

Stanford CS224v Course  
Conversational Virtual Assistants with Deep Learning

## Lecture 11

# Satisfying Natural Language Constraints with SMT

A Case Study in Clinical Trials Matching

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# Large-Scale Constraint Satisfaction

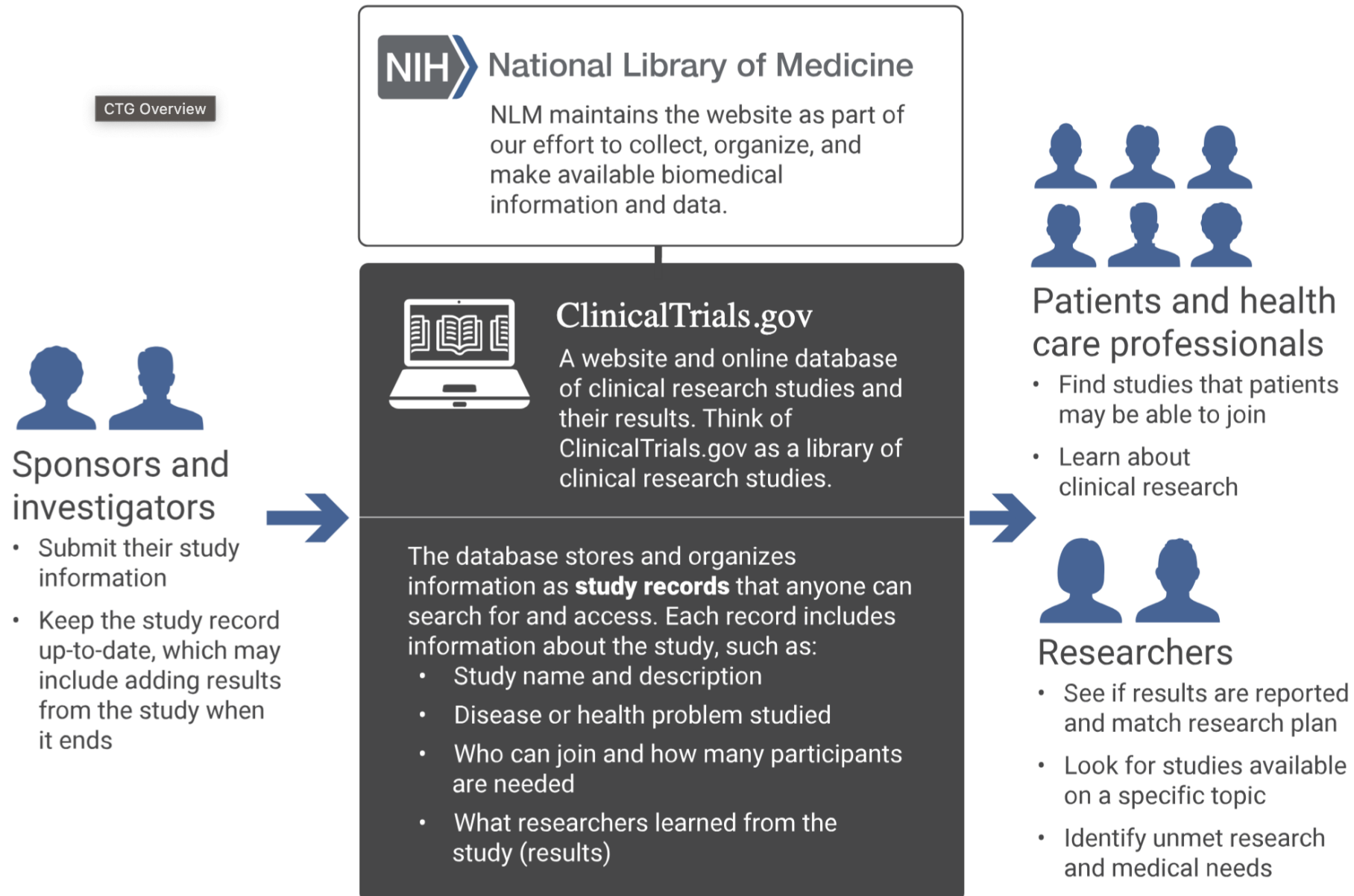
Many applications

- Education: Degree satisfaction
- Finance: Government regulations
- Medical: Clinical Trial matching

# Why is Clinical Trial Matching Important?

- Clinical trials are essential to modern medical progress.
  - Test out new treatments  
(drugs, procedures, behavioral therapies)
- Nearly **\$1.9 billion** is spent annually on recruitment efforts
- But still,  
around **80%** of trials fail to meet the initial enrollment target/timeline
  - Causing monetary loss
  - Blocking scientific progress

