CS 224S / LINGUIST 285
Spoken Language Processing

Andrew Maas

Stanford University

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Lecture 8: ASR: Noisy Channel, HMMs, Evaluation

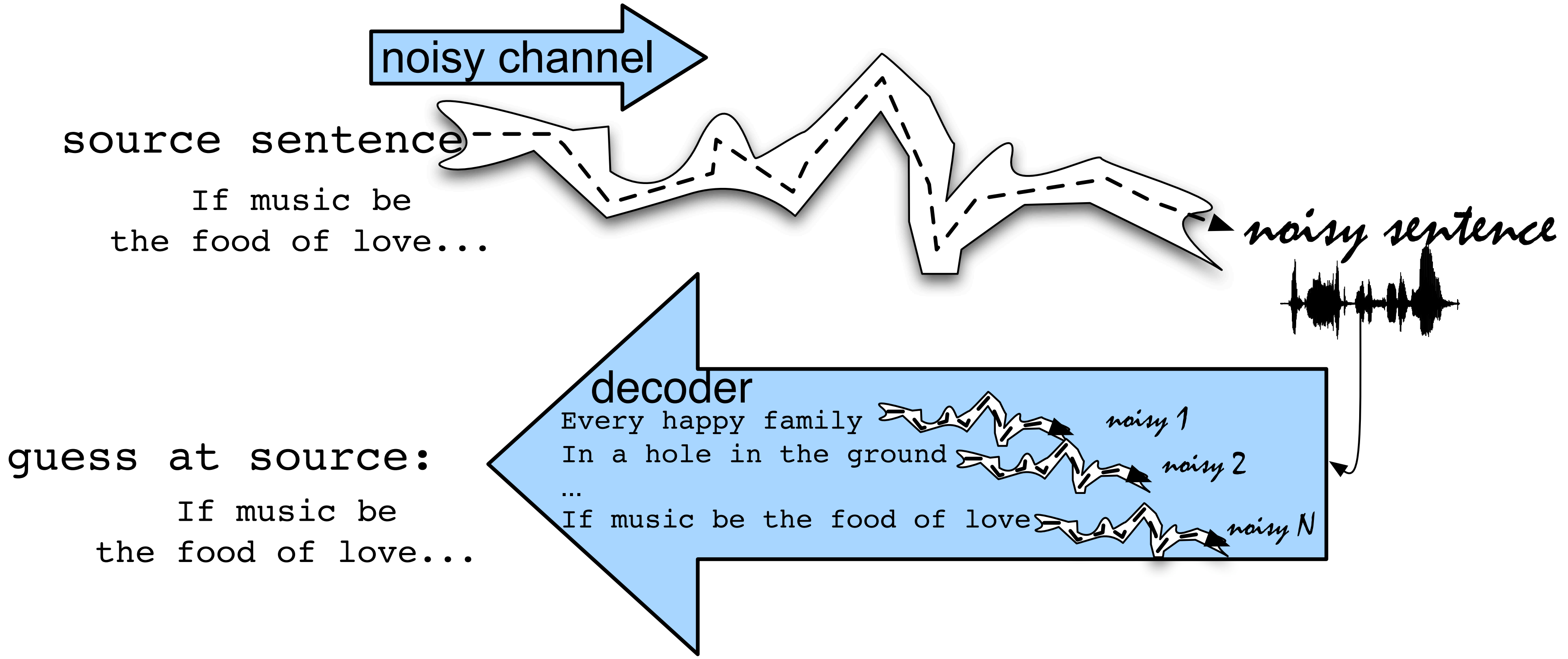
Original slides by Dan Jurafsky

Outline for Today

- ASR Architecture
- Hidden Markov Model (HMM) introduction
- HMMs for speech
- Evaluation with word error rate

The Noisy Channel Model

- Search through space of all possible sentences.
- Pick the one that is most probable given the waveform.



The Noisy Channel Model

- What is the most likely sentence out of all sentences in the language L given some acoustic input O?
- Treat acoustic input O as sequence of individual observations
  - $O = o_1, o_2, o_3, \ldots, o_t$
- Define a sentence as a sequence of words:
  - $W = w_1, w_2, w_3, \ldots, w_n$

Noisy Channel Model

- Probabilistic implication: Pick the highest prob word sequence W:

$$\hat{W} = \underset{W \in L}{\arg\max}\, P(W \mid O)$$

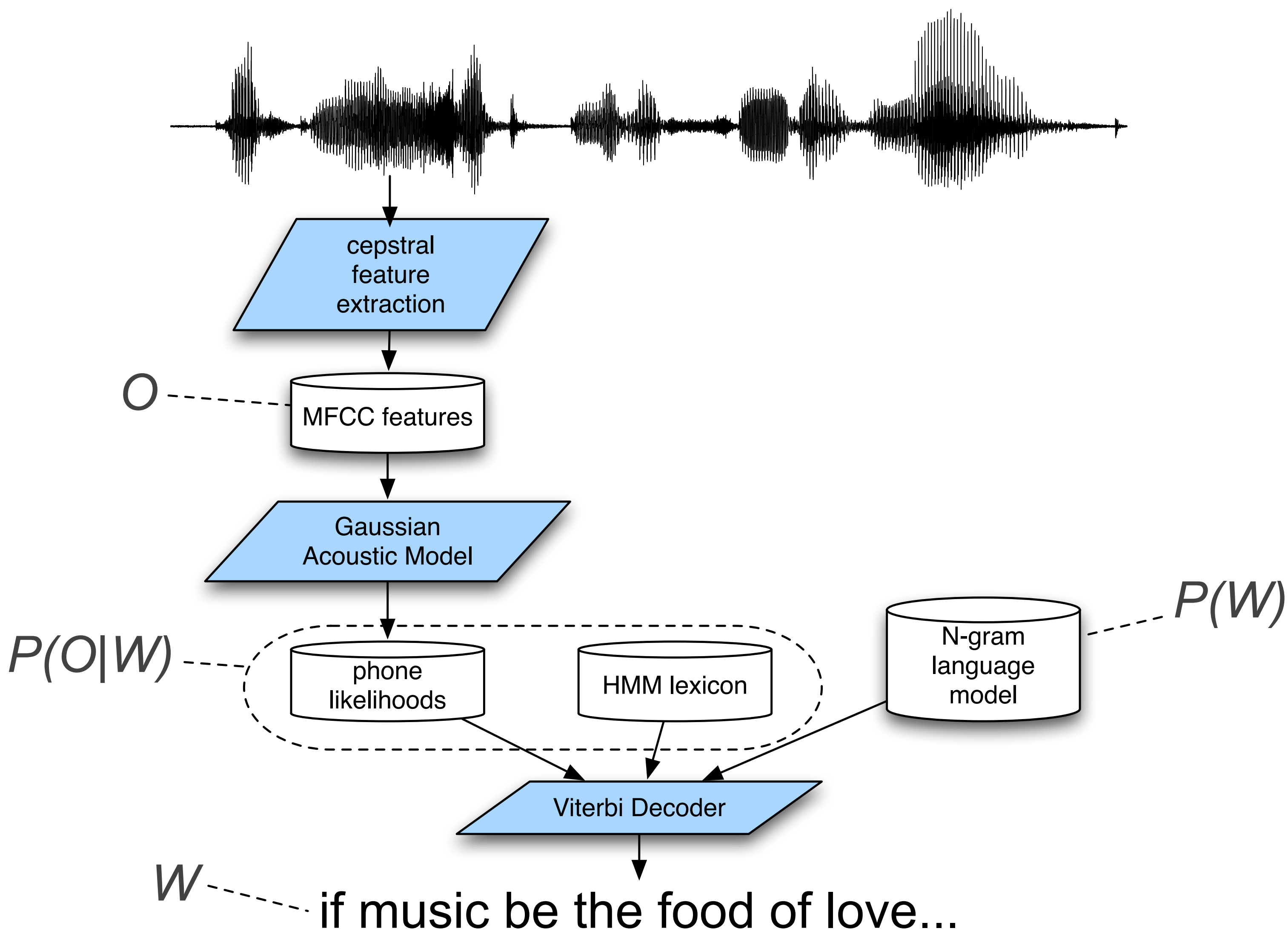
- We can use Bayes rule to rewrite this:

$$\hat{W} = \underset{W \in L}{\arg\max}\, \frac{P(O \mid W)P(W)}{P(O)}$$

- Since denominator is the same for each candidate sentence W, we can ignore it for the argmax:

$$\hat{W} = \underset{W \in L}{\arg\max}\, P(O \mid W)P(W)$$

Speech Recognition Architecture

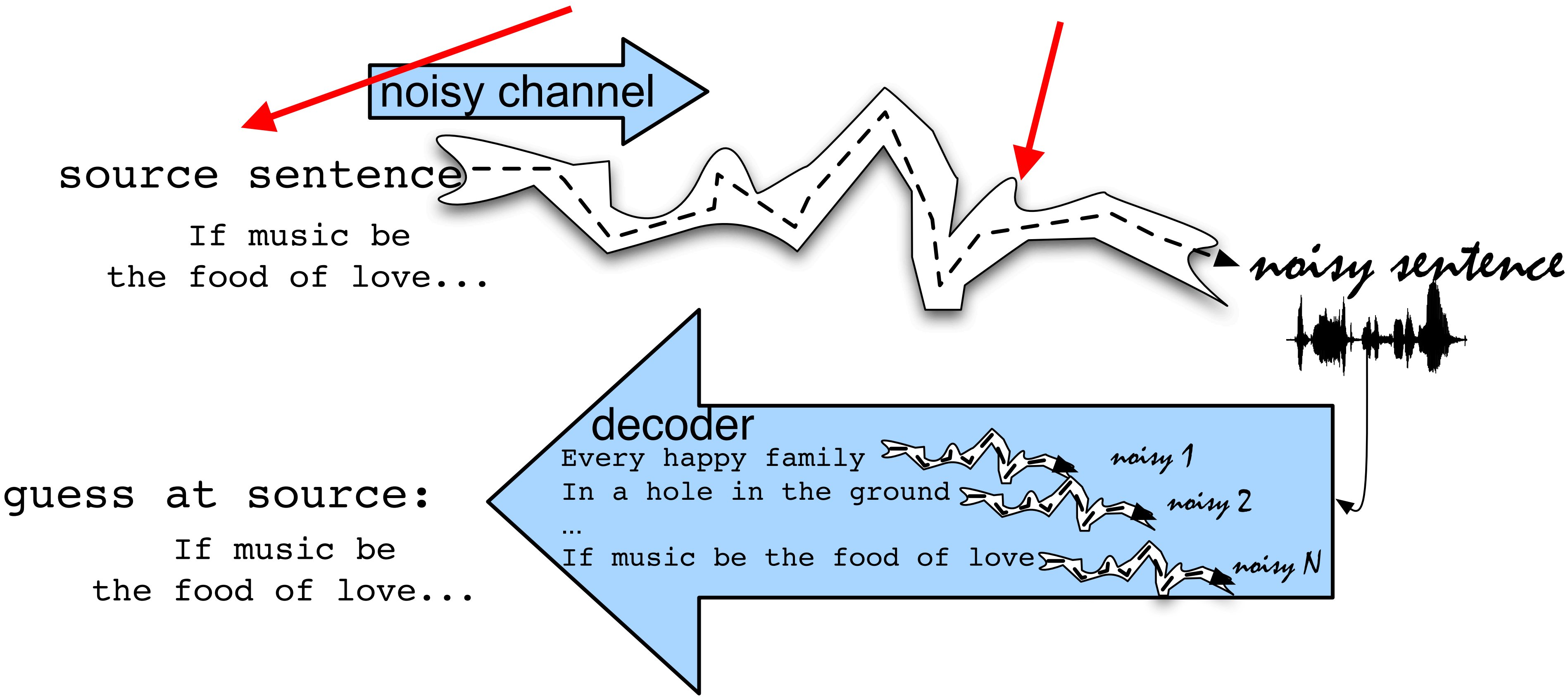


Noisy channel model

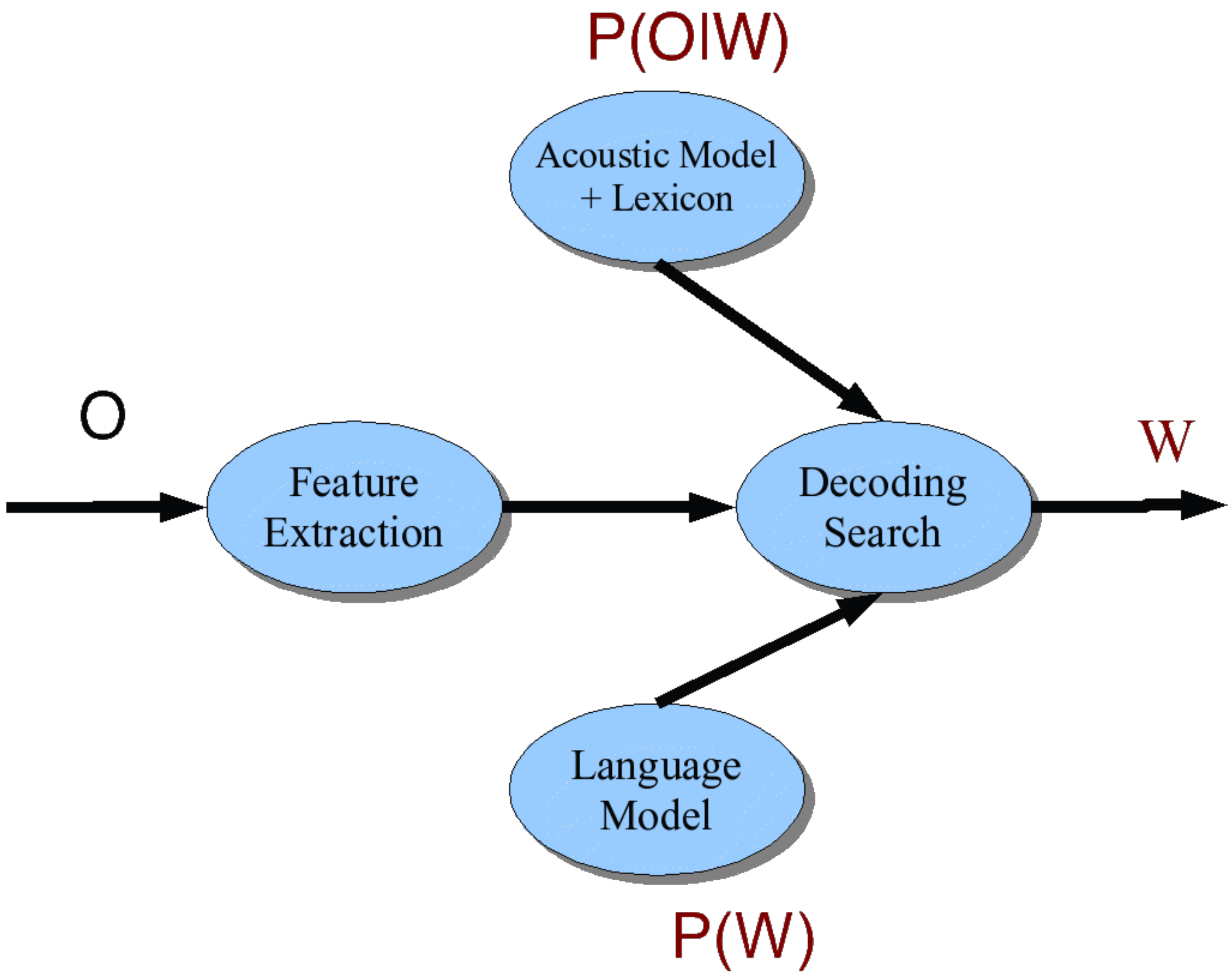
$$\hat{W} = \underset{W \in L}{\arg\max}\ \overset{\text{likelihood}}{P(O \mid W)}\ \overset{\text{prior}}{P(W)}$$

The noisy channel model

Ignoring the denominator leaves us with two factors: P(Source) and P(Signal|Source)



