{\pi \left\| x_i - x_j \right\|_2^2} A_i A_j$$



- $\hat{F}_{ij}$ represents how the light energy leaving one surface impacts the other surface, and vice versa (and only depends on the geometry, not on the light)

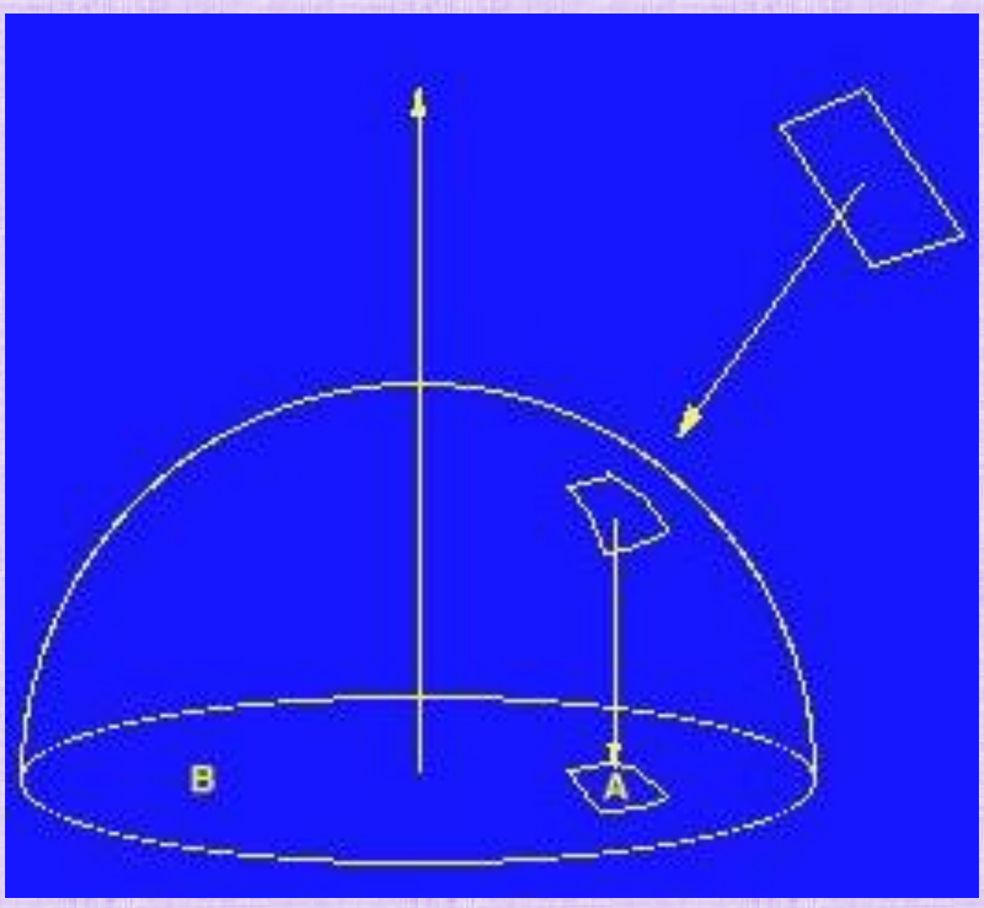
- The visibility between between x_i and x_j, i.e. $V(x_i, x_j)$, also only depends on the geometry (and can be included in $\hat{F}_{ij}$ if desired)

Understanding the Form Factor

- Place a unit hemisphere at a surface point x_i

- Project a surface onto the unit hemisphere, noting that $d\omega = \frac{dA \cos\theta}{r^2}$ gives $\frac{A_j \cos\theta_j}{\|x_i - x_j\|_2^2}$ as the result
- Project the result downwards onto the circular base of the hemisphere, which multiples by $\cos\theta_i$
  - Recall $\int_{i \in hemi} \cos\theta_i \, d\omega_i = \pi$, the area of the unit circle
- Divide the result by the total area π to get the fraction of the circle occupied