# Trials are in ClinicalTrials.gov!



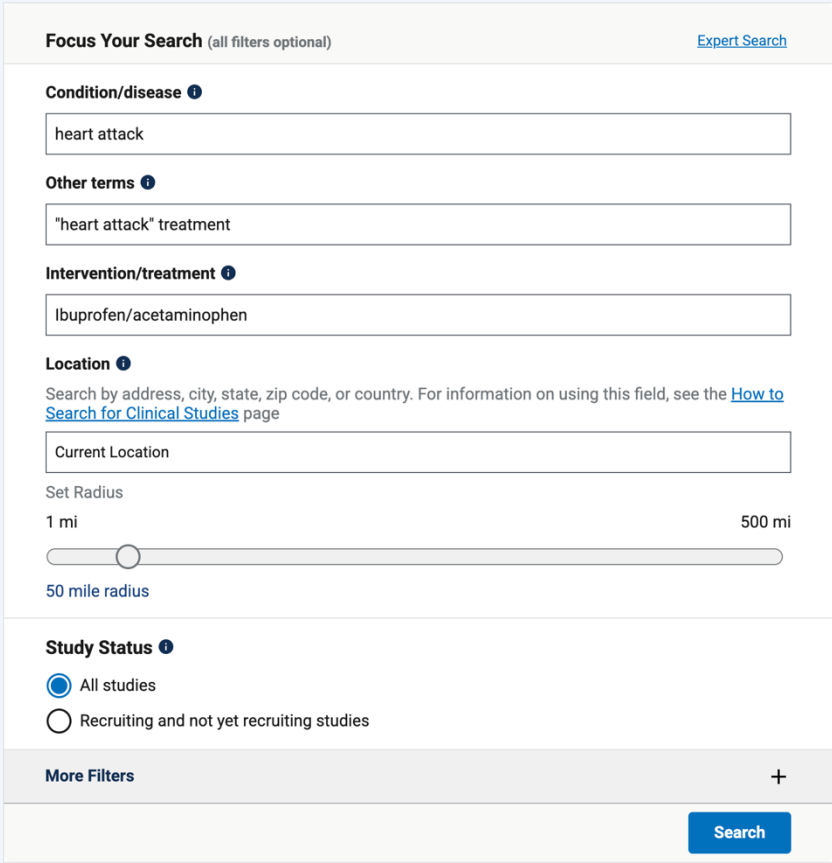
# ClinicalTrials.gov are Used Widely

- 485,000 studies listed from all 50 states and 223 countries (3/2024)
- Good coverage of trials: e.g.  
NIH-funded clinical trials are expected to register and submit results
- ClinicalTrials.gov receives about 4.5 million visitors monthly.

<https://www.bumc.bu.edu/ohra/clinicaltrials-gov/clinicaltrials-gov-what-why-which-studies-when>

# What is the Current Practice?

- Doctor referring patients to trials that they know of
- Patients looking for trials ..



The screenshot shows a search interface for clinical trials. It includes several filter sections: 'Condition/disease' with a text input containing 'heart attack'; 'Other terms' with a text input containing '"heart attack" treatment'; 'Intervention/treatment' with a text input containing 'Ibuprofen/acetaminophen'; 'Location' with a text input containing 'Current Location' and a radius slider set to 50 miles; and 'Study Status' with radio buttons for 'All studies' (selected) and 'Recruiting and not yet recruiting studies'. A 'More Filters' section is partially visible at the bottom. A blue 'Search' button is located at the bottom right of the form.

**Focus Your Search** (all filters optional) [Expert Search](#)

**Condition/disease** ⓘ

heart attack

**Other terms** ⓘ

"heart attack" treatment

**Intervention/treatment** ⓘ

Ibuprofen/acetaminophen

**Location** ⓘ

Search by address, city, state, zip code, or country. For information on using this field, see the [How to Search for Clinical Studies](#) page

Current Location

Set Radius

1 mi 500 mi

50 mile radius

**Study Status** ⓘ

☒ All studies

☐ Recruiting and not yet recruiting studies

**More Filters** +

**Search**

## MOTIVATION FOR OUR PROJECT

“I’M A PHYSICIAN,  
AND I CAN’T FIND A TRIAL FOR MYSELF”

LET'S TAKE A LOOK AT A CLINICAL TRIAL



# Study Overview

## Brief Summary

The purpose of the study is to assess the frequency and intensity of intramyocardial haemorrhage in patients with primary STEMI and different reperfusion strategies.

## Detailed Description

The study non-randomized, opened, controlled. In half of patients despite on carried in-time reperfusion therapy intramyocardial haemorrhage determined after a long-term period of severe ischemia. Earlier, definition of intramyocardial haemorrhage was possible only by autopsy. Nowadays, cardiac contrast MRI is the best diagnostic method, which allows to assess the regional and global function of the LV, structural changes in myocardial tissue and also in T2 mode it became assessable to reveal intramyocardial haemorrhage.

Taking into account the results of previous researches, it can be concluded that the intramyocardial haemorrhage was determined in half of patients with primary PCI [1]. An influence of fibrinolytic therapy to the intramyocardial haemorrhage was conducted in small group of patients in one trial, and therefore further data will be actual and useful [2].

It is planned to study 60 patients with primary STEMI using standard therapy. The patients will be divided into 2 groups. Patients of the 1st group will be treated by pharmaco-invasive strategy. The 2nd group will be treated by primary PCI. Patients in all groups after reperfusion strategies will be conducted cardiac contrast MRI for detection intramyocardial haemorrhage within 2 days onset. At day 7 and through 3 months, the clinical condition of the patients will be assessed and cardiac ultrasound will be performed for the evaluation of myocardial contractile function and 2D global longitudinal strain. Also, the incidence rate of secondary endpoints will be evaluated.

