In this pre-lecture exercise, we will remember a little bit of probability!

1. Let $X$ be a random variable which is 1 with probability $1/100$ and 0 with probability $99/100$.

   (a) What is the expected value $E[X]$?

   (b) Suppose you draw $n$ independent random variables, $X_1, X_2, \ldots, X_n$, distributed like $X$. What is the expected value $E[\sum_{i=1}^n X_i]$?

   (c) Suppose I draw independent random variables $X_1, X_2, \ldots$ and I stop when I see the first “1”. For example, if I draw

   
   \[ X_1 = 0, X_2 = 0, X_3 = 0, X_4 = 1 \]

   then I would stop at $X_4$. Let $N$ be the last index that we draw. (So in the previous example, $N = 4$). How big do you expect $N$ to be?

   \[ \text{Note: actually figuring out } E[N] \text{ from scratch is a bit tricky, although you may have seen it in CS109. But even if you don’t do it rigorously, intuitively how big do you expect } N \text{ to be?} \]

2. Consider the following pseudocode, which is an in-place sorting algorithm for an array $A$.

   ```python
   def bogosort(A):
       while A is not sorted:
           A.shuffle() # this randomly permutes A
       return A
   ```

   (a) Let $X_i$ be a random variable which is 1 if $A.shuffle()$ is sorted after the $i$'th call, and 0 otherwise.

   (b) What is $E[X_i]$?

   (c) What is the expected number of times that `bogosort` executes the `while` loop?