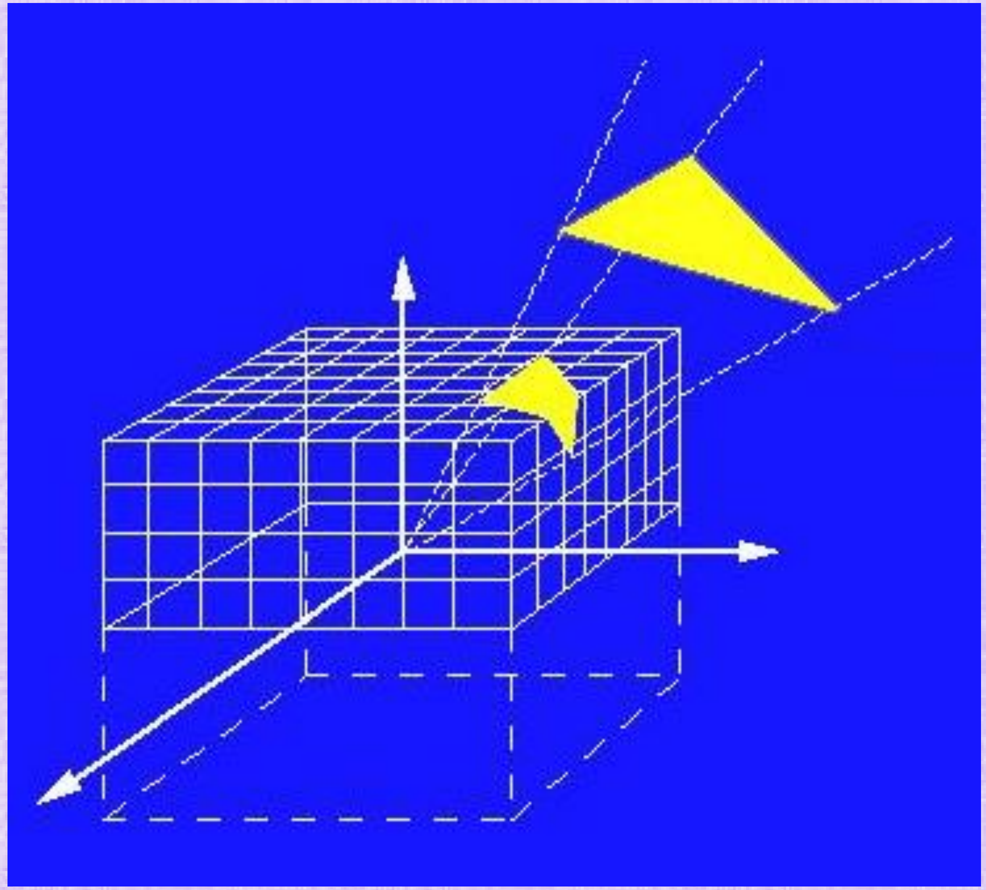
- Overall, this gives: $F_{ij} = \frac{\cos\theta_i \cos\theta_j}{\pi \|x_i - x_j\|_2^2} A_j$



Implementation

Precomputation:

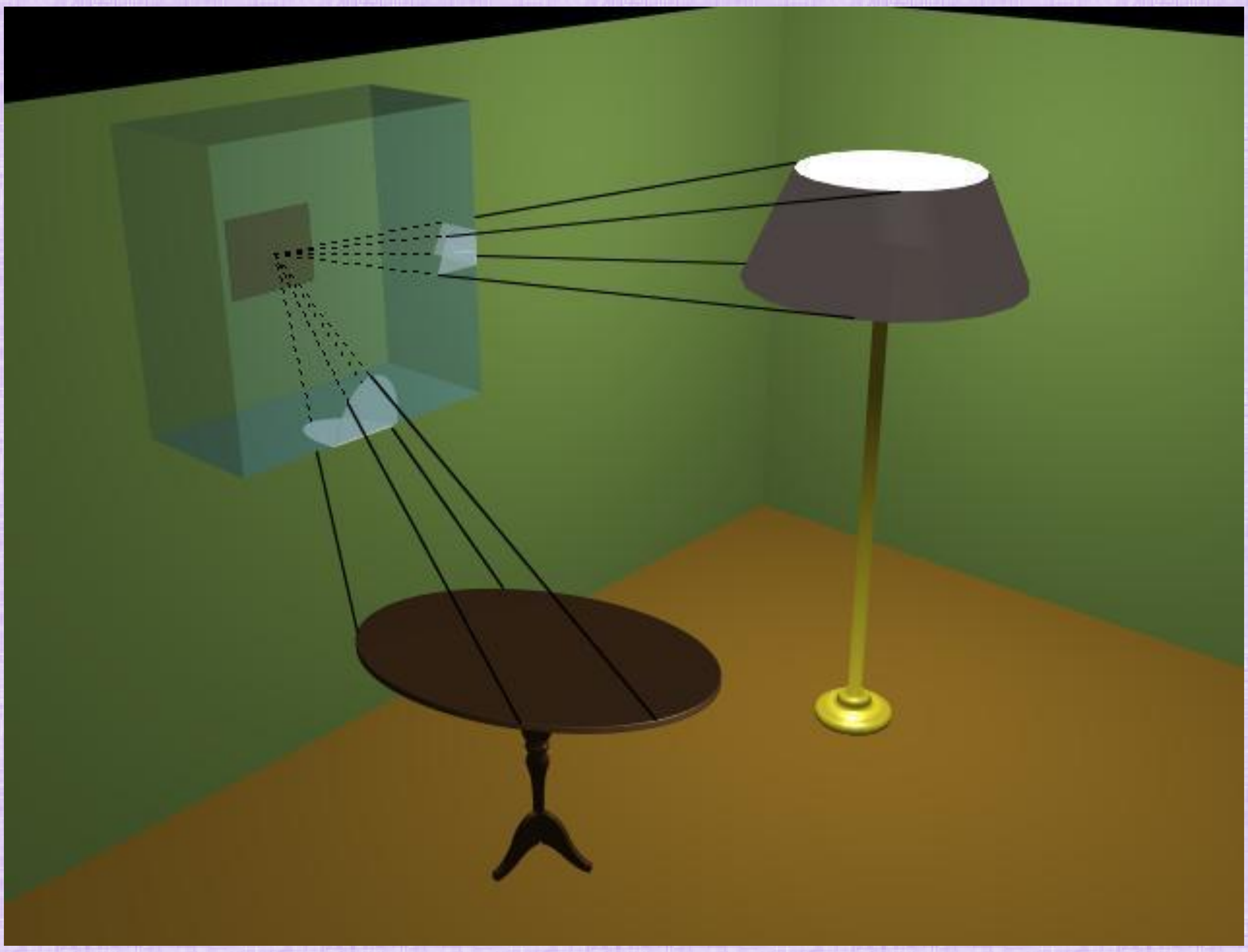
- Create a hemicube, and divide each face into sub-squares (as small as desired)
- For each sub-square, use the hemisphere projection (from the last slide) to pre-compute its contribution to F_{ij}



For each surface chunk:

- Place the hemicube at a surface point x_i
- Surface patches (from other objects) are projected onto the hemicube in order to approximate F_{ij} (using the pre-computed values for the sub-squares)
- The five hemicube faces can each be treated as a film plane where sub-squares are pixels
    - This is then just standard scanline rasterization
- The depth buffer is used for the visibility term

Hemicube Scanline Rasterization



Iterative Solvers

- For large matrices, iterative solvers are typically far more accurate than direct methods (like Cramer's rule for computing inverses)
- Iterative methods start with an initial guess, and subsequently iteratively improve it

- Consider $\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 8 \\ 10 \end{pmatrix}$ with solution $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$
- Start with an initial guess of $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

- <u>Jacobi</u> iteration (solve both equations, using the old values to get the new values):
  - $x^{new} = \frac{8-y^{old}}{2}$ and $y^{new} = \frac{10-x^{old}}{2}$

- <u>Gauss Seidal</u> iteration (always use the most up to date values):
  - $x^{current} = \frac{8-y^{current}}{2}$ and $y^{current} = \frac{10-x^{current}}{2}$