## Study Start (Actual) ⓘ

2018-01-25

## Primary Completion (Actual) ⓘ

2018-10-30

## Study Completion (Actual) ⓘ

2019-03-20

## Enrollment (Actual) ⓘ

60

## Study Type ⓘ

Interventional

## Phase ⓘ

Not Applicable

## Eligibility Criteria

### Description

#### Inclusion Criteria:

- Age  $\geq$  18 years at time of randomization (18 years and older);
- Acute myocardial infarction;
- Reperfusion of the infarct-related coronary artery in terms within 12 h of symptom onset;
- Written the informed consent to participate in research;

#### Exclusion Criteria:

- Inability to obtain informed consent;
- Patients previously undergone endovascular / surgical revascularization of coronary artery;
- Severe comorbidity;
- History of myocardial infarction;
- History of intracranial haemorrhage;
- Pulmonary edema, cardiogenic shock;
- Creatinine clearance  $<30$  mL/min or dialysis;
- Unable to undergo or contra-indications for MRI;
- Allergy for contrast agent;
- Indication or use of oral anticoagulant therapy;
- Major bleeding;
- Atrio-ventricular block II and III degree;
- Active gastroduodenal ulcer;
- Aortic dissection;
- Acute psychotic disorders

### Ages Eligible for Study

18 Years and older (Adult, Older Adult )

### Sexes Eligible for Study

All

### Accepts Healthy Volunteers

No

# What does a Real Patient Record Look Like?

## Hospital Medicine — Admission H&P

**Patient:** J.D. (F, 52) • **MRN:** 1xxxxxx • **Room:** 6W-6213  
**Date/Time:** 2025-09-21 18:42 PT • **Admitting Service:** Hospital Medicine-B  
**Attending:** Alex Kim, MD • **Resident:** Priya Shah, MD • **PCP:** Unknown  
**Code Status:** Full  
**Isolation:** None  
**Language:** English  
**Insurance:** Commercial PPO

### Chief Complaint

“Chest pressure and nausea.”

### History of Present Illness

52-year-old female with HTN and HLD presenting with acute central chest pressure starting ~12:30 PT while at rest, radiating to L arm, associated with diaphoresis and nausea, no syncope. Pain 8/10, non-pleuritic, non-positional. EMS EKG reportedly ST elevations anteriorly; ASA given by EMS? **unclear**. In ED: EKG with 2 mm STE V2–V4; initial trop-I 0.42 ng/mL ↑. Given ASA 325 mg PO, ticagrelor 180 mg PO, heparin 4,000 U IV bolus. Cath lab activated; patient transferred for emergent PCI at 13:35.

**Cath Lab (brief op note):** Proximal LAD 99% thrombotic lesion → drug-eluting stent ×1 with TIMI-3 flow restoration. LVDP mildly elevated. No complications. Total contrast 90 mL. Fluoro time 12.8 min.

Post-PCI pain resolved (0–1/10). Admitted to CCU then transferred to floor on 6W at 18:10 in stable condition on DAPT.

**Prior episodes:** Intermittent exertional chest tightness over last 2–3 months climbing stairs, self-limited; never evaluated.

**Precipitating factors today:** At rest; possible recent work stress; slept 4–5 h/night for past week. No recent illness, immobilization, or surgery.

**ED Course (chronological):** See nursing and ED MD notes copied below under *Copied Forward Content*.

### Problem List (active)

- 1. **Acute anterior STEMI s/p PCI to proximal LAD** (09/21/2025)
- 2. **Hypertension** (2016)
- 3. **Hyperlipidemia** (2018)

4. **Overweight** (BMI 29)

### Allergies

- **Penicillin** — rash (non-anaphylactic)

### Home Medications (patient report, med reconciliation completed by pharmacy)

- Lisinopril 10 mg PO daily (last fill 2 months ago; reports good adherence)
- Atorvastatin 40 mg PO nightly
- OTC: multivitamin daily
- Denies herbals/supplements

### Social History / SDOH

- Lives with spouse in apartment (elevator access); independent in ADLs/IADLs.
- Work: retail manager; high stress; on feet most of day.
- Tobacco: never.
- Alcohol: 1–2 drinks/week.
- Illicits: denies.
- Transportation: drives.
- Food security/housing security: stable.
- Prefers afternoon appointments due to work schedule.

### Family History

- Father MI at 58; deceased at 72 from stroke.
- Mother T2DM, HTN.

### Review of Systems (template — positives in bold)

- **Constitutional:** diaphoresis at onset; no fever/chills, no weight loss.
- **Cardiac:** chest pressure, radiation L arm; no palpitations; no LE edema.
- **Respiratory:** no SOB at rest, no cough, no wheeze.
- **GI:** nausea; no vomiting/diarrhea, no melena/hematochezia.
- **GU:** no dysuria, no hematuria.
- **Neuro:** no syncope, no focal weakness, no speech changes.
- **MSK:** no calf pain, no recent trauma.
- **Derm:** no rashes currently.
- **Endo:** no heat/cold intolerance.
- **Psych:** baseline stress; no SI/HI.

**Auto-imported ROS block (system default)** — 14-pt ROS negative except as noted above.

### Vitals (selected)

**ED Triage (12:48):** BP 162/94, HR 104, RR 20, Temp 36.8 °C, SpO<sub>2</sub> 98% RA, Wt 78 kg, Ht 164 cm (BMI 29.0)  
**Post-PCI (15:00):** BP 138/82, HR 88, RR 16, SpO<sub>2</sub> 98% RA  
**On transfer to floor (18:15):** BP 134/79, HR 82, RR 16, Temp 36.7 °C, SpO<sub>2</sub> 98% RA

**Intake/Output (last 24 h):** Intake 450 mL; Output 300 mL; Net +150 mL.

### Physical Exam (on floor 18:25)

- **General:** alert, oriented, comfortable, no distress.
- **HEENT:** PERRL, anicteric; MMM.
- **Neck:** supple; no JVD at 30°.
- **CV:** RRR, no murmurs/rubs/gallops.
- **Lungs:** CTAB, normal WOB.
- **Abd:** soft, NT/ND, +BS.
- **Ext:** warm, well-perfused; no edema; R radial access site clean/dry/intact with TR band removed at 17:30; distal pulses 2+.
- **Neuro:** AO×3, non-focal; speech clear; moves all extremities.
- **Skin:** no rash.