Speech Architecture meets Noisy Channel



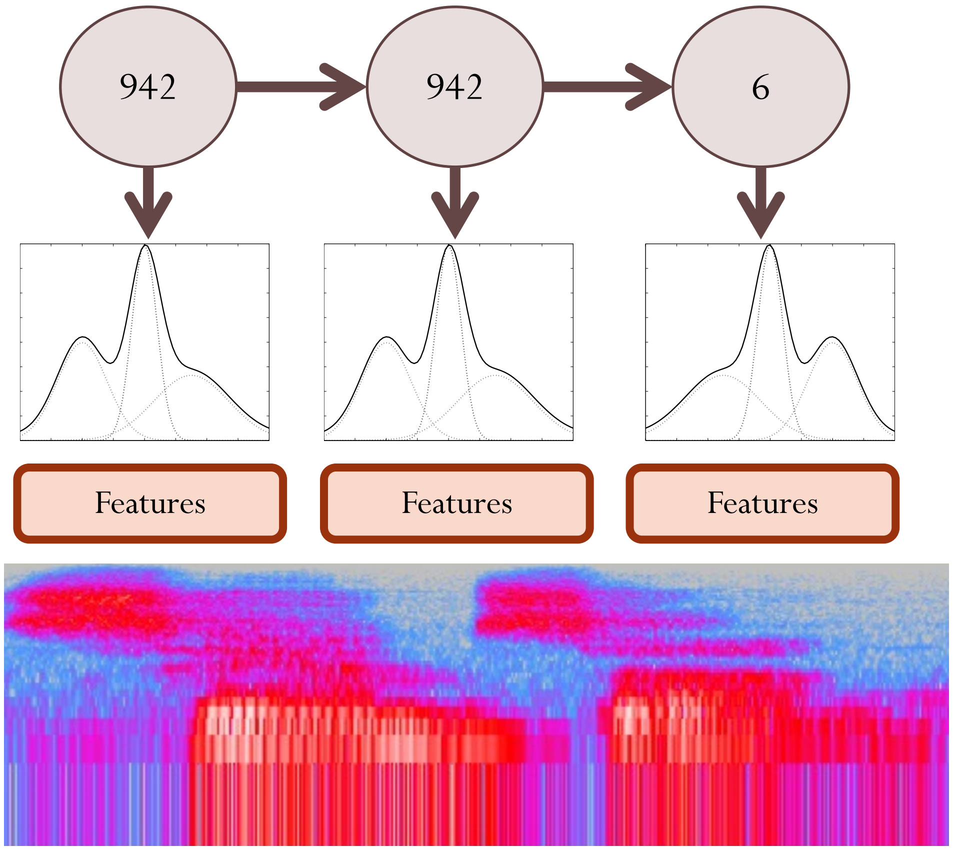
HMM-GMM System

Transcription: Samson
Pronunciation: S – AE – M – S –AH – N
Sub-phones : 942 – 6 – 37 – 8006 – 4422 …

Hidden Markov Model (HMM):

Acoustic Model:

Audio Input:



GMM models:
P(o|q)
o: input features
q: HMM state

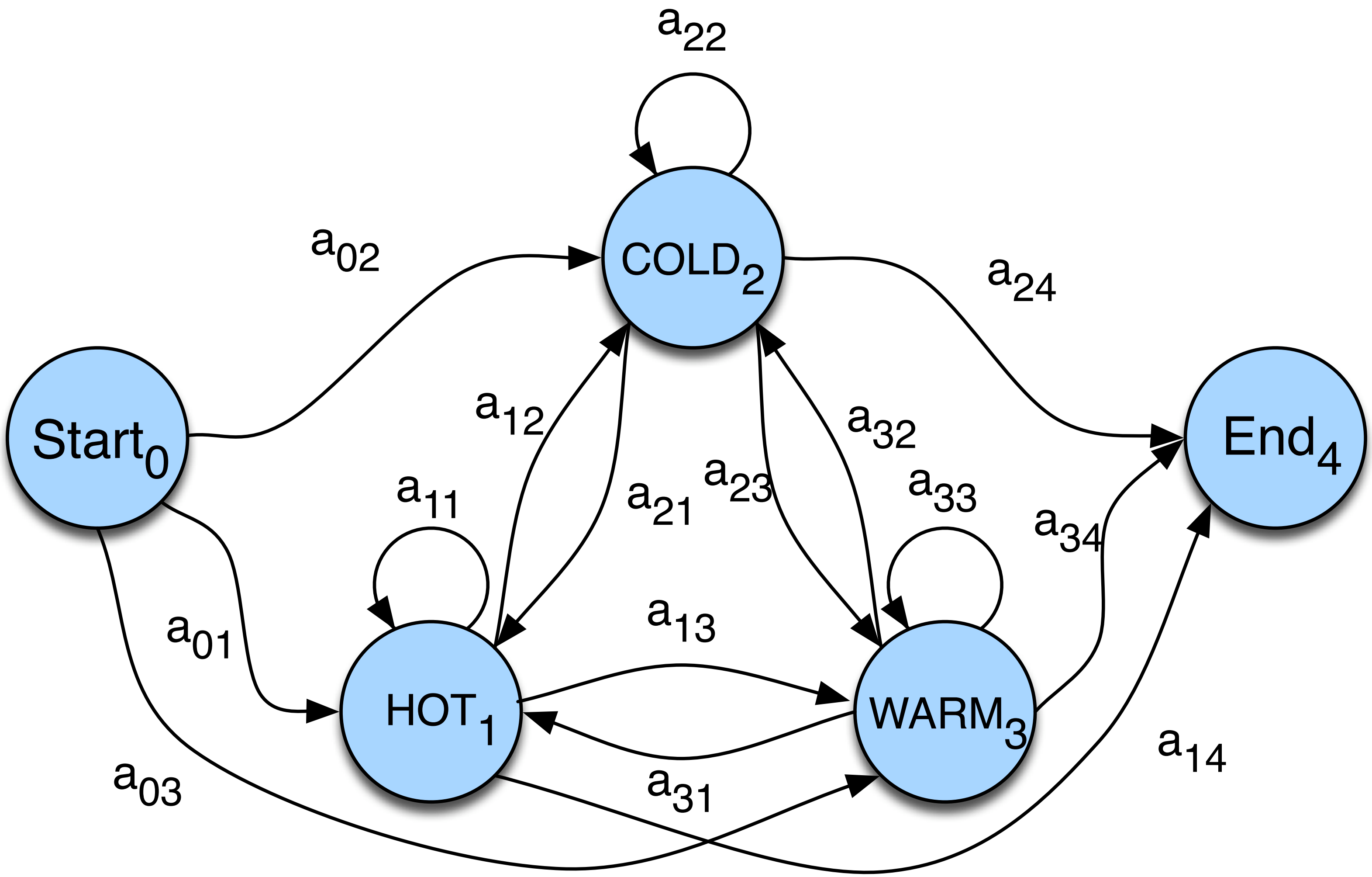
HMM-GMM decoding architecture: five easy pieces

- Feature Extraction:
  - 39 MFCC features
- Acoustic Model:
  - Gaussians for computing $p(o|q)$
- Lexicon/Pronunciation Model
  - HMM: what phones can follow each other
- Language Model
  - N-grams for computing $p(w_i|w_{i-1})$
- Decoder
  - Viterbi algorithm: dynamic programming for combining all these to get word sequence from speech

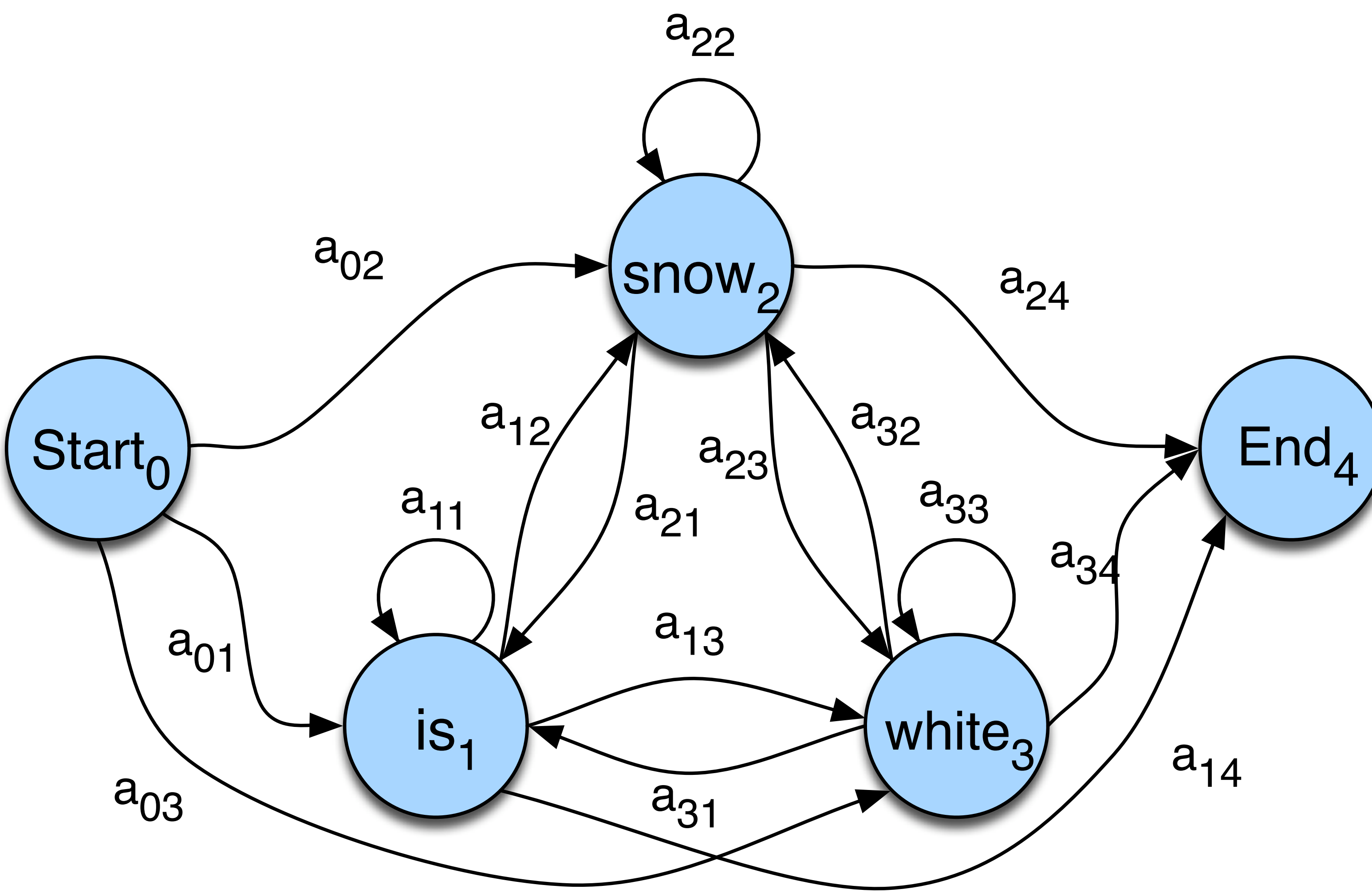
Lexicon

- A list of words
- Each one with a pronunciation in terms of phones
- We get these from an existing pronunciation dictionary
  - Default academic resource: CMU dictionary: 127K words
- We represent the lexicon as an HMM

Markov chain for weather



Markov chain for words



Markov chain = First-order observable Markov Model

- a set of states
  - $Q = q_1, q_2 \ldots q_{N;}$ the state at time t is q_t
- Transition probabilities:
  - a set of probabilities $A = a_{01}a_{02}\ldots a_{n1}\ldots a_{nn}$.
  - Each a_{ij} represents the probability of transitioning from state i to state j
  - The set of these is the transition probability matrix A

$$a_{ij} = P(q_t = j \mid q_{t-1} = i) \quad 1 \leq i, j \leq N$$

$$\sum_{j=1}^{N} a_{ij} = 1; \quad 1 \leq i \leq N$$

- Distinguished start and end states

Markov chain = First-order observable Markov Model

Current state only depends on previous state

$$\text{Markov Assumption}: \quad P(q_i \mid q_1 \cdots q_{i-1}) = P(q_i \mid q_{i-1})$$

Another representation for start state

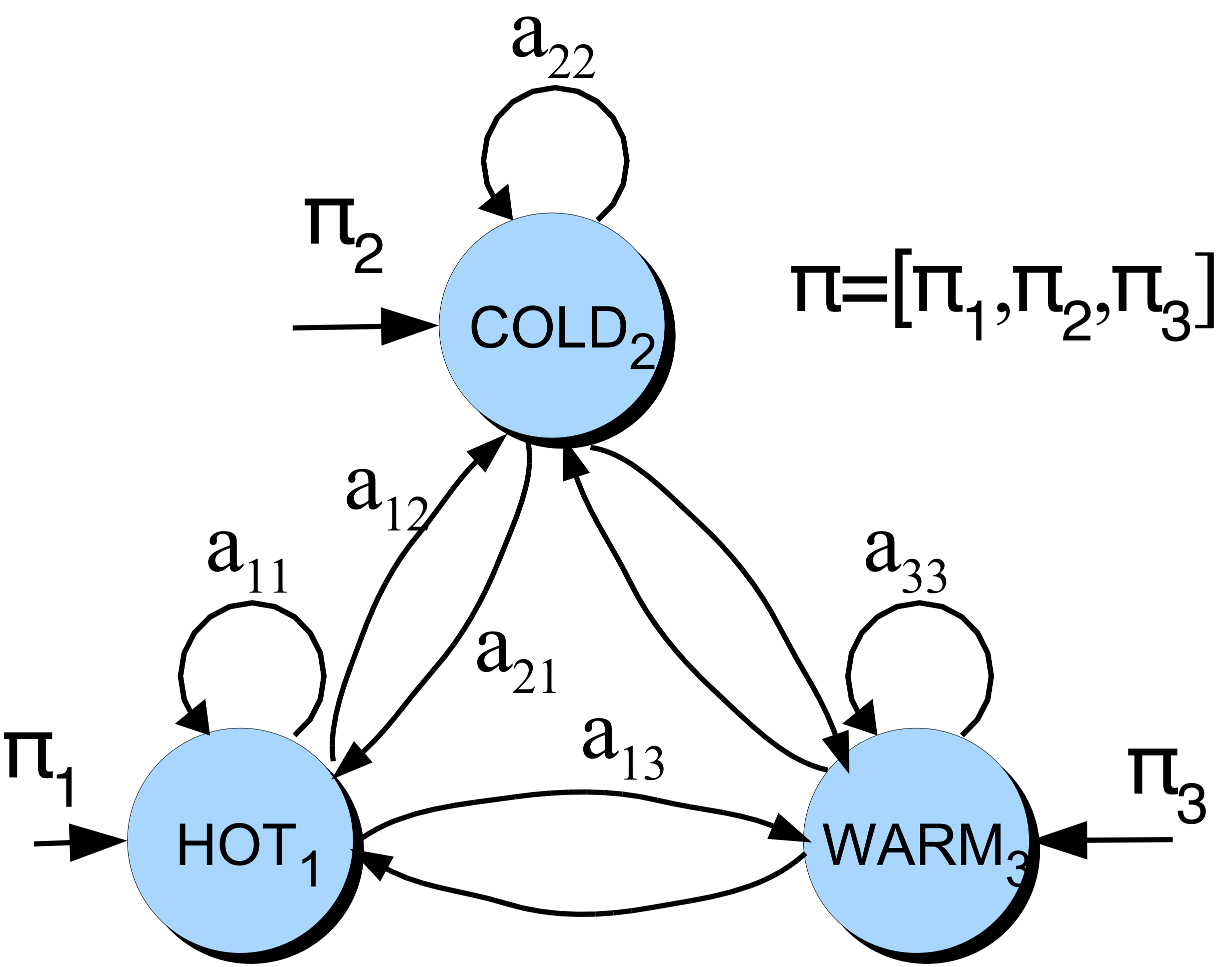
- Instead of start state
- Special initial probability vector π
  - An initial distribution over probability of start states

