More Texture Mapping
Recall: (Averaged) Vertex Normals

- Each vertex has a number of incident triangles, each with their own normal
- Averaging those face normals (or a weighted average based on: area, angle, etc.) gives a unique normal for each vertex
Recall: Smooth Shading

• faceted silhouette
Perturbing the Normal

• Store a normal vector in the texture (instead of a color)
• This perturbed normal can “fake” geometric details
Bump Map

- Single-channel (grey-scale) height map $h_{ij}$, representing the height at location $(u_i, v_j)$
- The tangent plane at a point $(u_i, v_j, h_{ij})$ is given by:
  \[
  \frac{\partial h(u_i, v_j)}{\partial u} (u - u_i) - \frac{\partial h(u_i, v_j)}{\partial v} (v - v_j) + (h - h_{ij}) = 0
  \]
- So, the outward (non-unit) normal is \(\left(-\frac{\partial h(u_i, v_j)}{\partial u}, -\frac{\partial h(u_i, v_j)}{\partial v}, 1\right)\)
- Partial derivatives are computed via finite differences:
  \[
  \frac{\partial h(u_i, v_j)}{\partial u} = \frac{h_{i+1,j} - h_{i-1,j}}{u_{i+1} - u_{i-1}} \quad \text{and} \quad \frac{\partial h(u_i, v_j)}{\partial v} = \frac{h_{i,j+1} - h_{i,j-1}}{v_{j+1} - v_{j-1}}
  \]
Normal Map

- A normalized vector has each component in $[-1,1]$, so one can convert back and forth to a color via:

$$ (R, G, B) = 255 \frac{\vec{N} + (1,1,1)}{2} \quad \text{and} \quad \vec{N} = \frac{2}{255} (R, G, B) - (1,1,1) $$

- Normal maps use more storage than bump maps, but require less computation.
Displacement Mapping

- Subdivide geometry at render time, and use a height map $h(u,v)$ to (actually) perturb vertices in the normal direction.
- Pros: self-occlusion, self-shadowing, correct **silhouettes**
- Cons: expensive, requires adaptive tessellation, still need bump/normal map for sub-triangle (fake) detail
Displacement Mapping

bump map
displacement map
Recall: Measuring Incoming Light

- **Light Probe**: a small reflective chrome sphere
- Photograph it, in order to record the incoming light (at its location) from all directions
Recall: Using the (measured) Incoming Light

- The (measured) incoming light can be used to render a synthetic object (with realistic lighting)
**Environment Mapping**

- Place a coordinate system at the center of the sphere, so the surface normal is: \( N = \frac{1}{\sqrt{x^2+y^2+z^2}} (x, y, z) \)
- \( R \) is the direction from the light probe to the camera
- Since \( I \) and \( R \) are equal-angle from \( N \) (because of mirror reflection), \( N \) has a one-to-one correspondence with \( I \)
- Note: assuming that the intensity of incoming light depends only on incoming direction \( I \) (not on position)
Environment Mapping

• Given a normal on geometry (to be rendered)
• Use $n_x$ and $n_y$ (which are in the range [-1, 1]) to obtain texture coordinates $(u, v) = \frac{1}{2}(n_x + 1, n_y + 1)$
• Then, look up the incoming light on a picture of the light probe
Environment Mapping
Sky Boxes

- Approximate the sky with a texture on the inside of geometry.
Texture Acquisition via Imaging
Texture Acquisition via Imaging
Texture Synthesis: Pixel Based

• Create a large non-repetitive texture (one pixel at a time) from a small sample (by using its structural content)
• Generate the texture in a raster scan ordering
• To generate the texture for pixel $p$
  • compare $p$’s neighboring pixels in the (red) stencil to all potential choices in the sample
  • choose the one with the smallest difference to fill pixel $p$
• When the stencil needs values outside the domain, use periodic boundary conditions (so, fill the last few rows/columns with random values)

![Stencil](image1)
![Texture Sample](image2)
![Raster Scan Ordering](image3)
Texture Synthesis: Pixel Based

Heeger and Bergen

Efros and Leung

Wei and Levoy
Texture Synthesis: Patch Based

- For each patch being considered:
  - search the original sample to find candidates which best match the overlap regions (on the boundaries)
  - choose the best candidates
  - blends overlapped regions to remove “seams”
Texture Synthesis: Patch Based
Don’t Stretch Textures!

• Stretching out 10 bricks to cover an entire wall of a building is going to look unrealistic!
• Can instead **tile textures** if the small tiles are made with periodic boundaries
Marble Texture

• Predefine layers of different colors
• Use a function to map \((u, v)\) texture locations to layers
• For example:

\[
marbleColor(u, v) = LayerColor(\sin(k_u u + k_v v))
\]
Marble Texture

- $k_u$ and $k_v$ are spatial frequencies
- $(k_u, k_v)$ determines the direction, and $\frac{2\pi}{\sqrt{k_u^2 + k_v^2}}$ determines the periodicity
- Too regular (need to add noise/randomness)
Perlin Noise

• Noise should have both coherency and structure in order to look more natural
• Ken Perlin proposed a specific (and amazing!) method for doing this
Perlin Noise

- Place a 2D grid over the image, and assign a random (unit) gradient $g(u_i, v_j)$ to each grid point.
- For each pixel, compute the dot-products between vectors from the grid corners and the corresponding gradient.
- Take a weighted average of the result:

$$\text{noise}(u, v) = \sum_{i=0,1;j=0,1} w\left(\frac{u - u_i}{\Delta u}\right) w\left(\frac{v - v_j}{\Delta v}\right) g(u_i, v_j) \cdot (u - u_i, v - v_j)$$