### Labs (auto-imported)

**CBC**  
09/21 13:10 — WBC 8.2, Hgb 13.6, Hct 40.3, Plt 248  
**BMP**  
09/21 13:10 — Na 139, K 4.2, Cl 103, CO<sub>2</sub> 24, BUN 15, Cr 0.86, Glu 146  
**Mg/Phos**  
09/21 13:10 — Mg 1.9, Phos 3.4  
**Cardiac markers**  
09/21 13:10 — Troponin-I **0.42** (↑)  
09/21 17:40 — Troponin-I **2.3** (↑↑)  
**Lipid panel (fasting pending)** ordered for 09/22 06:00  
**HbA1c** ordered for 09/22 06:00

**Note:** Full lab history available in chart. Above values auto-pulled; see results tab for reference ranges.

### Microbiology

- None pending. COVID/flu not indicated.

### Imaging / Studies

**EKG (ED 13:04):** Sinus tachy 102. 2 mm ST elevations V2–V4 with reciprocal depressions II/III/aVF.

# Clinical Matching is Complex

1. Basic matching problem: Is a patient eligible for a trial?
  - LLMs seem to do a reasonable job for most cases
2. From a pharmaceutical company's perspective:
  - Recruit from **millions** of patients for a trial
  - How to match against millions?
3. From a patient's perspective
  - Find eligible trials among **500K clinical trials**

*Key Questions: How accurate is the LLM approach?  
How to handle the scale?*

# Outline

- Motivation
- **Prior Research**
- SMT Approach
- SMT-Based Matching Algorithm
- Preliminary Results

# LLM-Based Matching

- TrialGPT: Matching patients to clinical trials with LLMs, Jin et al., Nov, 2024, Nature Communications
  - Synthetic patients
- Zero-Shot Clinical Trial Patient Matching with LLMs Wornow et al., Dec. 2024, New England Journal of Medicine
  - Automatic qualitative coding from patient records
- Cohort Discovery: A Survey on LLM-Assisted Clinical Trial Recruitment Ghosh, et al. June 2025, arXiv:2506.15301v1

# Datasets

## TrialGPT dataset

- Koopman, B. & Zuccon, G.  
A test collection for matching patients to clinical trials. In Proc. 39th International ACM SIGIR Conference on Research and Development in Information Retrieval 669-672 (2016).
- Roberts, K., Demner-Fushman, D., Voorhees, E. M., Bedrick, S. & Hersh, W. R.  
Overview of the TREC 2022 Clinical Trials Track. In Proc. Thirty-First Text REtrieval Conference (TREC 2022) (2022).
- Roberts, K., Demner-Fushman, D., Voorhees, E. M., Bedrick, S. & Hersh, W. R.  
Overview of the TREC 2021 Clinical Trials Track. In Proc. Thirtieth Text REtrieval Conference (TREC 2021) (2021).

## Warnow's paper datasets

- Stubbs, et al.,  
Cohort selection for clinical trials: n2c2 2018 shared task track 1.  
Journal of the American Medical Informatics Association, 26(11):1163–1171, 2019





# Matching patients to clinical trials with large language models

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# TrialGPT Dataset

- Number of trials: 75,000
- Synthesized patient notes: 183

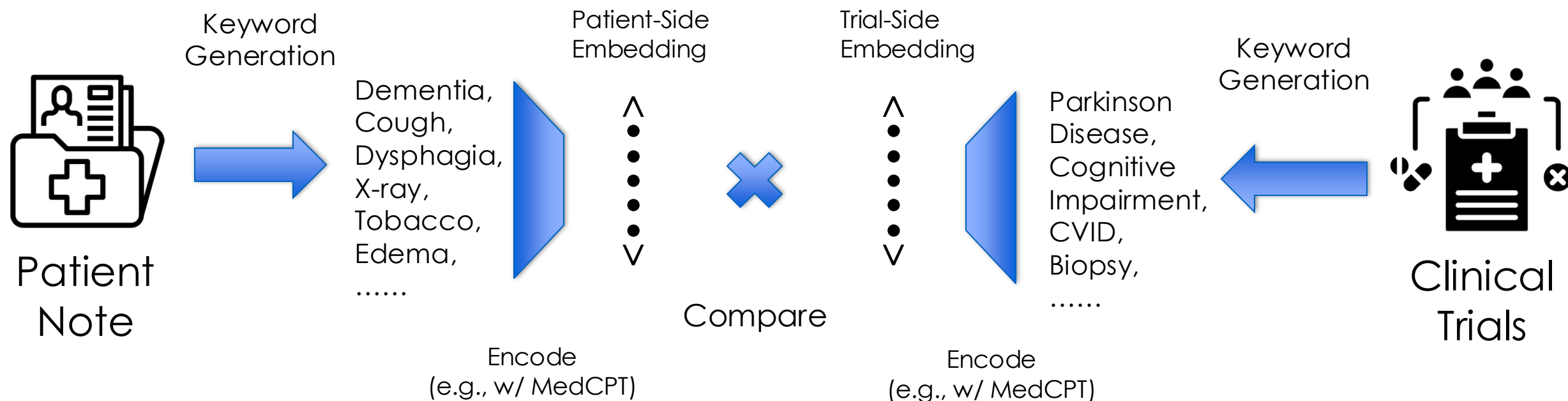
Cohort	SIGIR	TREC 2021 CT	TREC 2022 CT
N	58	75	50
Age (year)	38.5 ± 23.7	41.6 ± 19.4	35.3 ± 20.2
Sex (male: female)	29: 29	38: 37	28: 22
Note length (words)	88.7 ± 36.8	156.2 ± 45.4	109.9 ± 21.6
Eligible trials/patient	7.3 ± 6.7	74.3 ± 49.0	78.8 ± 67.3
Potential trials/ patient	11.7 ± 10.2	None	None
Excluded trials/ patient	None	80.3 ± 60.3	60.7 ± 65.5
Irrelevant trials/ patient	47.1 ± 19.5	323.2 ± 93.2	568.4 ± 164.1
Considered initial trials	3621	26149	26581