Jacobi vs. Gauss-Seidal

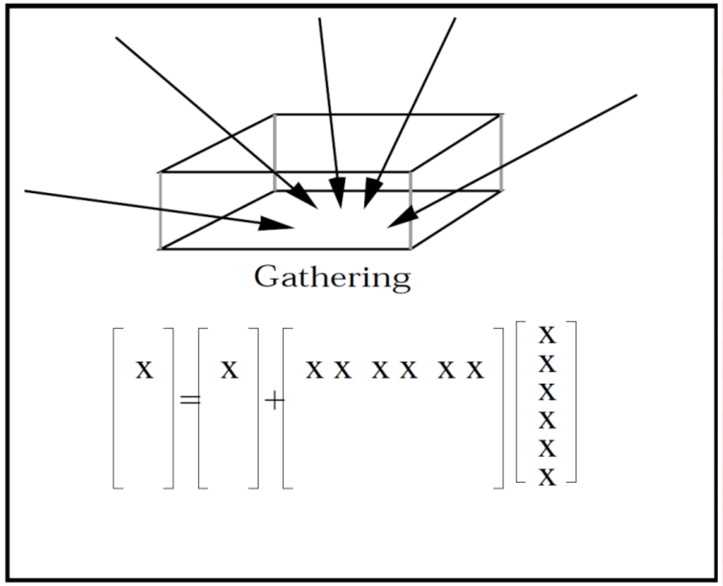
Iteration	Jacobi		Gauss Seidel	
	x	y	x	y
1	0	0	0	0
2	4	5	4	3
3	1.5	3	2.5	3.75
4	2.5	4.25	2.125	3.9375
5	1.875	3.75	2.03125	3.984375
6	2.125	4.0625	2.007813	3.996094
7	1.96875	3.9375	2.001953	3.999023
8	2.03125	4.015625	2.000488	3.999756
9	1.9921875	3.984375	2.000122	3.999939
10	2.0078125	4.00390625	2.000031	3.999985
11	1.998046875	3.99609375	2.000008	3.999996
12	2.001953125	4.000976563	2.000002	3.999999
13	1.999511719	3.999023438	2	4
14	2.000488281	4.000244141	2	4
15	1.99987793	3.999755859	2	4
16	2.00012207	4.000061035	2	4
17	1.999969482	3.999938965	2	4
18	2.000030518	4.000015259	2	4
19	1.999992371	3.999984741	2	4
20	2.000007629	4.000003815	2	4

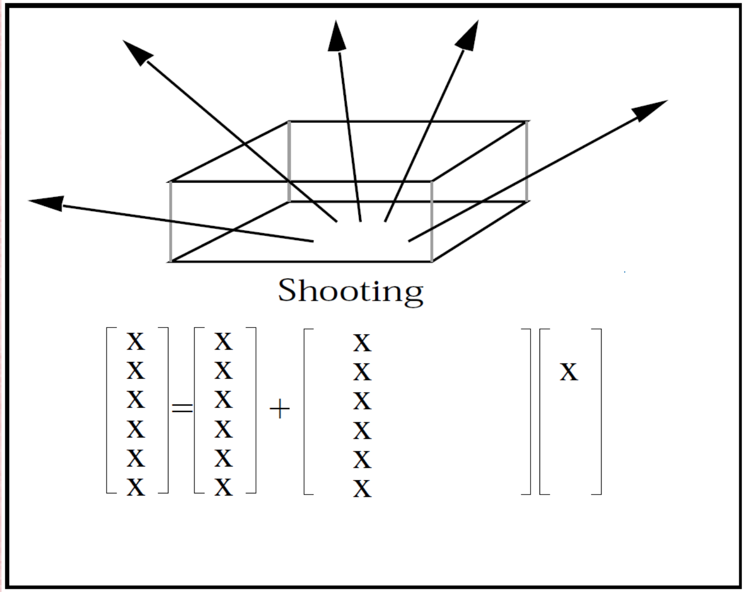
Better Initial Guess

Iteration	Jacobi		Gauss Seidal	
	x	y	x	y
1	2	3	2	3
2	2.5	4	2.5	3.75
3	2	3.75	2.125	3.9375
4	2.125	4	2.03125	3.984375
5	2	3.9375	2.007813	3.996094
6	2.03125	4	2.001953	3.999023
7	2	3.984375	2.000488	3.999756
8	2.0078125	4	2.000122	3.999939
9	2	3.99609375	2.000031	3.999985
10	2.001953125	4	2.000008	3.999996
11	2	3.999023438	2.000002	3.999999
12	2.000488281	4	2	4
13	2	3.999755859	2	4
14	2.00012207	4	2	4
15	2	3.999938965	2	4
16	2.000030518	4	2	4
17	2	3.999984741	2	4
18	2.000007629	4	2	4
19	2	3.999996185	2	4
20	2.000001907	4	2	4

Iterative Radiosity

- Gathering - update one surface by collecting light energy from all surfaces
- Shooting - update all surfaces by distributing light energy from one surface
- Sorting and Shooting - choose the surface with the greatest un-shot light energy and use shooting to distribute it to other surfaces
  - Start by shooting light energy out of the lights onto objects (the brightest light goes first)
  - Then, the object that would reflect the most light goes next, etc.
- Sorting and Shooting with Ambient - start with an initial guess for ambient lighting and do sorting and shooting afterwards





Iterative Radiosity