$$\pi_i = P(q_1 = i) \quad 1 \leq i \leq N$$

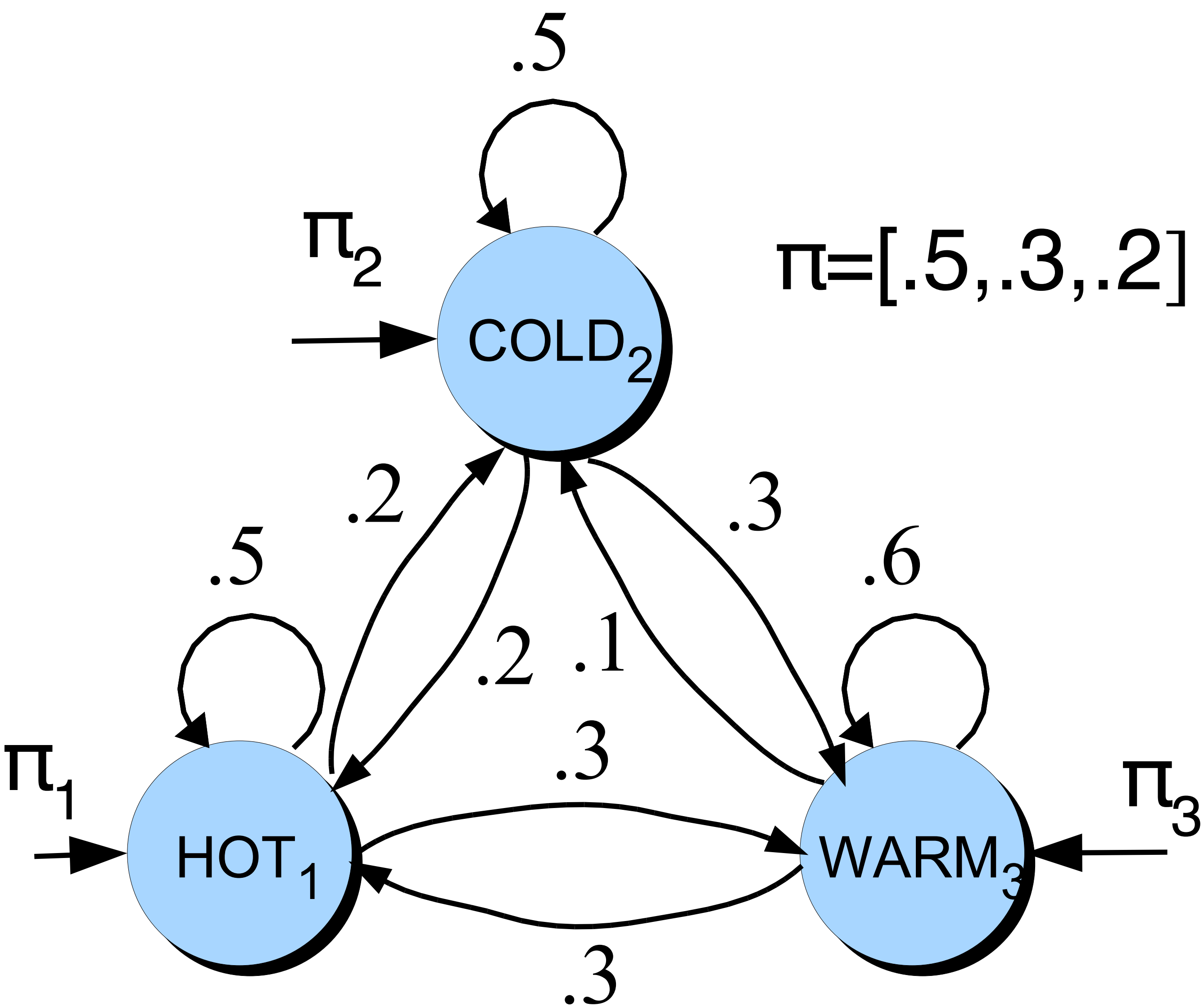
- Constraints:

$$\sum_{j=1}^{N} \pi_j = 1$$

The weather figure using pi



The weather figure: specific example



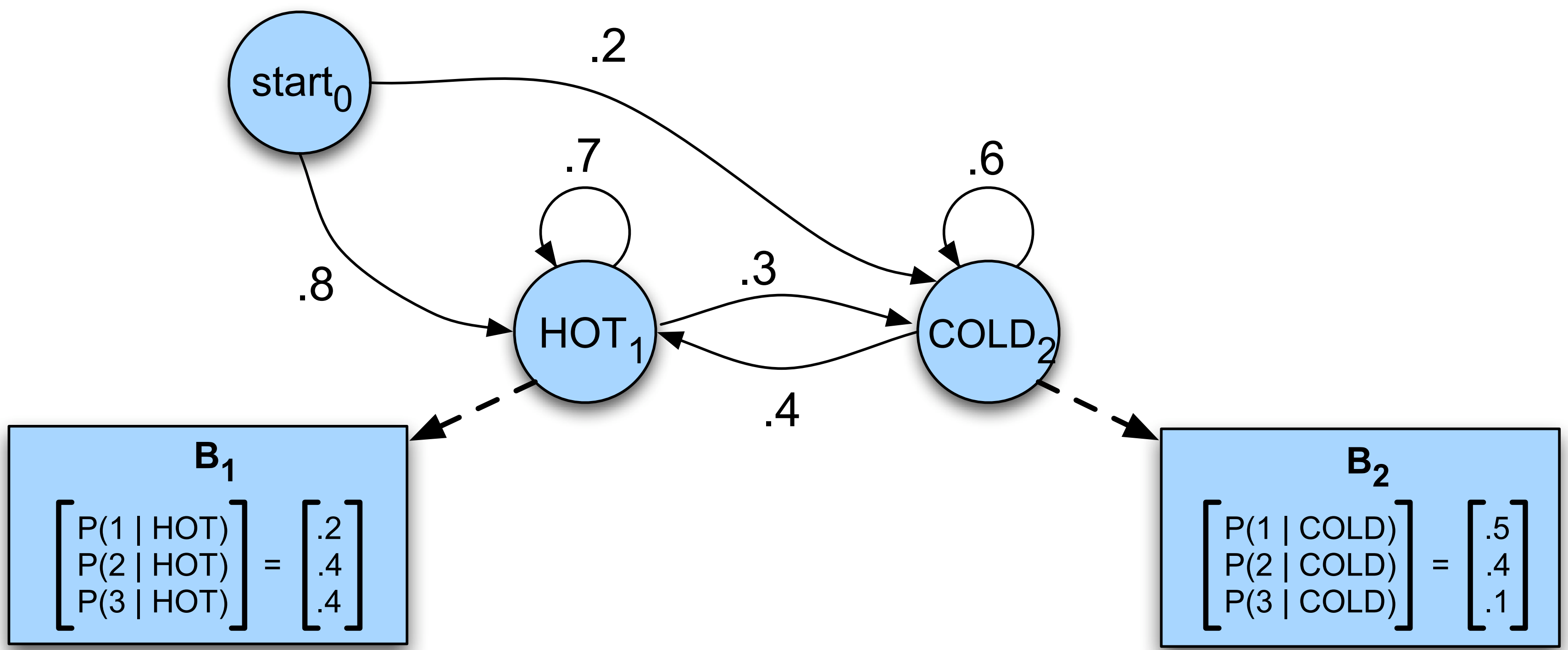
Hidden Markov Model

- For Markov chains, output symbols = state symbols
  - See **hot** weather: we're in state **hot**
- But not in speech recognition
  - Output symbols: vectors of acoustics (**cepstral features)**
  - Hidden states: **phones**
- So we need an extension!
- A Hidden Markov Model is an extension of a Markov chain in which the input symbols are not the same as the states.
- This means we don't know which state we are in.

HMM for Ice Cream

- You are a climatologist in the year 2799
- Studying global warming
- You can't find any records of the weather in Baltimore, MD for summer of 2008
- But you find Jason Eisner's diary
- Which lists how many ice-creams Jason ate every date that summer
- Our job: figure out how hot it was

HMM for ice cream



Hidden Markov Models

$Q = q_1 q_2 \ldots q_N$	a set of N **states**
$A = a_{11} a_{12} \ldots a_{n1} \ldots a_{nn}$	a **transition probability matrix** A, each a_{ij} representing the probability of moving from state i to state j, s.t. $\sum_{j=1}^{n} a_{ij} = 1 \quad \forall i$
$O = o_1 o_2 \ldots o_T$	a sequence of T **observations**, each one drawn from a vocabulary $V = v_1, v_2, ..., v_V$
$B = b_i(o_t)$	a sequence of **observation likelihoods**, also called **emission probabilities**, each expressing the probability of an observation o_t being generated from a state i
q_0, q_F	a special **start state** and **end (final) state** that are not associated with observations, together with transition probabilities $a_{01} a_{02} \ldots a_{0n}$ out of the start state and $a_{1F} a_{2F} \ldots a_{nF}$ into the end state

Assumptions

- **Markov assumption:**

$$P(q_i \mid q_1 \cdots q_{i-1}) = P(q_i \mid q_{i-1})$$

- **Output-independence assumption**

$$P(o_t \mid O_1^{t-1}, q_1^t) = P(o_t \mid q_t)$$

The Three Basic Problems for HMMs

Jack Ferguson at IDA in the 1960s

Problem 1 (**Evaluation**): Given the observation sequence $O=(o_1 o_2 \ldots o_T)$, and an HMM model $\Phi = (A,B)$, how do we efficiently compute $P(O|\Phi)$, the probability of the observation sequence, given the model?

Problem 2 (**Decoding**): Given the observation sequence $O=(o_1 o_2 \ldots o_T)$, and an HMM model $\Phi = (A,B)$, how do we choose a corresponding state sequence $Q=(q_1 q_2 \ldots q_T)$ that is optimal in some sense (i.e., best explains the observations)?

Problem 3 (**Learning**): How do we adjust the model parameters $\Phi = (A,B)$ to maximize $P(O|\Phi)$?

Decoding

- Given an observation sequence
  - 3 1 3
- And an HMM
- The task of the **decoder**
  - To find the best **hidden** state sequence
- Given the observation sequence $O=(o_1 o_2 \ldots o_T)$, and an HMM model $\Phi = (A,B)$, how do we choose a corresponding state sequence $Q=(q_1 q_2 \ldots q_T)$ that is optimal in some sense (i.e., best explains the observations)

HMM for ice cream Eisner task

Given

Observed Ice Cream Sequence:

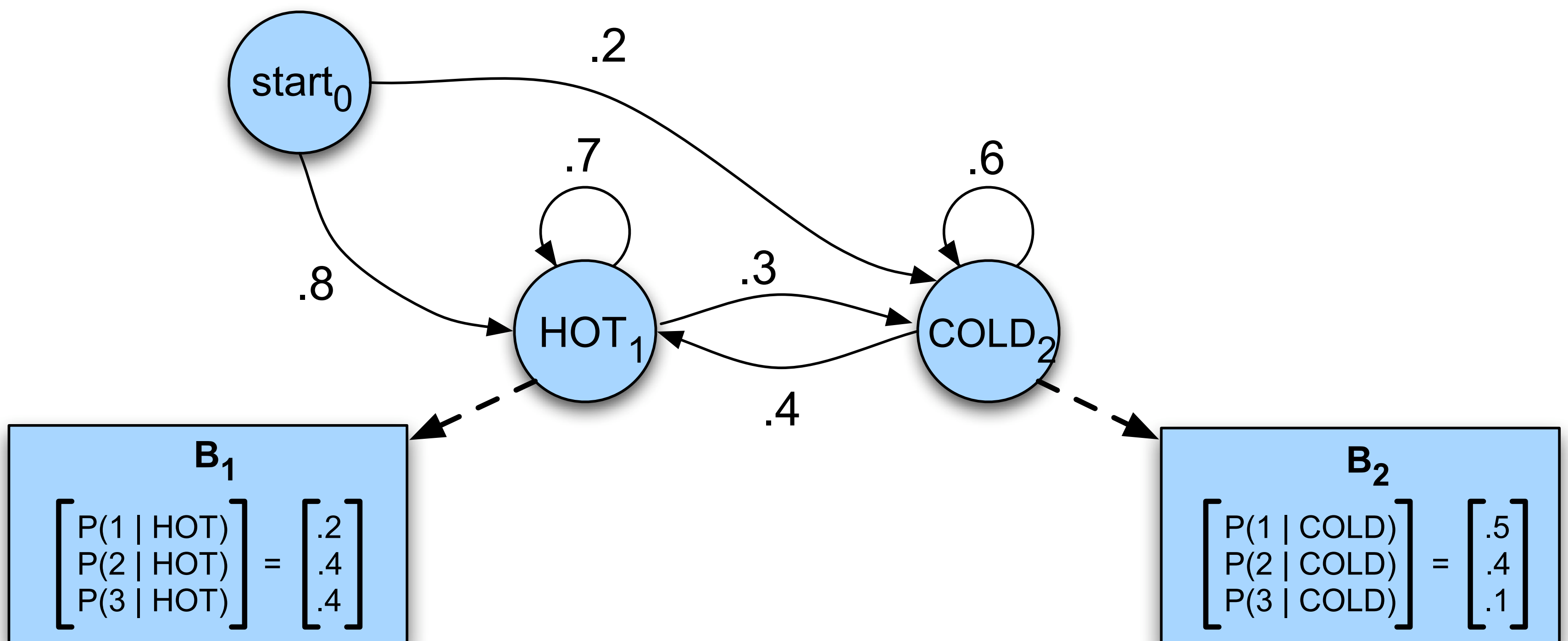
1,2,3,2,2,2,3…

Produce:

Hidden Weather Sequence:

H,C,H,H,H,C…

HMM for ice cream



Decoding

- One possibility:
  - For each hidden state sequence Q
    - HHH, HHC, HCH,
  - Compute P(O|Q)
  - Pick the highest one
- Why not?
  - N^T
- Instead:
  - The Viterbi algorithm
  - Is a **dynamic programming** algorithm
  - Uses a similar trellis to the Forward algorithm

Viterbi intuition

- We want to compute the joint probability of the observation sequence together with the best state sequence

$$v_t(j) = \max_{q_0,q_1,\ldots,q_{t-1}} P(q_0,q_1\ldots q_{t-1},o_1,o_2\ldots o_t,q_t = j|\lambda)$$

$$v_t(j) \;=\; \max_{i=1}^{N} v_{t-1}(i)\, a_{ij}\, b_j(o_t)$$

Viterbi Recursion

1. **Initialization:**

$$\begin{aligned} v_1(j) &= a_{0j} b_j(o_1) \;\; 1 \le j \le N \\ bt_1(j) &= 0 \end{aligned}$$

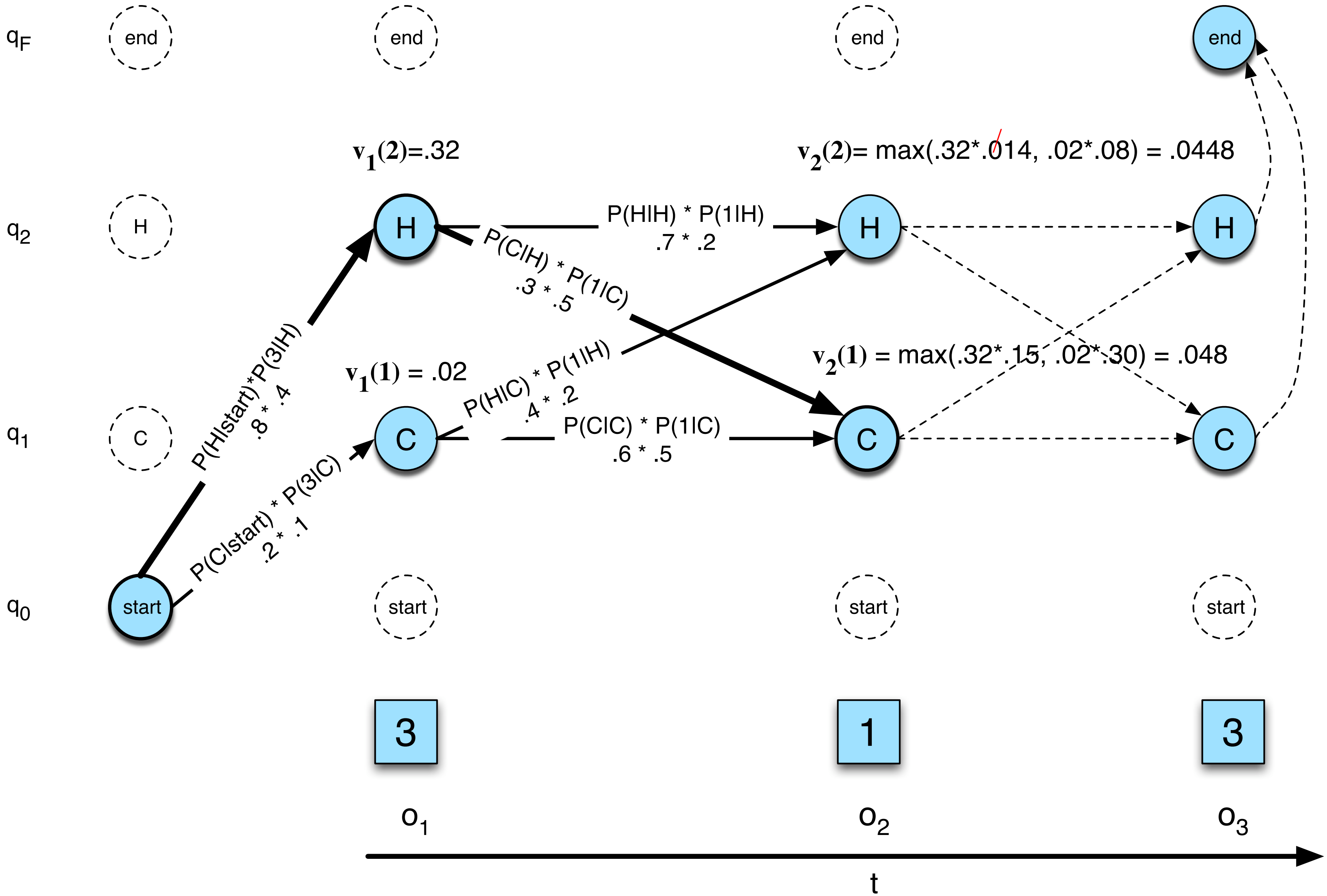
2. **Recursion** (recall that states 0 and q_F are non-emitting):

$$\begin{aligned} v_t(j) &= \max_{i=1}^{N} v_{t-1}(i)\, a_{ij}\, b_j(o_t); \quad 1 \le j \le N, 1 < t \le T \\ bt_t(j) &= \operatorname*{argmax}_{i=1}^{N} v_{t-1}(i)\, a_{ij}\, b_j(o_t); \quad 1 \le j \le N, 1 < t \le T \end{aligned}$$