We show the mean ± standard deviation for applicable variables. “None” denotes there is no such eligibility label in the corresponding cohort. SIGIR: the patient-trial matching cohort published at the Special Interest Group on Information Retrieval (SIGIR).

TREC the Text REtrieval Conference (TREC), CT the clinical trials track at TREC.

# TrialGPT: Finding Trials for Patients

## 1. Retrieval based on embedding similarity



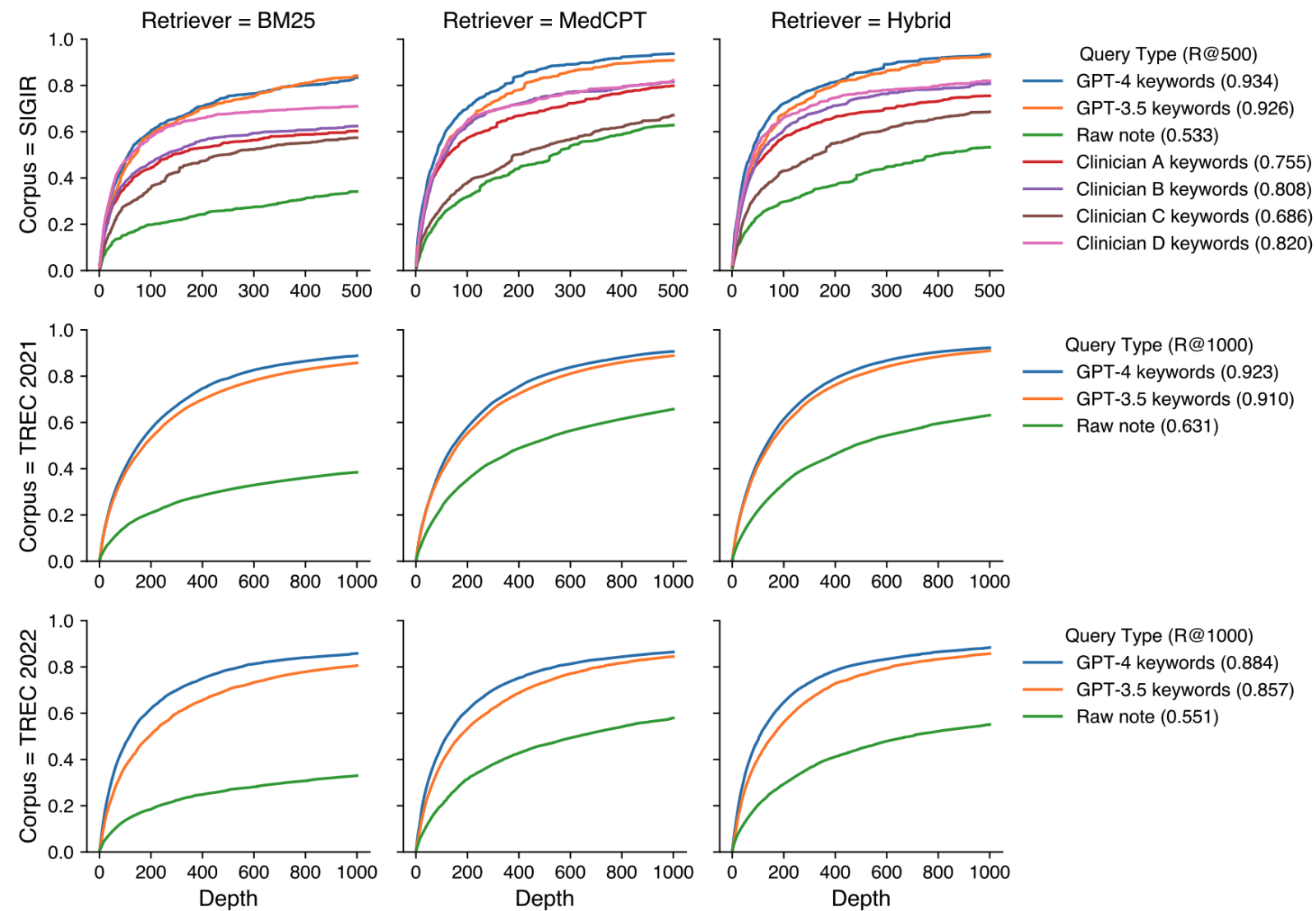
## 2. Matching: Use LLM to patch patient record with retrieved trials

# Recall of Dense Retrieval

	Recall	#Retrieved
SIGIR	93%	500
TREC 2021	92%	1000
TREC 2022	88%	1000

What if the life-saving trial is not retrieved?

