HW2 (Due: Wed Jan 20)

2.1 [5 points each]

2.1 (English and FOL). Encode the following English sentences into FOL.

(c) My mother's mother is my grandmother.
(d) The intersection of two convex sets is convex.

2.2 Use semantic arguments <u>with substitution</u>. [10 points each]

2.2 (FOL validity & satisfiability). For each of the following FOL formulae, identify whether it is valid or not. If it is valid, prove it with a semantic argument; otherwise, identify a falsifying interpretation.

(a) $(\forall x, y.\ p(x, y) \ \rightarrow\ p(y, x)) \ \rightarrow\ \forall z.\ p(z, z)$
(b) $\forall x, y.\ p(x, y) \ \rightarrow\ p(y, x) \ \rightarrow\ \forall z.\ p(z, z)$
(e) $\exists x, y.\ (p(x, y) \ \rightarrow\ (p(y, x) \ \rightarrow\ \forall z.\ p(z, z)))$

2.4 [10 points]

2.4 (Normal forms). Put the following formulae into prenex normal form.

(c) $\forall w.\ \neg(\exists x, y.\ \forall z.\ p(x, z) \ \rightarrow\ q(y, z)) \ \wedge\ \exists z.\ p(w, z)$

3.1 [10 points each]

3.1 (Semantic argument in T_E). Use the semantic method to argue the validity of the following Σ_E-formulae, or identify a counterexample (a falsifying T_E-interpretation).

(a) $f(x, y) = f(y, x) \ \rightarrow\ f(a, y) = f(y, a)$
(c) $f(f(f(a))) = f(f(a)) \ \wedge\ f(f(f(f(a)))) = a \ \rightarrow\ f(a) = a$
(e) $p(x) \ \wedge\ f(f(x)) = x \ \wedge\ f(f(f(x))) = x \ \rightarrow\ p(f(x))$

[20 points] Use semantic argument <u>with substitution</u> to prove the validity of the formula below:

$$(\forall x.\ p(x)) \leftrightarrow (\neg \exists x.\ \neg p(x))$$