3. **Termination:**

$$\begin{aligned} \text{The best score:} \quad P* = v_t(q_F) &= \max_{i=1}^{N} v_T(i) * a_{i,F} \\ \text{The start of backtrace:} \quad q_T* = bt_T(q_F) &= \operatorname*{argmax}_{i=1}^{N} v_T(i) * a_{i,F} \end{aligned}$$

The Viterbi trellis



Viterbi intuition

- Process observation sequence left to right
- Filling out the trellis
- Each cell:

$$v_t(j) = \max_{q_0,q_1,\ldots,q_{t-1}} P(q_0,q_1\ldots q_{t-1},o_1,o_2\ldots o_t,q_t=j|\lambda)$$

$$v_t(j) \ = \ \max_{i=1}^{N} v_{t-1}(i)\, a_{ij}\, b_j(o_t)$$

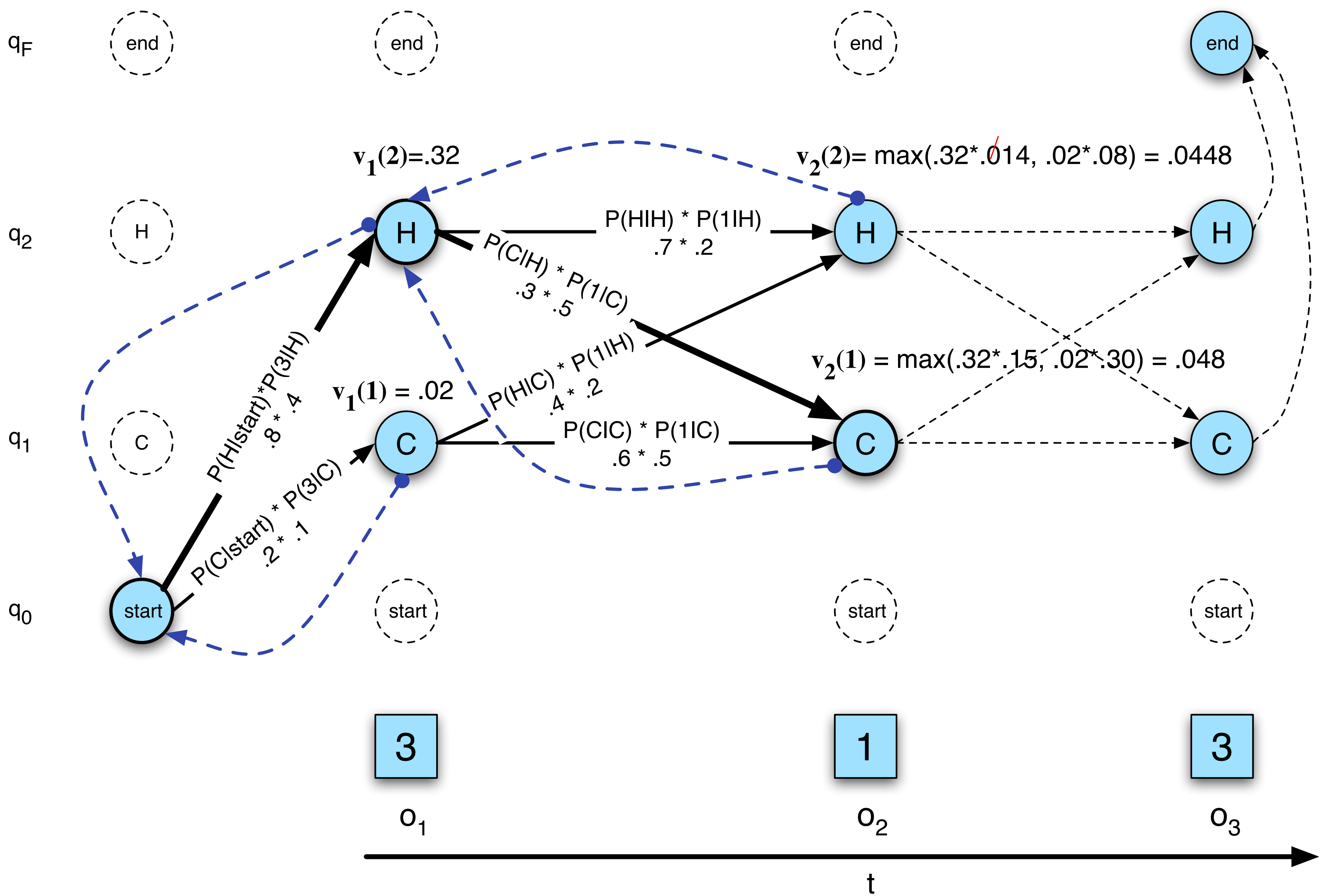
$v_{t-1}(i)$	the **previous Viterbi path probability** from the previous time step
a_{ij}	the **transition probability** from previous state q_i to current state q_j
$b_j(o_t)$	the **state observation likelihood** of the observation symbol o_t given the current state j

Viterbi Algorithm

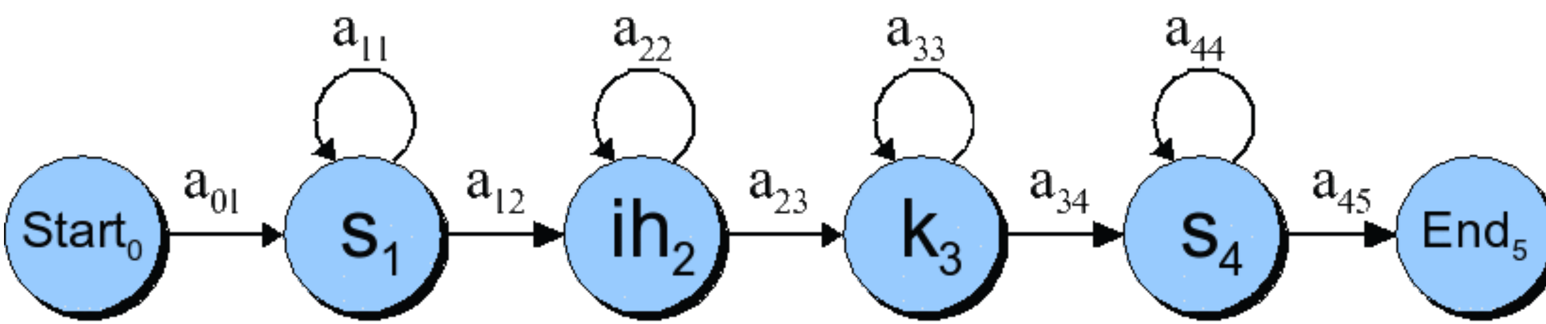
function VITERBI(*observations* of len T, *state-graph* of len N) **returns** *best-path*

create a path probability matrix *viterbi[N+2,T]*
for each state s **from** 1 **to** N **do** ; initialization step
$\quad viterbi[s,1] \leftarrow a_{0,s} * b_s(o_1)$
$\quad backpointer[s,1] \leftarrow 0$
for each time step t **from** 2 **to** T **do** ; recursion step
$\quad$ **for** each state s **from** 1 **to** N **do**
$$viterbi[s,t] \leftarrow \max_{s'=1}^{N} viterbi[s',t-1] * a_{s',s} * b_s(o_t)$$
$$backpointer[s,t] \leftarrow \operatorname*{argmax}_{s'=1}^{N} viterbi[s',t-1] * a_{s',s}$$
$$viterbi[q_F,T] \leftarrow \max_{s=1}^{N} viterbi[s,T] * a_{s,q_F}$$; termination step
$$backpointer[q_F,T] \leftarrow \operatorname*{argmax}_{s=1}^{N} viterbi[s,T] * a_{s,q_F}$$; termination step
return the backtrace path by following backpointers to states back in
$\quad$ time from $backpointer[q_F,T]$

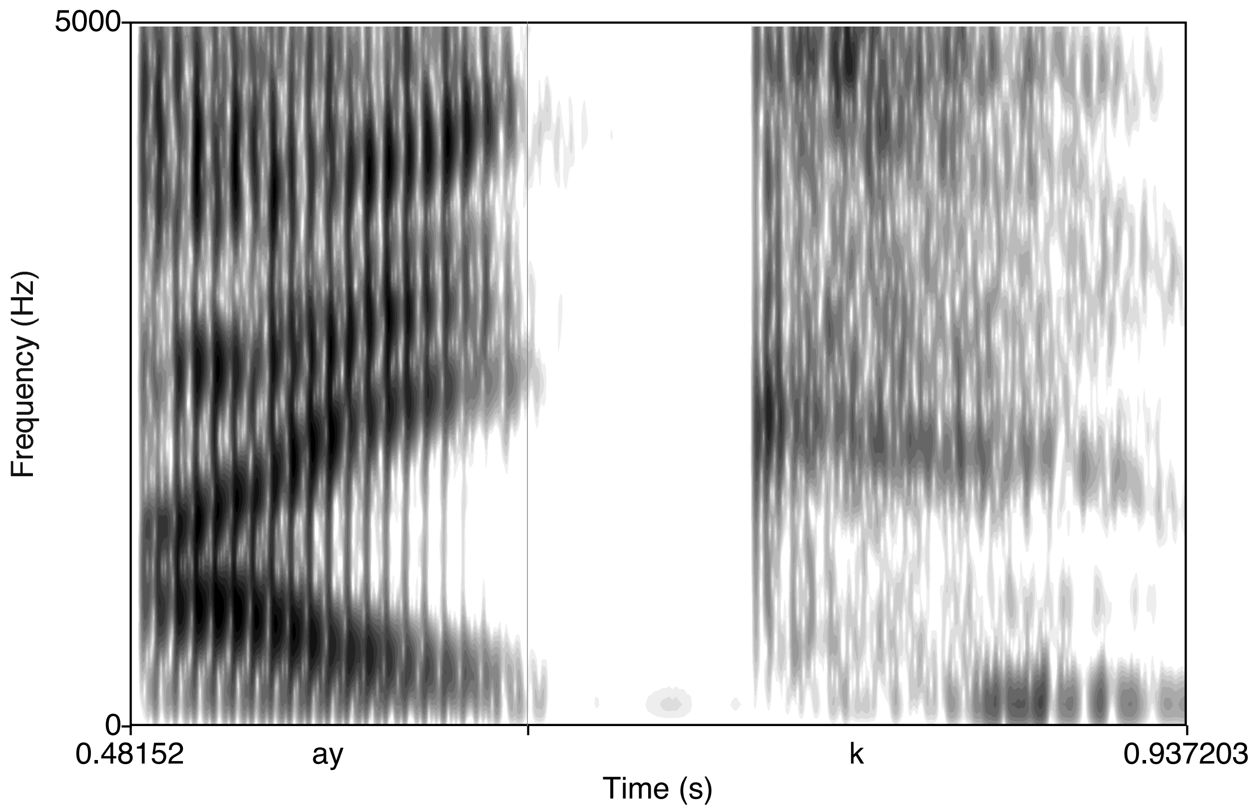
Viterbi backtrace



HMMs for speech

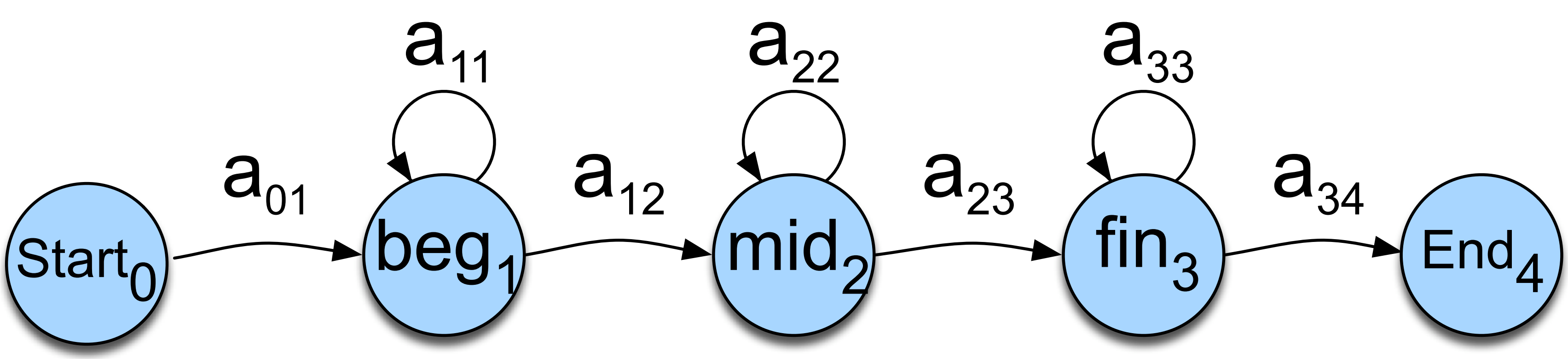


Phones are not homogeneous! Phone-level HMMs not enough

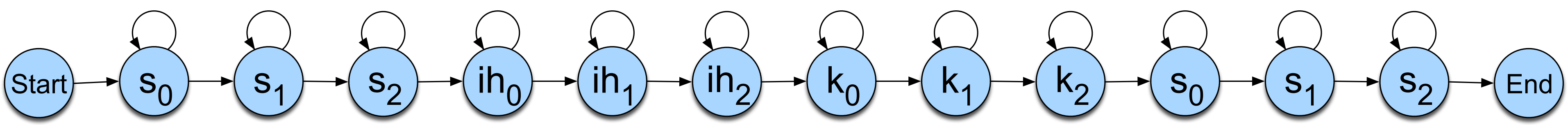


ay k

Each phone has 3 subphones



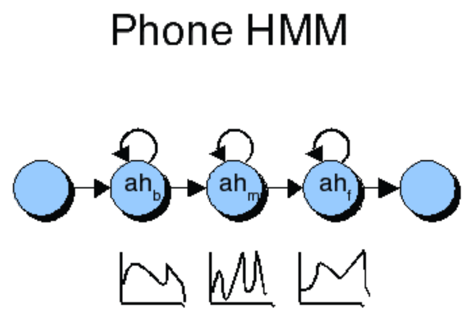
Resulting HMM word model for "six"