False negatives can be fatal



**Fig. 2 | First-stage retrieval results. a** Overview of TrialGPT-Retrieval. LLMs first generate a list of keywords for a given patient note. These keywords are used to derive the keyword-level relevant clinical trials, which are then fused to generate a final ranking. **b** Recalls of relevant clinical trials at different depths for various query

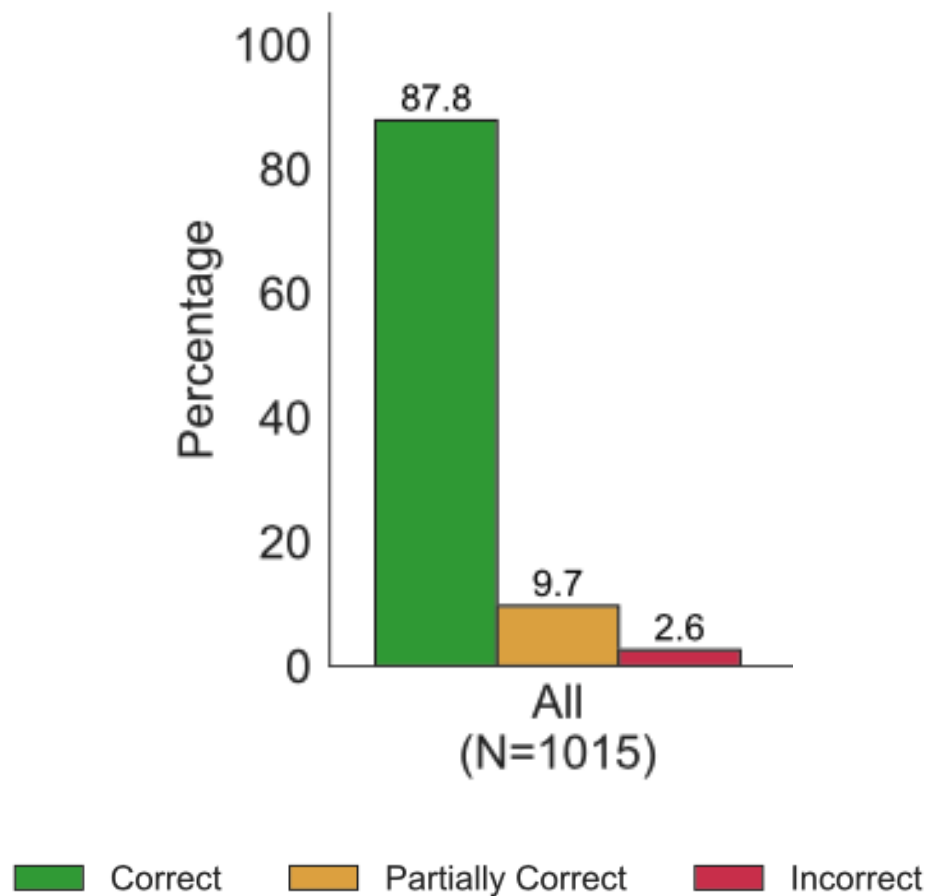
types and retrievers. The hybrid retriever combines the results of the BM25 (lexical matching) and the MedCPT (semantic matching) retrievers. Source data are provided as a Source Data file.

# Precision of Dense Retrieval

## (Embedding-Similarity Based)

- Imprecise – many false positives
- Based on “Semantic Similarity” not “Logical Compatibility”
- E.g. If a patient has diabetes
  - Retrieved trials:
    - Those that *include* diabetes
    - Those that *exclude* diabetes

