1 Grade-school multiplication

Suppose we multiply two \( n \)-digit integers \((x_1x_2\ldots x_n)\) and \((y_1y_2\ldots y_n)\) using the grade-school multiplication algorithm. How many pairs of digits \(x_i\) and \(y_j\) get multiplied in this algorithm?

- \(n^3\)
- \(2n - 1\)
- \(n^2\)

Correct

What is the smallest exponent \(x\) such that the number of one-digit operations in grade-school multiplication is always at most \(10000 \cdot n^x\)?

\[2\]

Correct

2 Divide-and-conquer multiplication

Suppose that we have a divide-and-conquer algorithm \(A\) that multiplies two \( n \)-digit integers by recursively calling itself to perform \( t \) number of \( \lceil n/2 \rceil \)-digit integer multiplications; when \( n \leq 1 \), it performs single-digit multiplication.

If \( t = 4 \), what is the smallest exponent \(x\) such that the number of one-digit multiplications is always at most \(10000 \cdot n^x\)?

\[2\]

Correct

For what values of \( t \) does the algorithm perform fewer one-digit multiplications than the grade-school multiplication algorithm for inputs that have \( n > 10000 \) digits?

- For all values of \( t \)
- \( t = 1, 2 \)
- \( t = 1, 2, 3 \)
- \( t = 1, 2, 3, 4 \)

Correct

What is the value of \( t \) for Karatsuba integer multiplication algorithm?

\[3\]

Correct