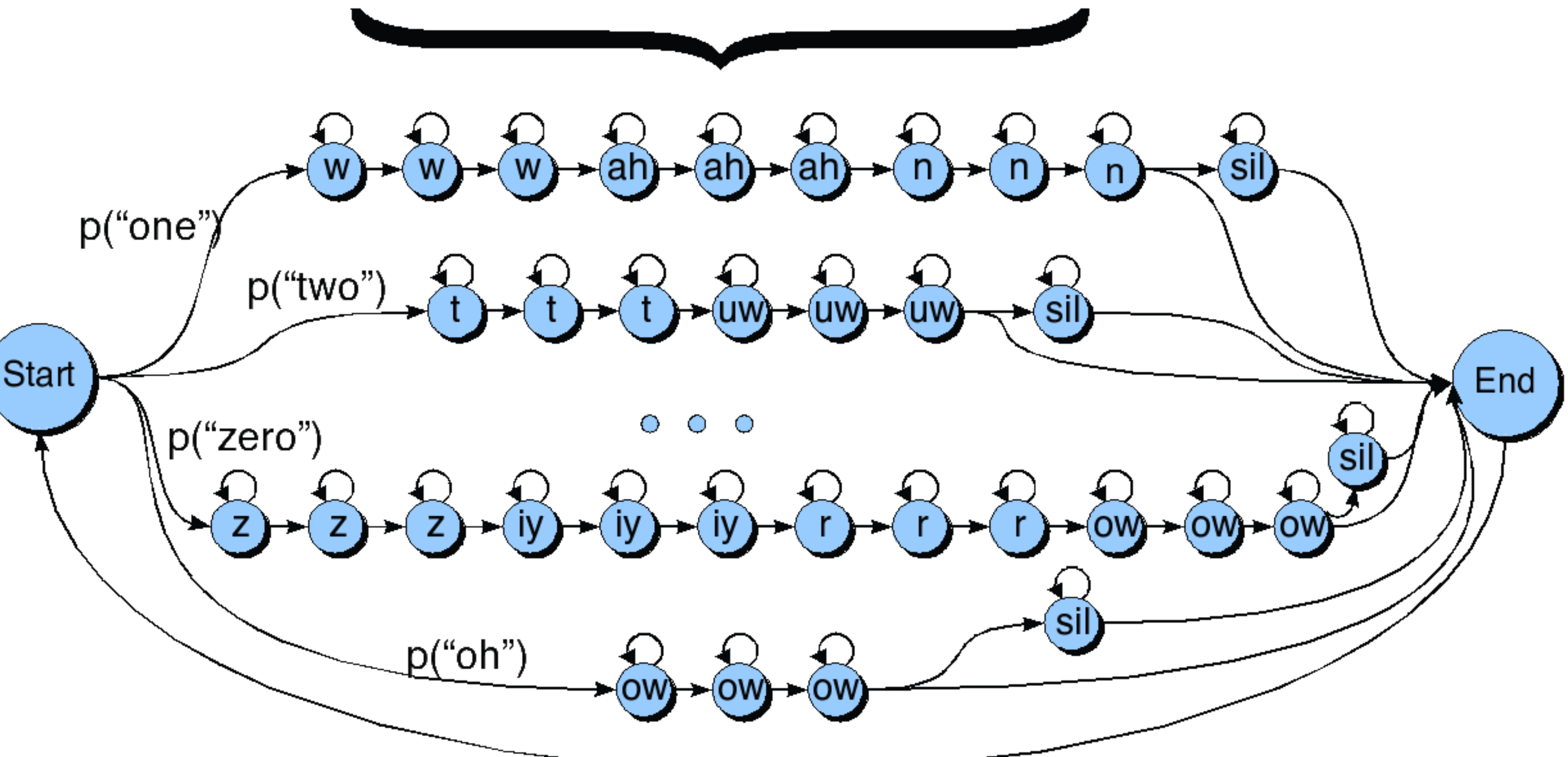


HMM for a digit recognition task

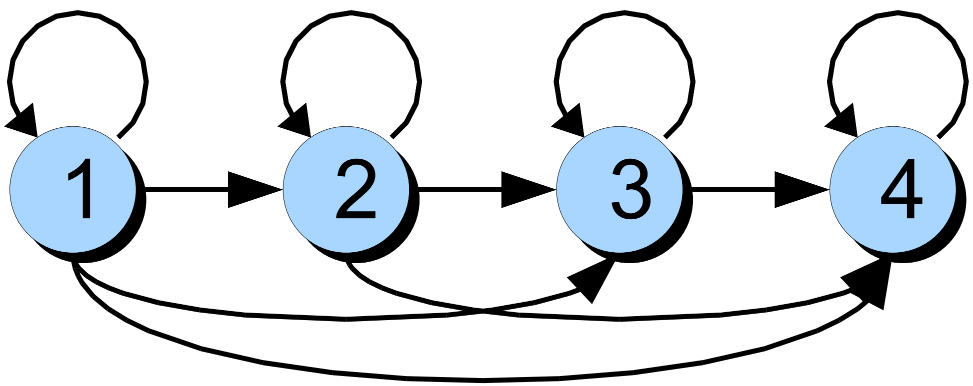
Lexicon

one	w ah n
two	t uw
three	th r iy
four	f ao r
five	f ay v
six	s ih k s
seven	s eh v ax n
eight	ey t
nine	n ay n
zero	z iy r ow
oh	ow

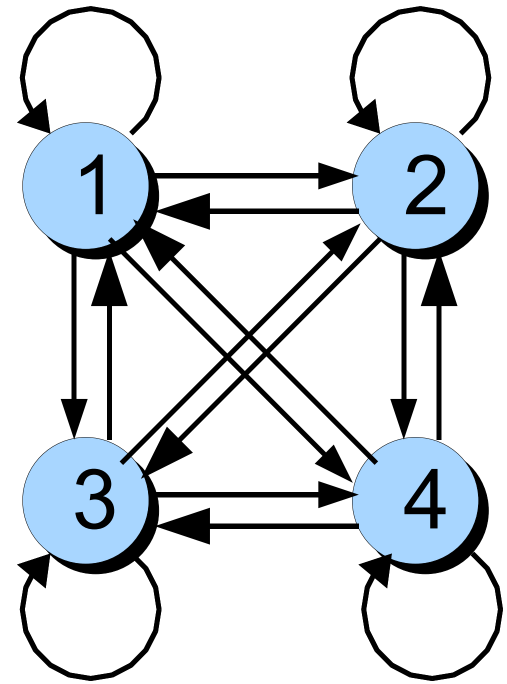




Different types of HMM structure

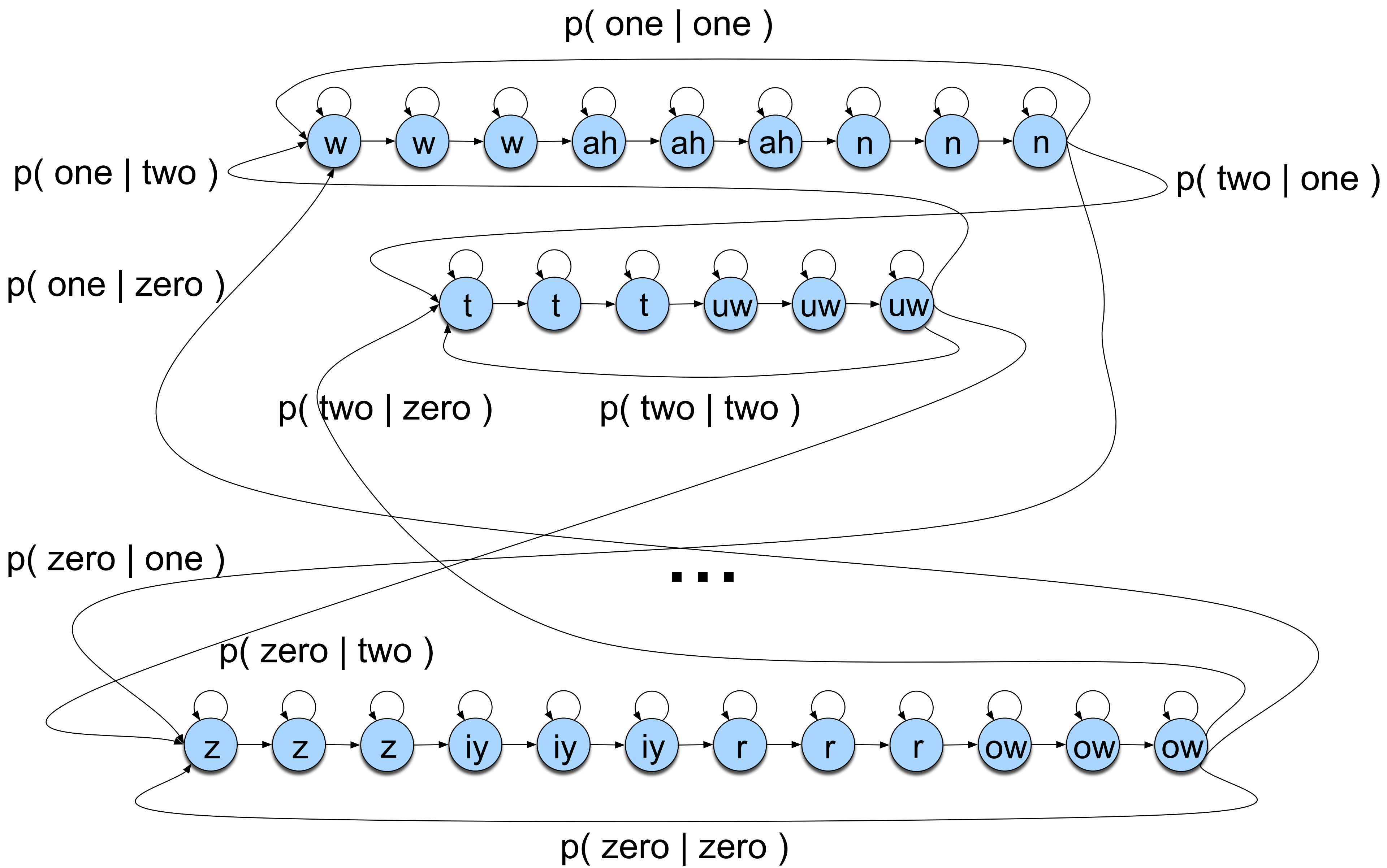


Bakis = left-to-right



Ergodic = fully-connected

Search space with bigrams



HMM-GMM System

Transcription: Samson

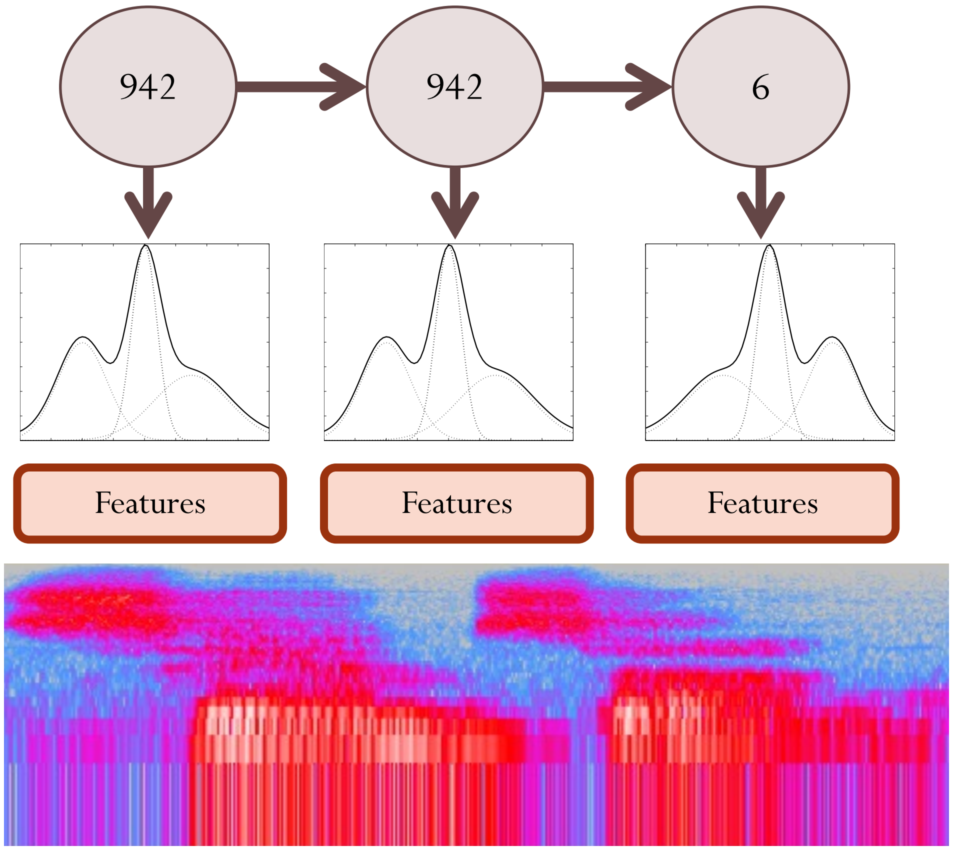
Pronunciation: S – AE – M – S –AH – N

Sub-phones : 942 – 6 – 37 – 8006 – 4422 …

Hidden Markov Model (HMM):

Acoustic Model:

Audio Input:



GMM models:
P(x|s)
x: input features
s: HMM state

Outline for Today

- ASR Architecture
- Hidden Markov Model (HMM) introduction
- HMMs for speech
- **Evaluation with word error rate**

Evaluation

- How to evaluate the word string output by a speech recognizer?

Word Error Rate

- Word Error Rate =

$$\frac{100\ (\text{Insertions}+\text{Substitutions} + \text{Deletions})}{\text{Total Word in Correct Transcript}}$$

- Aligment example:

```
REF:  portable ****  PHONE UPSTAIRS last night so
HYP:  portable FORM  OF    STORES   last night so
Eval           I     S     S
```

- WER = 100 (1+2+0)/6 = 50%

NIST sctk scoring software: Computing WER with sclite

- http://www.nist.gov/speech/tools/
- Sclite aligns a hypothesized text (HYP) (from the recognizer) with a correct or reference text (REF) (human transcribed)

```
id: (2347-b-013)
Scores: (#C #S #D #I) 9 3 1 2
REF:  was an engineer SO I   i was always with **** **** MEN UM
and they
HYP:  was an engineer ** AND i was always with THEM THEY ALL THAT
and they
Eval:                 D  S                     I    I    S   S
```

Sclite output for error analysis

```
CONFUSION PAIRS                  Total                    (972)
                                 With >=  1 occurrances (972)
   1:    6  ->  (%hesitation) ==> on
   2:    6  ->  the ==> that
   3:    5  ->  but ==> that
   4:    4  ->  a ==> the
   5:    4  ->  four ==> for
   6:    4  ->  in ==> and
   7:    4  ->  there ==> that
   8:    3  ->  (%hesitation) ==> and
   9:    3  ->  (%hesitation) ==> the
  10:    3  ->  (a-) ==> i
  11:    3  ->  and ==> i
  12:    3  ->  and ==> in
  13:    3  ->  are ==> there
  14:    3  ->  as ==> is
  15:    3  ->  have ==> that
  16:    3  ->  is ==> this
```

Better metrics than WER?

- WER is useful, but <10% or so and errors might be function words (and, of, uh-huh,…)
- But should we be more concerned with meaning ("semantic error rate")?
  - Good idea, but hard to agree on
  - Has been applied in dialogue systems, where desired semantic/task output is more clear

Appendix

Computing total likelihood of 3 1 3

- We would need to sum over
  - Hot hot cold
  - Hot hot hot
  - Hot cold hot
  - ….

$$P(O) = \sum_{Q} P(O,Q) = \sum_{Q} P(O|Q)P(Q)$$

- How many possible hidden state sequences are there for this sequence?

$$P(3\ 1\ 3) = P(3\ 1\ 3, \text{cold cold cold}) + P(3\ 1\ 3, \text{cold cold hot}) + P(3\ 1\ 3, \text{hot hot cold}) + \ldots$$

- How about in general for an HMM with N hidden states and a sequence of T observations?
  - N^T
- So we can't just do separate computation for each hidden state sequence.