# LLM's Precision in Matching

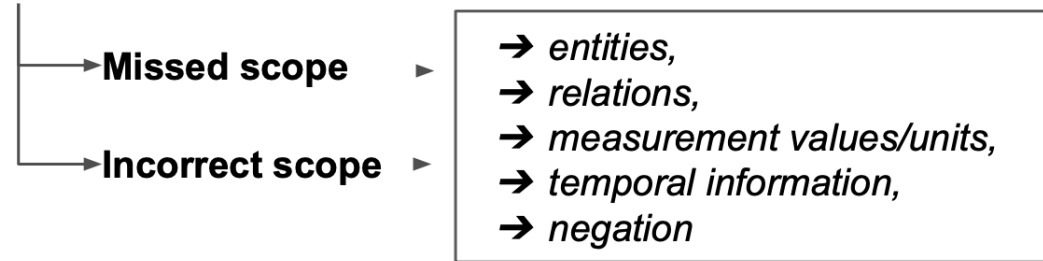


- Is LLM precise in the match?
- Human evaluators also differ

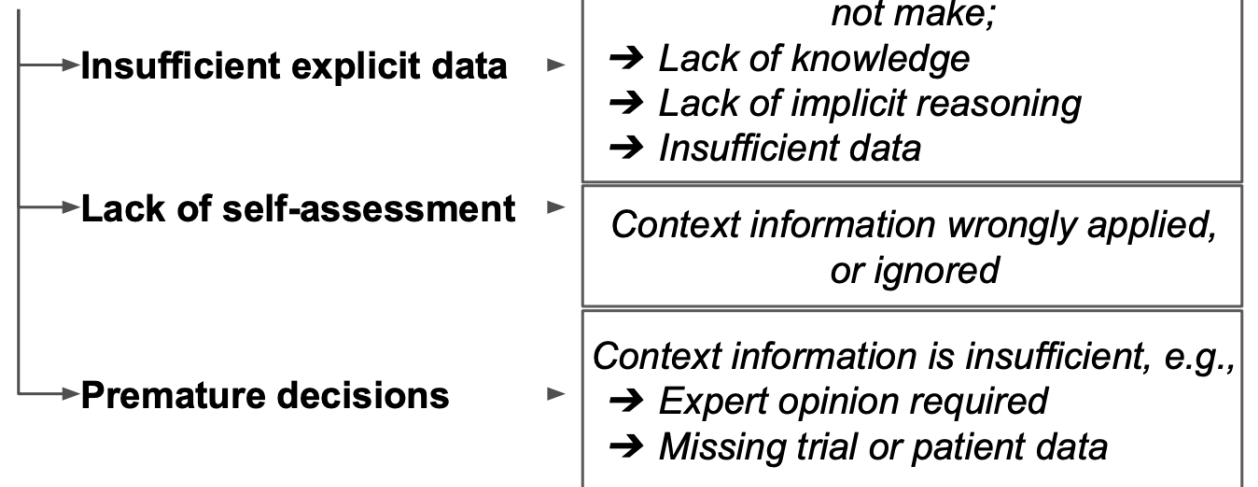
# Summary of Errors

- Dense retrieval may have false negatives
- LLM matching may be erroneous

## 1. Information Isolation Errors



## 2. Reasoning Errors



## 3. Inconsistency Errors

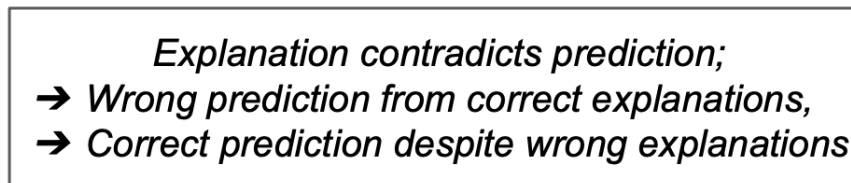


Figure 2: Taxonomy of errors in LLM-generations.

# Outline

- Motivation
- Prior Work
- **SMT Approach**
- SMT-Based Matching Algorithm
- Preliminary Results

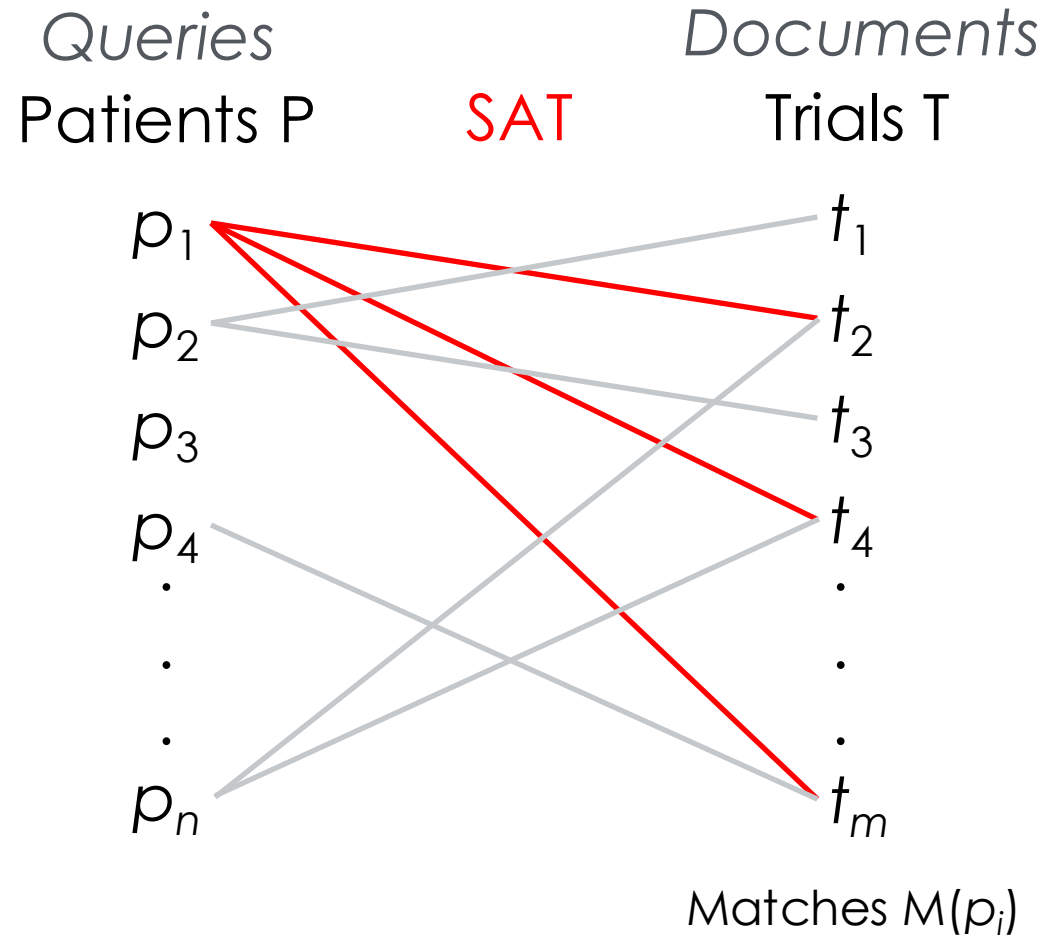
# Goal: No False Negatives & High Precision

Aim to prevent:

- Patients miss life-saving opportunities
- Trials miss recruit targets
  - Stall scientific progress
  - Incur monetary loss
  - Waste researchers' time



# Problem Statement



For every patient,  
find all trials the patient satisfies (**SAT**)  
→ pairwise operation  $O(N^2)$

Two-step approach:

1. Retrieve

- With **no false negatives**
- As few false positives as possible
- Quickly

2. Match patient with retrieved trials

- Accurately and quickly

LECTURE OBJECTIVE

CAN WE LEVERAGE

FORMAL NOTATIONS AND A THEOREM SOLVER  
TO IMPROVE PRECISION AND RECALL?

