Lecture 16.5
Tying up some loose ends, and answering your questions!
Announcements

• Final exam is a week from today!
  • *Monday June 12, 3:30-6:30pm.*

• Answering a question from last time: yes, EthiCS content will be on the exam.

• The exam is technically cumulative, but with a heavy focus on the second half of the quarter (starting with graph stuff)

• Practice exam is out now! We hope that this is similar in terms of length/difficulty/format to the actual exam.
  • Disclaimer: different things are differently difficult for different people.

• Past exams to be posted soon.
How to study for exam?

• Review sessions!
  • Some have already happened, videos are available on Canvas
  • Ford-Fulkerson/Max-Flow/Min-Cut practice problem session!
    • Today (Monday June 5) 3-5pm with Robbie. (Check Ed for location updates)
  • Hashing and Universal Hash Families!
    • Tomorrow (Tuesday June 6) 3-5pm in 200-203 with Sophia.
  • Going over old exams!
    • Saturday June 10, 12-2pm AND 4-6pm in Hewlett 201 with Rishu.
  • Check Ed before you go for any location/time updates.
How to study for exam?

• Resources available (other than review sessions + OH + Ed):
  • Practice exam (and previous years’ exams) on website
  • Textbook(s): Algs. Illuminated, CLRS
  • Problem-solving guide (posted on website, under Resources)
  • Concept-check questions from previous years (on website under Resources)

• Suggestions:
  • Do as many practice problems as you can!
  • Try to come up with practice problems!
  • Try to teach a concept or practice problem solution to a friend!
  • Making your cheat sheet is also a great study opportunity!
    • And, uh, maybe print a draft of your cheat sheet out the day before just in case....
Agenda

• Answering a few lingering questions about Ford-Fulkerson
• ... I got nothing. Answering your questions!
Question from last time

• Ford-Fulkerson (using BFS to find augmenting paths) runs in time $O(nm^2)$, which might be as bad as $O(n^5)$.

• Can we do better?
Answer 1: Yes!

- Ford-Fulkerson also runs in time \( O(mf) \), where \( f \) is the max flow value, if all the capacities are integers.
  - We increase the flow at each step and start from 0, so there are at most \( f + 1 \) steps.
  - Each step takes time \( O(m) \).
Answer 1: Yes! (if the max flow is small)

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For this application, the max flow is $O(n)$ (number of swag items), so $O(mf) = O(n^3)$ is better than $O(n^5)$.
**Answer 2: Yes!** (no matter what the max flow is)

<table>
<thead>
<tr>
<th>Authors</th>
<th>Year</th>
<th>Running time</th>
<th>Fine print</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ford-Fulkerson/Edmonds-Karp/Dinitz</td>
<td>1955 (and 1970/72)</td>
<td>$O(nm^2)$</td>
<td></td>
</tr>
<tr>
<td>Goldberg-Tarjan</td>
<td>1986</td>
<td>$O(n^2m)$</td>
<td></td>
</tr>
<tr>
<td>Goldberg-Rao</td>
<td>1997</td>
<td>$\tilde{O}(\min{n^{2/3},m^{1/2}} \cdot m)$</td>
<td>There are some logarithmic factors suppressed.</td>
</tr>
<tr>
<td>Kelner, Lee, Orecchia, Sidford / Sherman</td>
<td>2013</td>
<td>$O(m^{1.001})$</td>
<td>Returns a flow that is within 0.9999 of maximum; only for undirected graphs; can make “0.001” arbitrarily close to 0.</td>
</tr>
<tr>
<td>Orlin</td>
<td>2013</td>
<td>$O(nm)$</td>
<td></td>
</tr>
<tr>
<td>Chen, Kyng, Liu, Peng, Gutenberg, Sachdeva</td>
<td>2022</td>
<td>$O(m^{1.001})$</td>
<td>Can make “0.001” arbitrarily close to 0.</td>
</tr>
</tbody>
</table>
Question

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Give this person 1 scoop of this ice cream.
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Let’s look at the residual graph!

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Who has questions?
Technical, non-technical, whatever