Instead: the Forward algorithm

- A **dynamic programming** algorithm
  - Just like Minimum Edit Distance or CKY Parsing
  - Uses a table to store intermediate values
- Idea:
  - Compute the likelihood of the observation sequence
  - By summing over all possible hidden state sequences
  - But doing this efficiently
    - By folding all the sequences into a single **trellis**

The forward algorithm

- The goal of the forward algorithm is to compute

$$P(o_1, o_2, \ldots, o_T, q_T = q_F \mid \lambda)$$

- We'll do this by recursion

The forward algorithm

- Each cell of the forward algorithm trellis *alpha*$_t$(*j*)
  - Represents the probability of being in *state j*
  - After seeing the first *t* observations
  - Given the automaton
- Each cell thus expresses the following probability

$$\alpha_t(j) = P(o_1, o_2 \ldots o_t, q_t = j|\lambda)$$

The Forward Recursion

1. Initialization:

$$\alpha_1(j) = a_{0j} b_j(o_1) \quad 1 \le j \le N$$

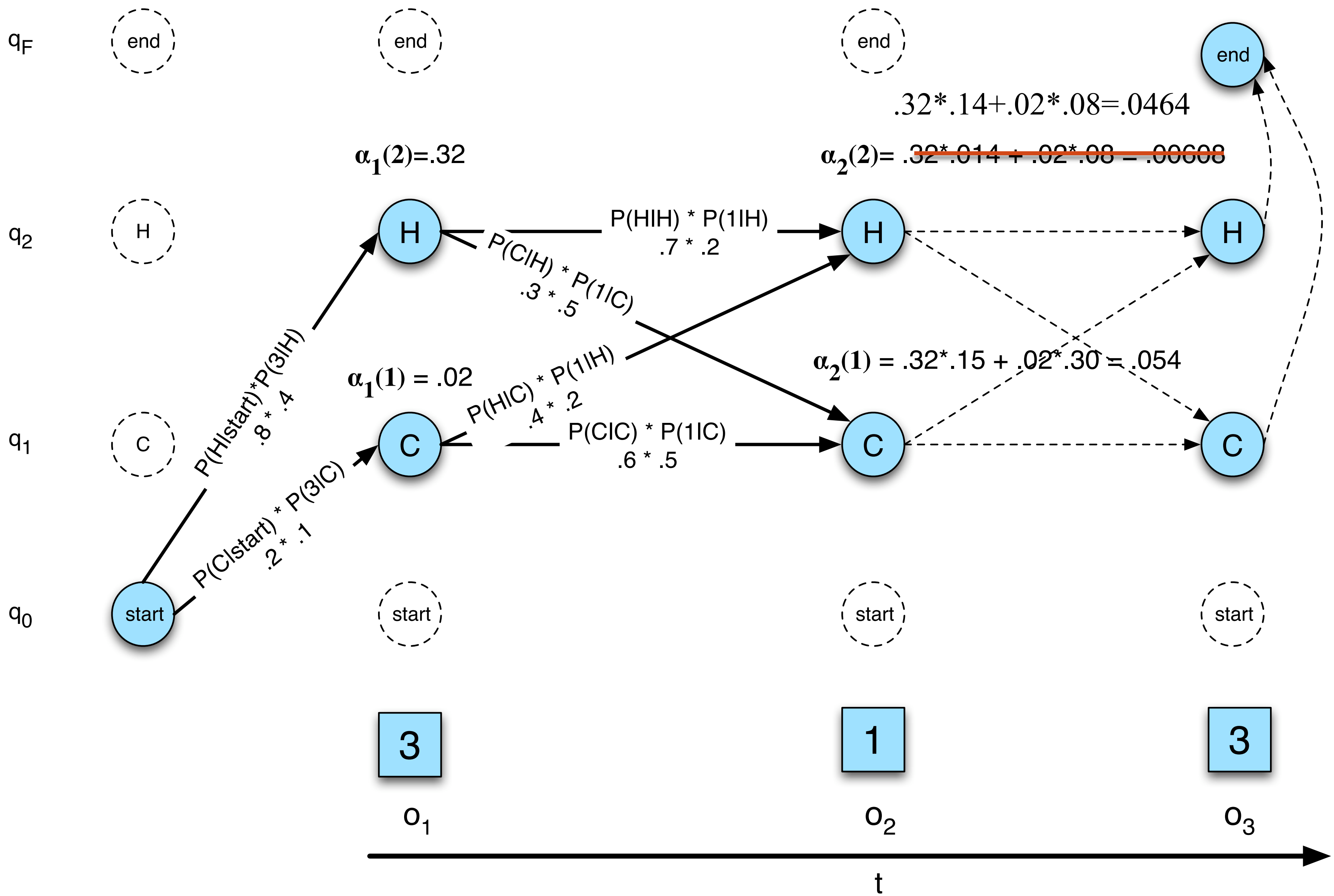
2. Recursion (since states 0 and F are non-emitting):

$$\alpha_t(j) = \sum_{i=1}^{N} \alpha_{t-1}(i) a_{ij} b_j(o_t); \quad 1 \le j \le N, 1 < t \le T$$

3. Termination:

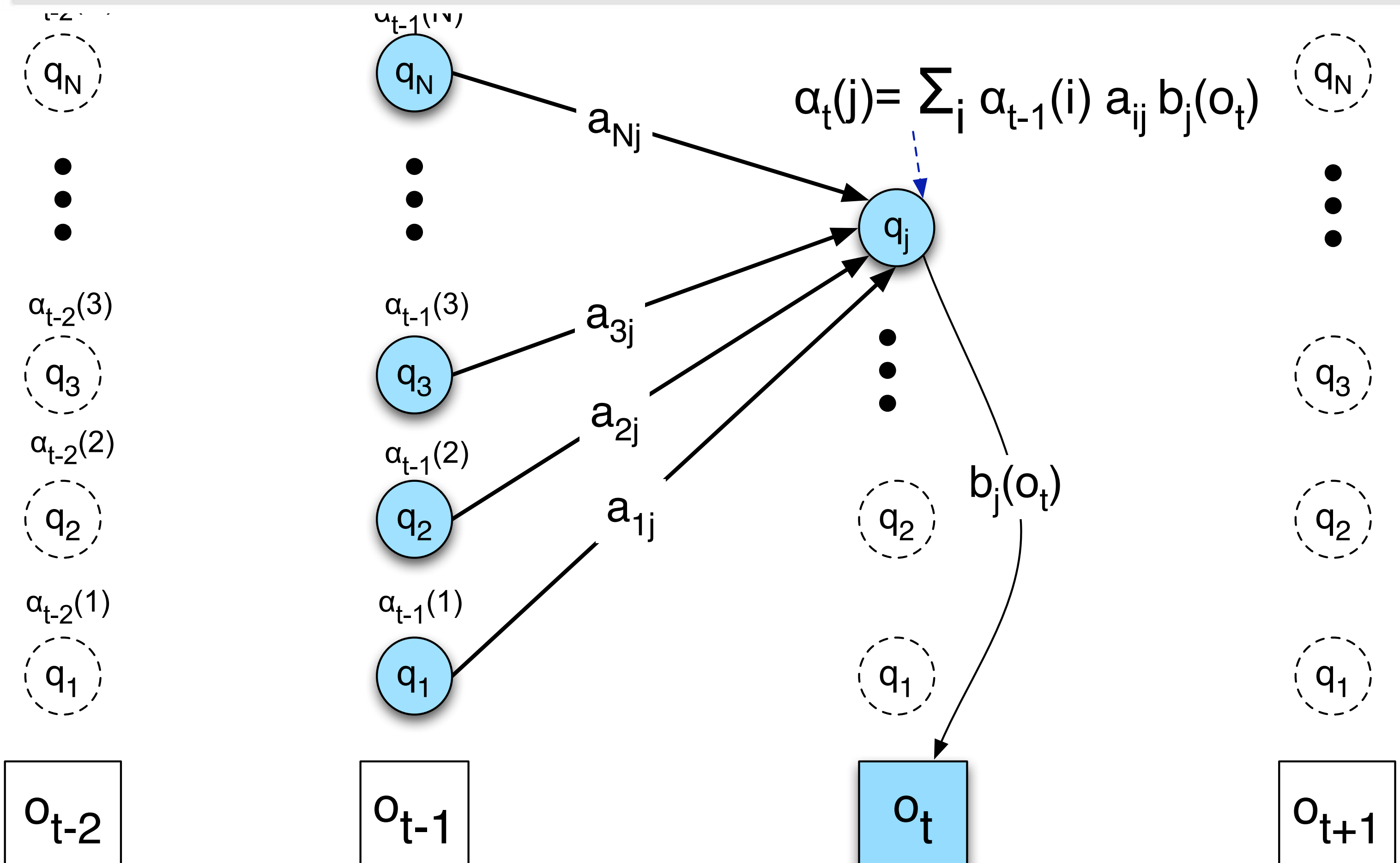
$$P(O|\lambda) = \alpha_T(q_F) = \sum_{i=1}^{N} \alpha_T(i)\, a_{iF}$$

The Forward Trellis



We update each cell

$\alpha_{t-1}(i)$	the **previous forward path probability** from the previous time step
a_{ij}	the **transition probability** from previous state q_i to current state q_j
$b_j(o_t)$	the **state observation likelihood** of the observation symbol o_t given the current state j



The Forward Algorithm

function FORWARD(*observations* of len T, *state-graph* of len N) **returns** *forward-prob*

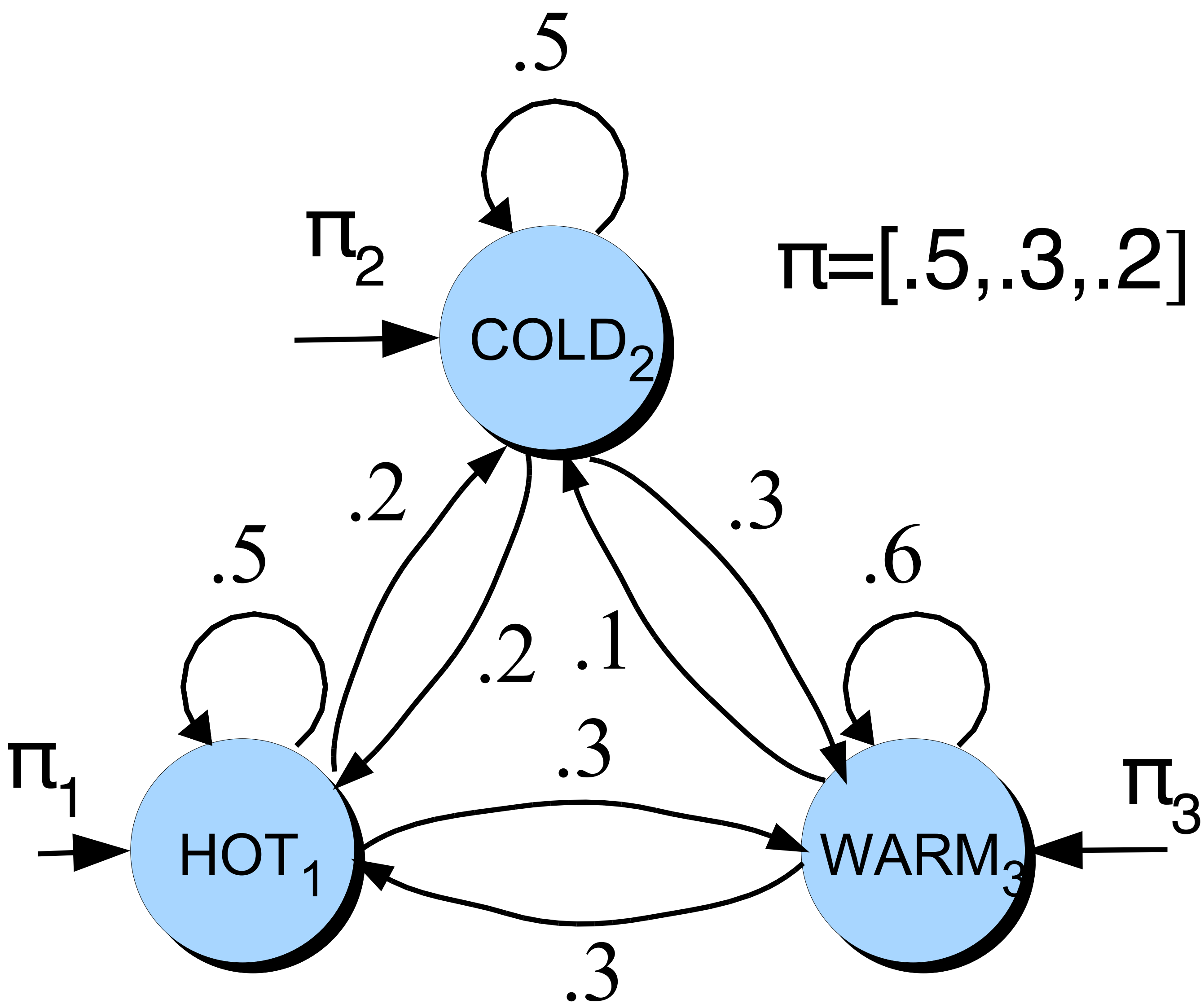
create a probability matrix *forward[N+2,T]*
for each state s **from** 1 **to** N **do** ; initialization step
$forward[s,1] \leftarrow a_{0,s} * b_s(o_1)$
for each time step t **from** 2 **to** T **do** ; recursion step
for each state s **from** 1 **to** N **do**

$$forward[s,t] \leftarrow \sum_{s'=1}^{N} forward[s',t-1] * a_{s',s} * b_s(o_t)$$

$$forward[q_F,\mathrm{T}] \leftarrow \sum_{s=1}^{N} forward[s,T] * a_{s,q_F}$$; termination step

return $forward[q_F,T]$

The weather figure: specific example

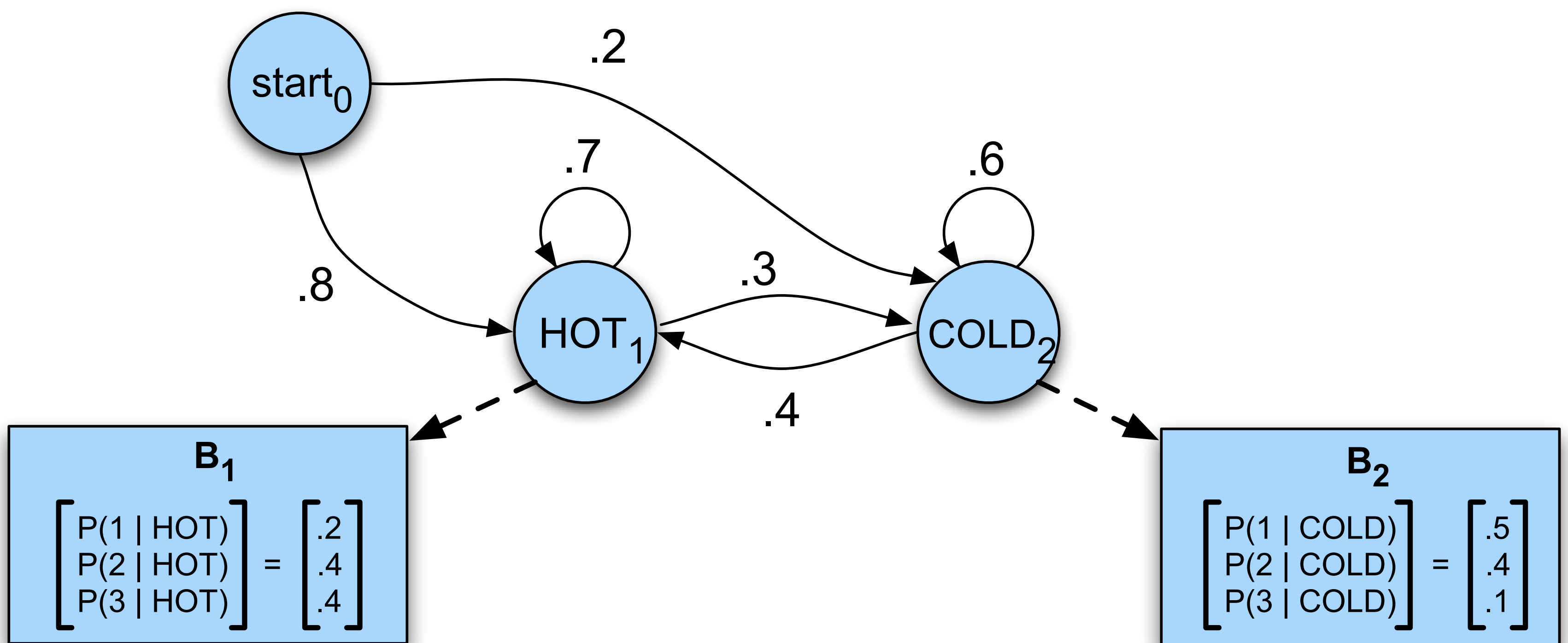


Markov chain for weather

- What is the probability of 4 consecutive warm days?
- Sequence is warm-warm-warm-warm
- I.e., state sequence is 3-3-3-3
- P(3,3,3,3) =
  - $\pi_3 a_{33} a_{33} a_{33} a_{33} = 0.2 \text{ x } (0.6)^3 = 0.0432$

Problem 1: computing the observation likelihood

Computing Likelihood: Given an HMM $\lambda = (A, B)$ and an observation sequence O, determine the likelihood $P(O|\lambda)$.



