Exercise 0
Suppose you are investing. You want to buy low, then sell high. You have an array $A$ of $n$ integers representing future prices, and can make one buy followed by one sell. Regardless of the price at which you buy, you will only purchase a single unit. What is the maximum profit you can make on this investment?

(a) Design an $O(n \log n)$ divide-and-conquer algorithm to solve this problem, and justify its runtime.
(b) Design an $O(n)$ algorithm to solve this problem, and justify its runtime.

Exercise 1
A string is a palindrome if it is the same both forwards and backwards. For example, “kayak” is a palindrome, but “canoe” is not. Similarly, “aa” is a palindrome, but “abaa” is not. (”a” is also a palindrome.)

A subsequence of a string is any sequence of characters that can be derived from the original string by deleting characters from that string. For example, the subsequences of the string “aid” are “aid”, “ai”, “ad”, “a”, “id”, “i”, “d”, and “” (the empty string).

Design an algorithm that takes a string $S$ and returns the length of the longest subsequence that is a palindrome. Analyze the runtime of your algorithm.

Exercise 2
Suppose you want to start a petting zoo, and you’ve identified $n$ animals you’d like to buy, each of which currently costs $100. However, it takes time to set up each exhibit, so you can only buy one animal each month. Additionally, for each month you wait to buy animal $i$, its price goes up by a factor of $r_i > 1$ due to inflation, so if you buy animal $i$ in month $m$ it will cost $100 \times r_i^m$.

Design an algorithm which takes in the inflation rates $r_i$ and returns the order in which you should buy animals to minimize your total cost, and prove that it is correct.

Exercise 3
Suppose that you’re playing a simplified version of minesweeper where there are $n$ grid squares, and only one of them contains a mine. When you click on a square, it either reveals an empty slot, or reveals the mine and explodes. (No additional information is revealed, e.g., there are no numbers telling you how many adjacent slots contain mines, etc.) If you explode, you start over with a new board of size $n$ and worst-case location of the mine. You decide to use a randomized algorithm—choosing uniformly at random from the remaining squares—to make your decisions about which square to click next. The game ends when you reveal all $n - 1$ safe slots on a single board without exploding.

(a) What is the probability of succeeding in one run without ever exploding?
(b) What is the expected number of squares visited until you win?
(c) How does your answer to (b) change if, after you explode, you are allowed to restart using the randomized algorithm at the point right before you exploded (e.g. if you explode with 10 slots (9 safe and 1 mine) left, your next move will be chosen uniformly at random from the same 10 slots)?
(d) Advanced (Take Home) - What if you only find out at the end of the game whether you exploded or succeeded, but if you exploded you can restart from any point in history?