Pre-lecture exercises will not be collected for credit. However, you will get more out of each lecture if you do them, and they will be referenced during lecture. We recommend writing out your answers to pre-lecture exercises before class. Pre-lecture exercises usually should not take you more than 30 minutes.

In this pre-lecture exercise, you’ll get acquainted with the $k$-SELECT problem which we’ll see in Lecture 4.

Consider the problem $k$-SELECT: When given an array $A$ of $n$ distinct numbers, find the $k$'th smallest one. For example, if the input were

\[ A = [6, 4, 8, 9, 5, 2, 1], \]

then the output to 3-select should be “$A[1] = 4,” since 4 is the 3rd smallest item in this array.

1. Give an $O(n \log(n))$-time algorithm for $k$-SELECT (for any fixed $k$).
2. Give an $O(n)$-time algorithm for 1-SELECT. (That is, $k$-SELECT for $k = 1$).
3. Give an $O(n)$-time algorithm for 2-SELECT.
4. Question to ponder before Lecture 4: can you come up with an $O(n)$-time algorithm for $k$-SELECT for general $k$? (It’s okay if not—as we’ll see in Lecture 4, this is tricky!)