How likely is the sequence 3 1 3?

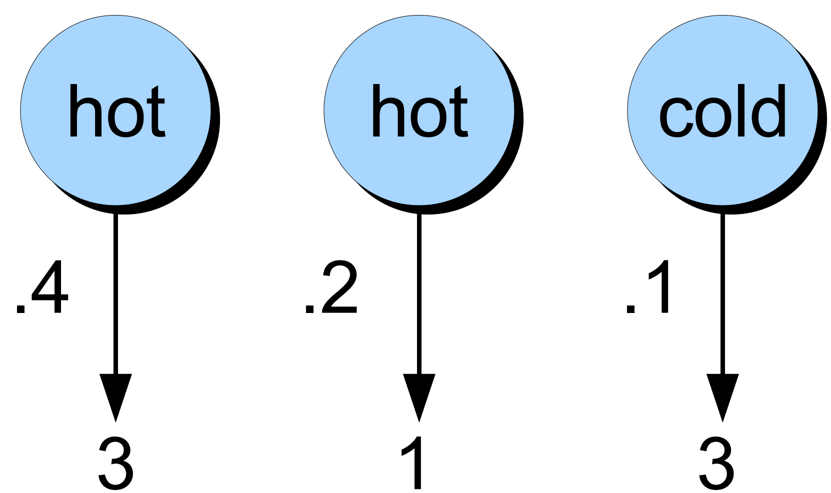
How to compute likelihood

- For a Markov chain, we just follow the states 3 1 3 and multiply the probabilities
- But for an HMM, we don't know what the states are!
- So let's start with a simpler situation.
- Computing the observation likelihood for a **given** hidden state sequence
  - Suppose we knew the weather and wanted to predict how much ice cream Jason would eat.
  - i.e., $P(3\ 1\ 3 \mid H\ H\ C)$

Computing likelihood of 3 1 3 given hidden state sequence

$$P(O|Q) = \prod_{i=1}^{T} P(o_i|q_i)$$

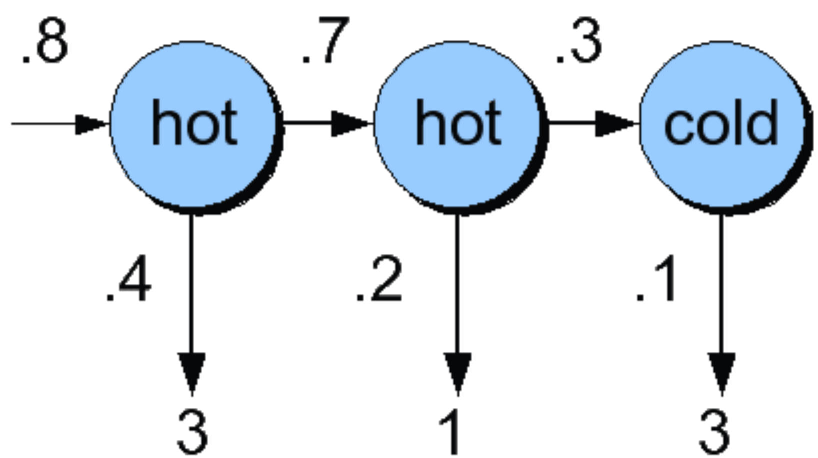
$$P(3\ 1\ 3|\text{hot hot cold}) \;=\; P(3|\text{hot}) \times P(1|\text{hot}) \times P(3|\text{cold})$$



Computing joint probability of observation and state sequence

$$P(O,Q) = P(O|Q) \times P(Q) = \prod_{i=1}^{n} P(o_i|q_i) \times \prod_{i=1}^{n} P(q_i|q_{i-1})$$

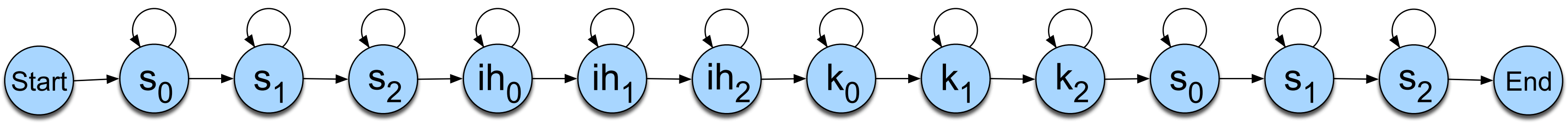
$$\begin{aligned} P(3\ 1\ 3, \text{hot hot cold}) \ = \ & P(\text{hot}|\text{start}) \times P(\text{hot}|\text{hot}) \times P(\text{cold}|\text{hot}) \\ & \times P(3|\text{hot}) \times P(1|\text{hot}) \times P(3|\text{cold}) \end{aligned}$$



HMMs for Speech

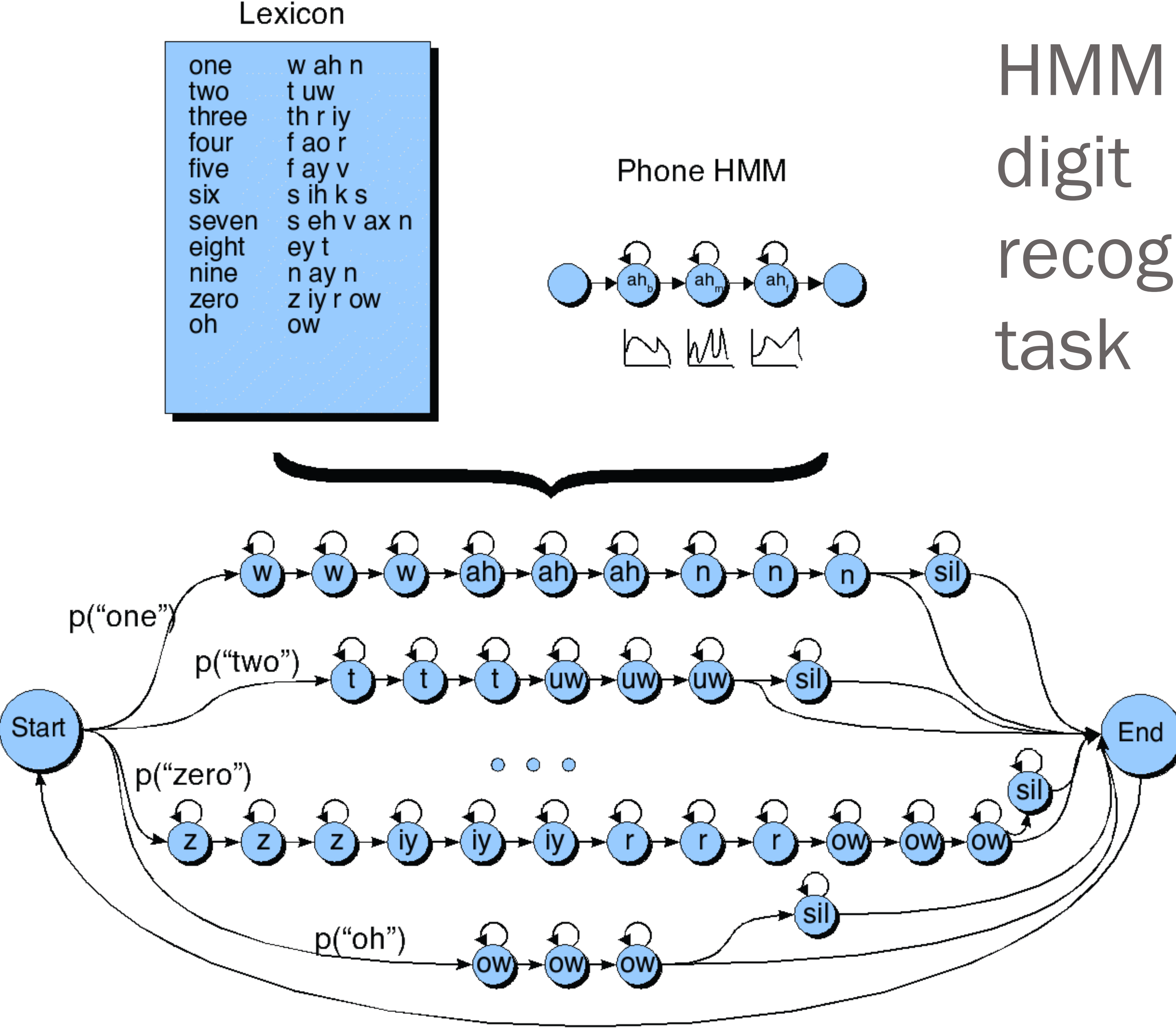
- We haven't yet shown how to **learn** the A and B matrices for HMMs;
  - The Baum-Welch (Forward-Backward alg)
- But let's return to think about speech

Reminder: a word looks like this:



$Q = q_1 q_2 \ldots q_N$	a set of **states** corresponding to **subphones**
$A = a_{01} a_{02} \ldots a_{n1} \ldots a_{nn}$	a **transition probability matrix** A, each a_{ij} representing the probability for each subphone of taking a **self-loop** or going to the next subphone. Together, Q and A implement a **pronunciation lexicon**, an HMM state graph structure for each word that the system is capable of recognizing.
$B = b_i(o_t)$	A set of **observation likelihoods:**, also called **emission probabilities**, each expressing the probability of a cepstral feature vector (observation o_t) being generated from subphone state i.

HMM for digit recognition task



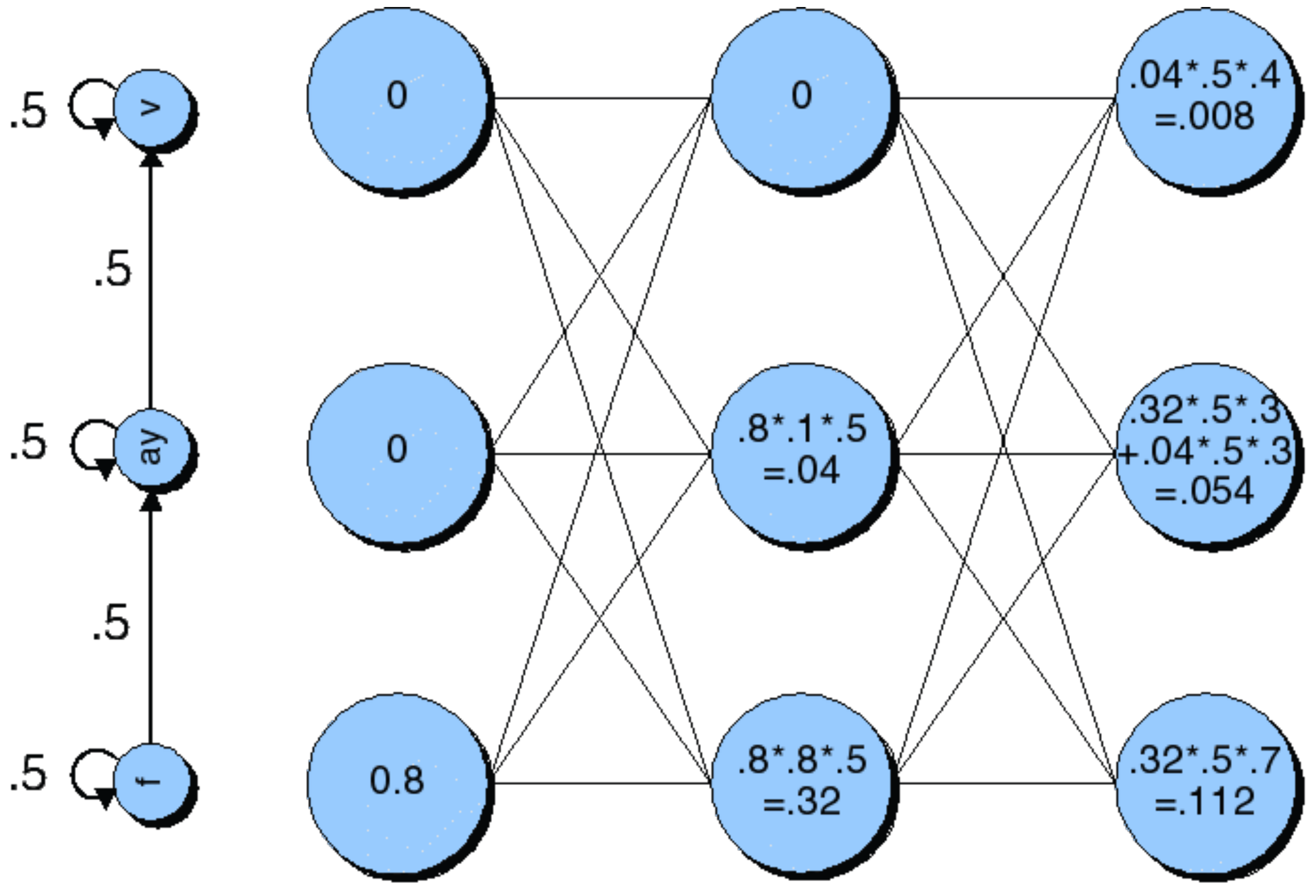
The Evaluation (forward) problem for speech

- The observation sequence O is a series of MFCC vectors
- The hidden states W are the phones and words
- For a given phone/word string W, our job is to evaluate $P(O|W)$
- Intuition: how likely is the input to have been generated by just that word string W?

Evaluation for speech: Summing over all different paths!

- f ay ay ay ay v v v v
- f f ay ay ay ay v v v
- f f f f ay ay ay ay v
- f f ay ay ay ay ay ay v
- f f ay ay ay ay ay ay ay ay v
- f f ay v v v v v v v

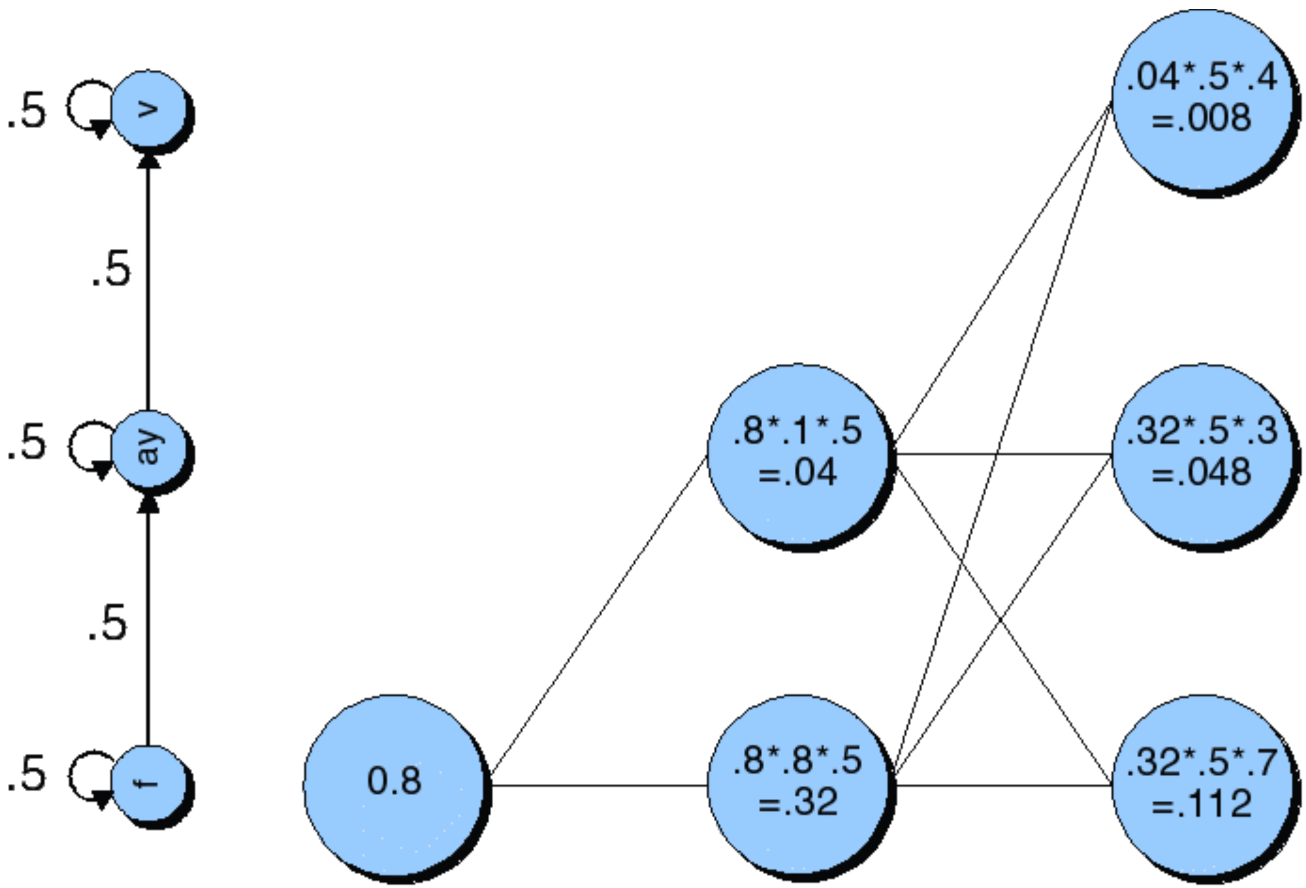
The forward lattice for “five”



