Fair Division

You have a cake sliced into \( n \) differently sized pieces. You need to split it among \( k \) students in your class where \( k \leq n \) and you want to give each student exactly 1 piece of cake. However, you know that since the pieces are differently sized, the students who end up with smaller pieces will most likely complain about unfairness. You want to minimize the complaints by minimizing unfairness. If you select \( k \) cake pieces of size \( S_1, \ldots, S_k \) then unfairness is defined as \( \max(S_1, \ldots, S_k) - \min(S_1, \ldots, S_k) \).

(a) Describe a very simple algorithm to maximize unfairness.

(b) Describe an algorithm to minimize unfairness.

Restricted MST

Given an undirected weighted graph \( G = (V, E) \), we have a set \( U \subset V \). We wish to find a minimum spanning tree such that all nodes in \( U \) are leaf nodes. (The result may not be a MST of the original graph \( G \).)

Pareto Points

Given a set of 2d points \( P \), a Pareto optimal point is a point \((x, y)\) such that \( x > x' \) or \( y > y' \), \( \forall (x', y') \neq (x, y) \in P \). Develop an algorithm to find all Pareto optimal points.