Encoding

Suppose we encode lowercase letters into a numeric string as follows: we encode a as 1, b as 2 . . . and z as 26. Given a numeric string $S$ of length $n$, develop an $O(n)$ algorithm to find how many letter strings this can correspond to.

Dice Probabilities

We wish to find the probability that rolling $k$ 6-sided fair dice will result in a sum $S$. Devise an algorithm to find this probability.

Knight Moves

Given an $8 \times 8$ chessboard and a knight that starts at position $a1$, devise an algorithm that returns how many ways the knight can end up at position $xy$ after $k$ moves. Knights move $\pm 1$ squares in one direction and $\pm 2$ squares in the other direction.

Rod Cutting

Suppose we have a rod of length $k$, where $k$ is a positive integer. We would like to cut the rod into integer-length segments such that we maximize the product of the resulting segments’ lengths. Multiple cuts may be made. Write an algorithm to determine the maximum product possible.

Egg Dropping

We have $e$ eggs and $f$ floors. We want to find the minimum number of drops needed in the worst case to find the highest floor in which an egg will not break after dropping. Assume that all of the eggs are the same strength; if one egg breaks after dropping from a floor, all eggs will break after dropping from that floor. Design an algorithm that returns the minimum number of drops needed to accomplish this task in the worst case, without actually dropping any eggs.