1. In the basic linear temporal logic, show that for any given frame, $\mathcal{F} \models PF\varphi \rightarrow (P\varphi \lor \varphi \lor F\varphi)$, if and only if $\mathcal{F}$ is linear in the future, that is, if $x < y$ and $x < z$, then either $y < z$, $z < y$, or $y = z$.

2. (a) Write a program in the program language of Propositional Dynamic Logic (PDL) that means $\text{Do } a \text{ until } \varphi$.

(b) Show that the following two programs are equivalent: $(a \cup b)^*$ and $(a^*b)^*a^*$.

3. Using the “induction axiom” for the Kleene star in PDL, that is, the formula:

$$\varphi \land [a^*](\varphi \rightarrow [a]\varphi) \rightarrow [a^*]\varphi,$$

show that the following rule is admissible in PDL:

From $\varphi \rightarrow [a]\varphi$, infer $\varphi \rightarrow [a^*]\varphi$.

4. Show that in the Public Announcement Logic (PAL) that the following holds: $\models [\varphi \land [\varphi]\psi] \chi \rightarrow [\varphi][\psi]\chi$.

5. A formula in PAL is successful just in case $\models [\varphi]\varphi$.

   (a) Show the successful formulas are not closed under negation. (That is, show there is a successful formula $\varphi$ such that $\neg\varphi$ is not successful.)

   (b) Show there are successful formulas $\psi$ and $\chi$ such that $[\psi]\chi$ is not successful.

6. **Bonus:** (+10) Give an example of two successful formulas $\psi$ and $\chi$ such that $\psi \lor \chi$ is not successful.