Assignment #4

Due: Thursday, November 8

Problem 1 – Tracing Turing machine execution
Solve the problems in the puzzle box on page 162 of the reader.

Problem 2 – Calculating remainders
Write the program for a Turing machine $M_{\%3}$ that computes the remainder of its input when divided by 3. Given, for example, an input tape containing the number 8:

```
... 0 1 1 1 1 1 1 1 0 ...
```

executing $M_{\%3}$ should leave the number 2 on the tape, because 2 is the remainder of 8 when divided by 3.

```
... 0 1 1 0 ...
```

Note that your program must leave the tape head at the beginning of the remainder.

Problem 3 – Duplicating a number
Implement a Turing machine $M_{\text{copy}}$ that copies an input value on the tape, leaving two identical values on the output separated by a single 0. Thus, if the input tape is:

```
... 0 1 1 1 0 0 0 0 0 ...
```

the final configuration should be

```
... 0 1 1 1 0 1 1 1 0 0 ...
```
Problem 4 – the Busy Beaver problem

The solution for the BB(4) problem, which produces thirteen 1s, is included below. To get a feel for how challenging this problem is, write a four-state Turing machine that produces at least eight but no more than twelve 1s before stopping. It will probably help to look at the solution to BB(3) for ideas.

BB(4) solution:

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1R2</td>
<td>1L2</td>
</tr>
<tr>
<td>2</td>
<td>1L1</td>
<td>0L3</td>
</tr>
<tr>
<td>3</td>
<td>1L0</td>
<td>1L4</td>
</tr>
<tr>
<td>4</td>
<td>1R4</td>
<td>0R1</td>
</tr>
</tbody>
</table>

BB(3) solution:

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1R2</td>
<td>1L0</td>
</tr>
<tr>
<td>2</td>
<td>0R3</td>
<td>1R2</td>
</tr>
<tr>
<td>3</td>
<td>1L3</td>
<td>1L1</td>
</tr>
</tbody>
</table>