- Cubic weighting function: $w(t) = 2|t|^3 - 3|t|^2 + 1$ for $-1 < t < 1$
Multiple Scales

• Many natural textures contain a variety of feature sizes
• Mimic this by adding together noises with different frequencies and amplitudes:
  \[ \text{perlin}(u, v) = \sum_k \text{noise}(\text{frequency}(k) \times (u, v)) \times \text{amplitude}(k) \]

• Each successive noise function is twice the frequency of the previous one:
  \[ \text{frequency}(k) = 2^k \]

• The amplitude of higher frequencies is measured by a persistence parameter (\( \leq 1 \))
• Higher frequencies have a diminished contribution:
  \[ \text{amplitude}(k) = \text{persistence}^k \]
**1D Examples**

- Smaller persistence -> less higher frequency noise -> smoother result

<table>
<thead>
<tr>
<th>Frequency</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>8</th>
<th>16</th>
<th>32</th>
</tr>
</thead>
<tbody>
<tr>
<td>Persistence = 1/4</td>
<td>![Graph](from: <a href="http://freespace.virgin.net/hugo.elias/models/m_perlin.htm">http://freespace.virgin.net/hugo.elias/models/m_perlin.htm</a>)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Amplitude:</td>
<td>1</td>
<td>1/4</td>
<td>1/16</td>
<td>1/64</td>
<td>1/256</td>
<td>1/1024</td>
</tr>
<tr>
<td>Persistence = 1/2</td>
<td>![Graph](from: <a href="http://freespace.virgin.net/hugo.elias/models/m_perlin.htm">http://freespace.virgin.net/hugo.elias/models/m_perlin.htm</a>)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Amplitude:</td>
<td>1</td>
<td>1/2</td>
<td>1/4</td>
<td>1/8</td>
<td>1/16</td>
<td>1/32</td>
</tr>
<tr>
<td>Persistence = 1 / \sqrt{2}</td>
<td>![Graph](from: <a href="http://freespace.virgin.net/hugo.elias/models/m_perlin.htm">http://freespace.virgin.net/hugo.elias/models/m_perlin.htm</a>)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Amplitude:</td>
<td>1</td>
<td>1/1.414</td>
<td>1/2</td>
<td>1/2.828</td>
<td>1/4</td>
<td>1/5.656</td>
</tr>
<tr>
<td>Persistence = 1</td>
<td>![Graph](from: <a href="http://freespace.virgin.net/hugo.elias/models/m_perlin.htm">http://freespace.virgin.net/hugo.elias/models/m_perlin.htm</a>)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Amplitude:</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

(from: http://freespace.virgin.net/hugo.elias/models/m_perlin.htm)
2D Examples
Set the value of $A$ to scale the amount of noise:

$$marbleColor(u, v) = LayerColor\left(\sin(k_u u + k_v v + A \cdot \text{perlin}(u, v))\right)$$
3D Marble Texture

• Carve an object out of a 3D texture (eliminating the need to gift-wrap a complex 3D object)
• Marble texture function w/Perlin noise (for 3D):

\[ \text{marbleColor}(u, v, w) = \text{LayerColor} \left( \sin(k_u u + k_v v + k_w w + A * \text{perlin}(u, v, w)) \right) \]
3D Wood Texture

- Procedurally generate tree rings (and cut the object out of the 3D texture)
- Cylindrical coordinates for \((x, y, z)\) object points: 
  
  \[H = y, \quad R = \sqrt{x^2 + z^2}, \quad \theta = \tan^{-1}\left(\frac{z}{x}\right)\]
3D Wood Texture

- rings
- added eccentricity
- added twist
- added tilt
Neural Texture Synthesis: Gram Matrix

Each layer of CNN gives $C \times H \times W$ tensor of features; $H \times W$ grid of $C$-dimensional vectors.

Outer product of two $C$-dimensional vectors gives $C \times C$ matrix measuring co-occurrence.

Average over all $HW$ pairs of vectors, giving **Gram matrix** of shape $C \times C$.

Efficient to compute; reshape features from $C \times H \times W$ to $C \times HW$, then compute $G = FF^T$.
Machine Learning

Neural Texture Synthesis

1. Pretrain a CNN on ImageNet (VGG-19)
2. Run input texture forward through CNN, record activations on every layer; layer $i$ gives feature map of shape $C_i \times H_i \times W_i$
3. At each layer compute the Gram matrix giving outer product of features:
   \[ G_{ij}^l = \sum_k F_{ik}^l F_{jk}^l \]  
   (shape $C_i \times C_i$)
4. Initialize generated image from random noise
5. Pass generated image through CNN, compute Gram matrix on each layer
6. Compute loss: weighted sum of L2 distance between Gram matrices
   \[ \mathcal{L}(\vec{x}, \hat{\vec{x}}) = \sum_{l=0}^{L} w_l E_l \]
7. Backprop to get gradient on image
8. Make gradient step on image
9. GOTO 5

Figure copyright Leon Gatys, Alexander S. Ecker, and Matthias Bethge, 2015. Reproduced with permission.
Neural Texture Synthesis

Reconstructing texture from higher layers recovers larger features from the input texture.

Gatys, Ecker, and Bethge, "Texture Synthesis Using Convolutional Neural Networks", NIPS 2015
Figure copyright Leon Gatys, Alexander S. Ecker, and Matthias Bethge, 2015. Reproduced with permission.
Machine Learning