The forward trellis for "five"

V	0	0	0.008	0.0093	0.0114	0.00703	0.00345	0.00306	0.00206	0.00117
AY	0	0.04	0.054	0.0664	0.0355	0.016	0.00676	0.00208	0.000532	0.000109
F	0.8	0.32	0.112	0.0224	0.00448	0.000896	0.000179	4.48e-05	1.12e-05	2.8e-06
Time	1	2	3	4	5	6	7	8	9	10
	f 0.8	*f 0.8*	*f 0.7*	*f 0.4*	*f 0.4*	*f 0.4*	*f 0.4*	*f 0.5*	*f 0.5*	*f 0.5*
	ay 0.1	*ay 0.1*	*ay 0.3*	*ay 0.8*	*ay 0.8*	*ay 0.8*	*ay 0.8*	*ay 0.6*	*ay 0.5*	*ay 0.4*
B	*v 0.6*	*v 0.6*	*v 0.4*	*v 0.3*	*v 0.3*	*v 0.3*	*v 0.3*	*v 0.6*	*v 0.8*	*v 0.9*
	p 0.4	*p 0.4*	*p 0.2*	*p 0.1*	*p 0.1*	*p 0.1*	*p 0.1*	*p 0.1*	*p 0.3*	*p 0.3*
	iy 0.1	*iy 0.1*	*iy 0.3*	*iy 0.6*	*iy 0.6*	*iy 0.6*	*iy 0.6*	*iy 0.5*	*iy 0.5*	*iy 0.4*

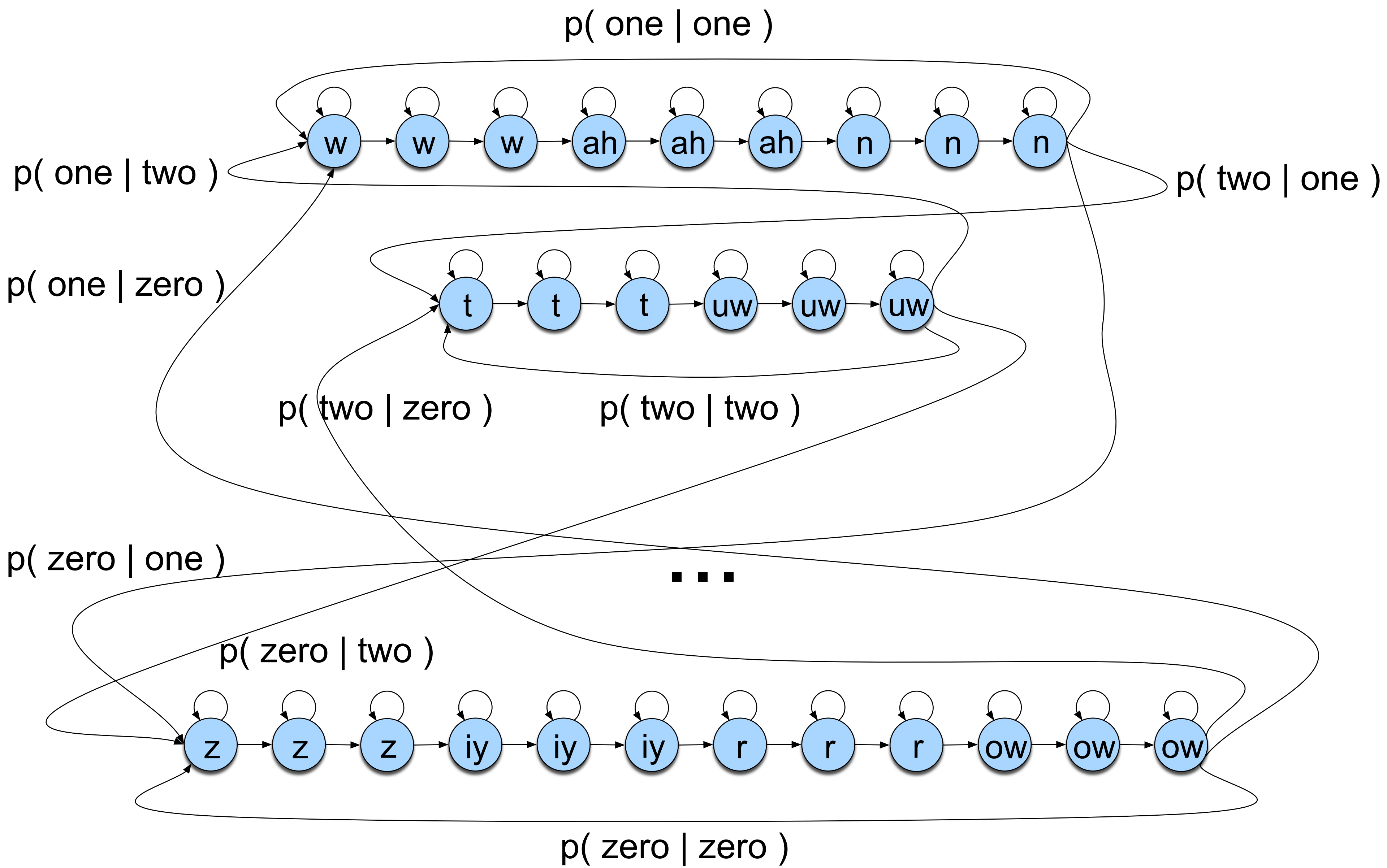
Viterbi trellis for “five”



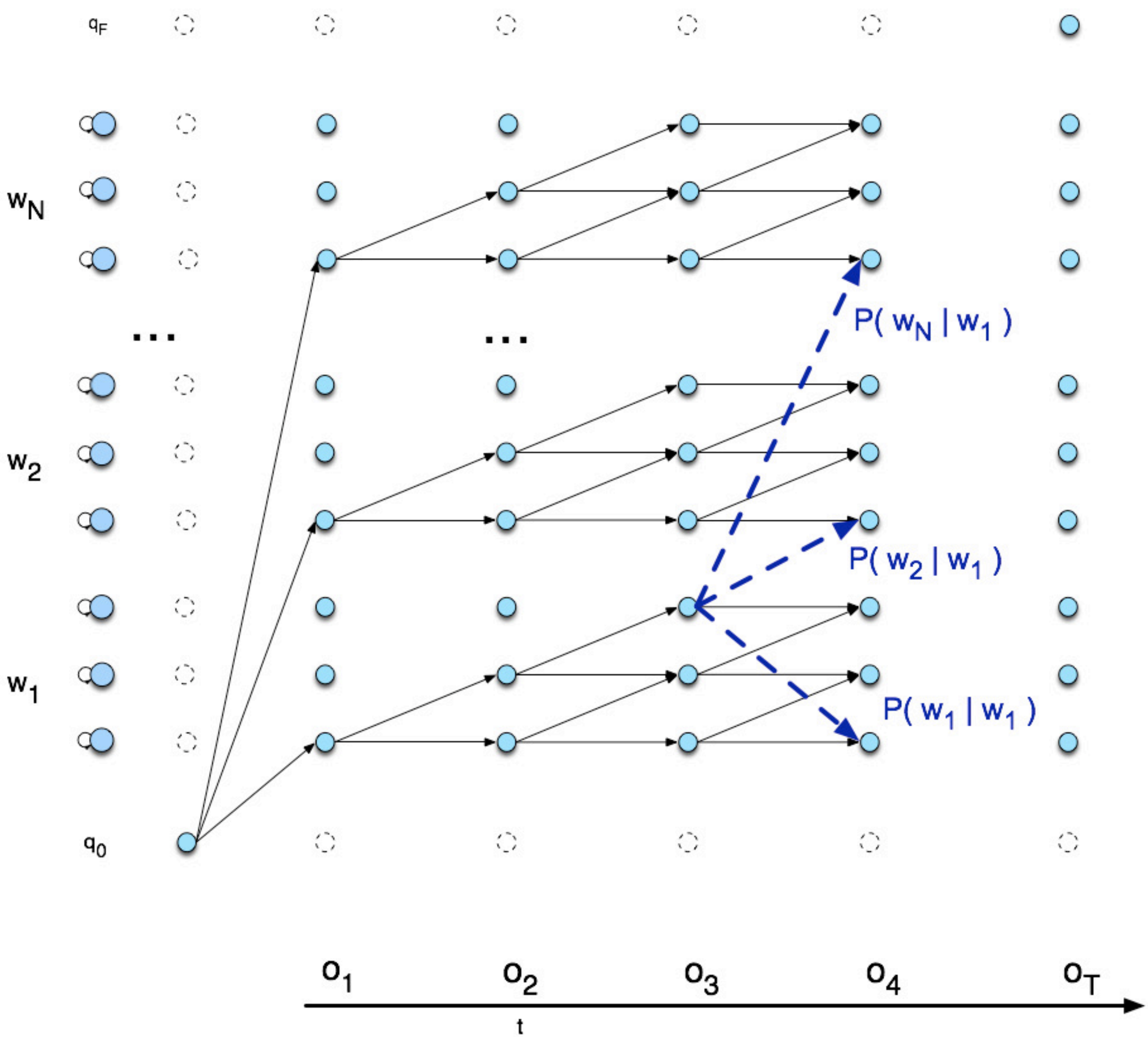
Viterbi trellis for “five”

V	0	0	0.008	0.0072	0.00672	0.00403	0.00188	0.00161	0.000667	0.000493
AY	0	0.04	0.048	0.0448	0.0269	0.0125	0.00538	0.00167	0.000428	8.78e-05
F	0.8	0.32	0.112	0.0224	0.00448	0.000896	0.000179	4.48e-05	1.12e-05	2.8e-06
Time	1	2	3	4	5	6	7	8	9	10
	f 0.8	*f 0.8*	*f 0.7*	*f 0.4*	*f 0.4*	*f 0.4*	*f 0.4*	*f 0.5*	*f 0.5*	*f 0.5*
	ay 0.1	*ay 0.1*	*ay 0.3*	*ay 0.8*	*ay 0.8*	*ay 0.8*	*ay 0.8*	*ay 0.6*	*ay 0.5*	*ay 0.4*
B	*v 0.6*	*v 0.6*	*v 0.4*	*v 0.3*	*v 0.3*	*v 0.3*	*v 0.3*	*v 0.6*	*v 0.8*	*v 0.9*
	p 0.4	*p 0.4*	*p 0.2*	*p 0.1*	*p 0.1*	*p 0.1*	*p 0.1*	*p 0.1*	*p 0.3*	*p 0.3*
	iy 0.1	*iy 0.1*	*iy 0.3*	*iy 0.6*	*iy 0.6*	*iy 0.6*	*iy 0.6*	*iy 0.5*	*iy 0.5*	*iy 0.4*

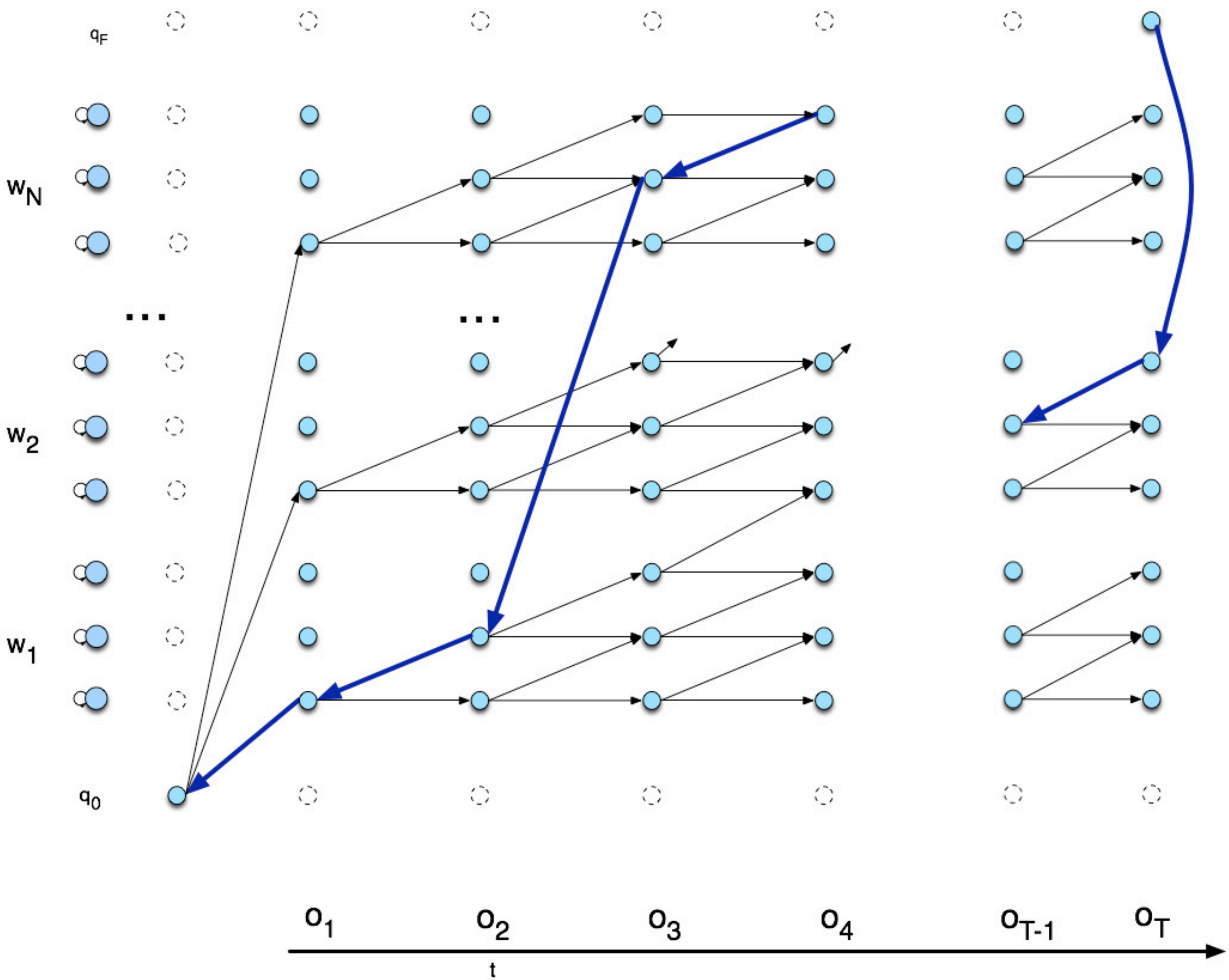
Search space with bigrams



Viterbi trellis



Viterbi backtrace



Summary: ASR Architecture

- Five easy pieces: ASR Noisy Channel architecture
  - Feature Extraction:
    - 39 "MFCC" features
  - Acoustic Model:
    - Gaussians for computing $p(o|q)$
  - Lexicon/Pronunciation Model
    - HMM: what phones can follow each other
  - Language Model
    - N-grams for computing $p(w_i|w_{i-1})$
  - Decoder
    - Viterbi algorithm: dynamic programming for combining all these to get word sequence from speech