# 2 Formal Notations

- **Propositional Logic (PL)**
  - Variables: Boolean
  - Operators: and, or, not
  - Conjunctive Normal Form:  $A \text{ and } (B \text{ or } C) \text{ and } (A \text{ or } D) \dots$
  - SAT solver: checks if a set of PL statements are satisfiable
- **SMT (Satisfiability Modulo Theories)** ← More expressive than PL,  
SMT solver more expensive than SAT solver
  - Variables: Boolean, number, ...
  - Operators: and, or, not, arithmetic, numeric comparison
  - Conjunctive Normal Form:  
 $A \text{ and } (B > 10 \text{ or } C + B > D) \text{ and } (A \text{ or } E) \dots$
  - SMT solver: checks if a set of SMT statements are satisfiable

# Example

<https://www.clinicaltrials.gov/study/NCT00092885>

**An Approved Drug to Study a New Indication for Seasonal Allergic Rhinitis in Patients With Asthma (0476-269)**

**ClinicalTrials.gov ID** ⓘ NCT00092885

**Sponsor** ⓘ Organon and Co

**Information provided by** ⓘ Organon and Co (Responsible Party)

**Last Update Posted** ⓘ 2024-08-15

# Eligibility

## Description

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### Inclusion Criteria:

- Non-smoker
- A 2-year documented history of seasonal allergic rhinitis
- A 1-year documented history of chronic asthma
- Positive allergy testing

### Exclusion Criteria:

- Medical history of a lung disorder (other than asthma) or a recent upper respiratory tract infection.

## Ages Eligible for Study ⓘ

---

15 Years to 85 Years (Child, Adult, Older Adult )

## Sexes Eligible for Study ⓘ

---

All

## Accepts Healthy Volunteers ⓘ

---

No

# Inclusion Constraints in SMT

## Free-text Requirements

Non-smoker  
A 2-year documented history of seasonal allergic rhinitis  
A 1-year documented history of chronic asthma  
Positive allergy testing

## Requirements as SMT Constraints

```
(declare-const patient_has_finding_of_tobacco_smoking_behavior_now Bool)
(assert (not patient_has_finding_of_tobacco_smoking_behavior_now))

(declare-const patient_has_finding_of_seasonal_allergic_rhinitis_inthehistory Bool)
(assert (and patient_has_finding_of_seasonal_allergic_rhinitis_inthehistory
  (>= duration_of_documented_history_of_seasonal_allergic_rhinitis_in_years 2.0)))

(declare-const duration_of_chronic_asthma_in_years Real)
(declare-const patient_has_diagnosis_of_asthma_inthehistory Bool)
(declare-const patient_has_diagnosis_of_asthma_inthehistory@@chronic Bool)
(declare-const patient_has_diagnosis_of_asthma_inthehistory@@duration_at_least_1_year Bool)
(assert (=> patient_has_diagnosis_of_asthma_inthehistory@@chronic
  patient_has_diagnosis_of_asthma_inthehistory))
(assert (=> patient_has_diagnosis_of_asthma_inthehistory@@duration_at_least_1_year
  patient_has_diagnosis_of_asthma_inthehistory))
(assert (= patient_has_diagnosis_of_asthma_inthehistory@@duration_at_least_1_year
  (>= duration_of_chronic_asthma_in_years 1.0)))
(assert (and patient_has_diagnosis_of_asthma_inthehistory@@chronic
  patient_has_diagnosis_of_asthma_inthehistory@@duration_at_least_1_year))
```

# Patient Record as SMTs (Example 1)

A 58-year-old African-American woman presents to the ER with episodic pressing/burning anterior chest pain that began two days earlier for the first time in her life.

The pain started while she was walking, radiates to the back, and is accompanied by nausea, diaphoresis and mild dyspnea, but is not increased on inspiration.

The latest episode of pain ended half an hour prior to her arrival.

(assert patient\_has\_undergone\_emergency\_room\_admission\_inthehistory)

(assert patient\_has\_symptoms\_of\_anterior\_chest\_wall\_pain\_now)

(assert patient\_has\_finding\_of\_anterior\_chest\_wall\_pain\_inthepast2days)

(assert patient\_has\_symptoms\_of\_dyspnea\_inthepast2days)

# Patient Record as SMTs (Example 2)

She is known to have hypertension and obesity.

She denies smoking, diabetes, hypercholesterolemia, or a family history of heart disease.

She currently takes no medications.

Physical examination is normal.

The EKG shows nonspecific changes.

```
(assert patient_has_diagnosis_of_hypertensive_disorder_now)
```

```
(assert (not patient_has_diagnosis_of_hypercholesterolemia_now))
```

```
(assert patient_has_undergone_electrocardiographic_procedure)
```

```
(assert (not patient_has_finding_of_diabetes_procedure_now))
```



# Trial Matching as a Satisfiability Problem

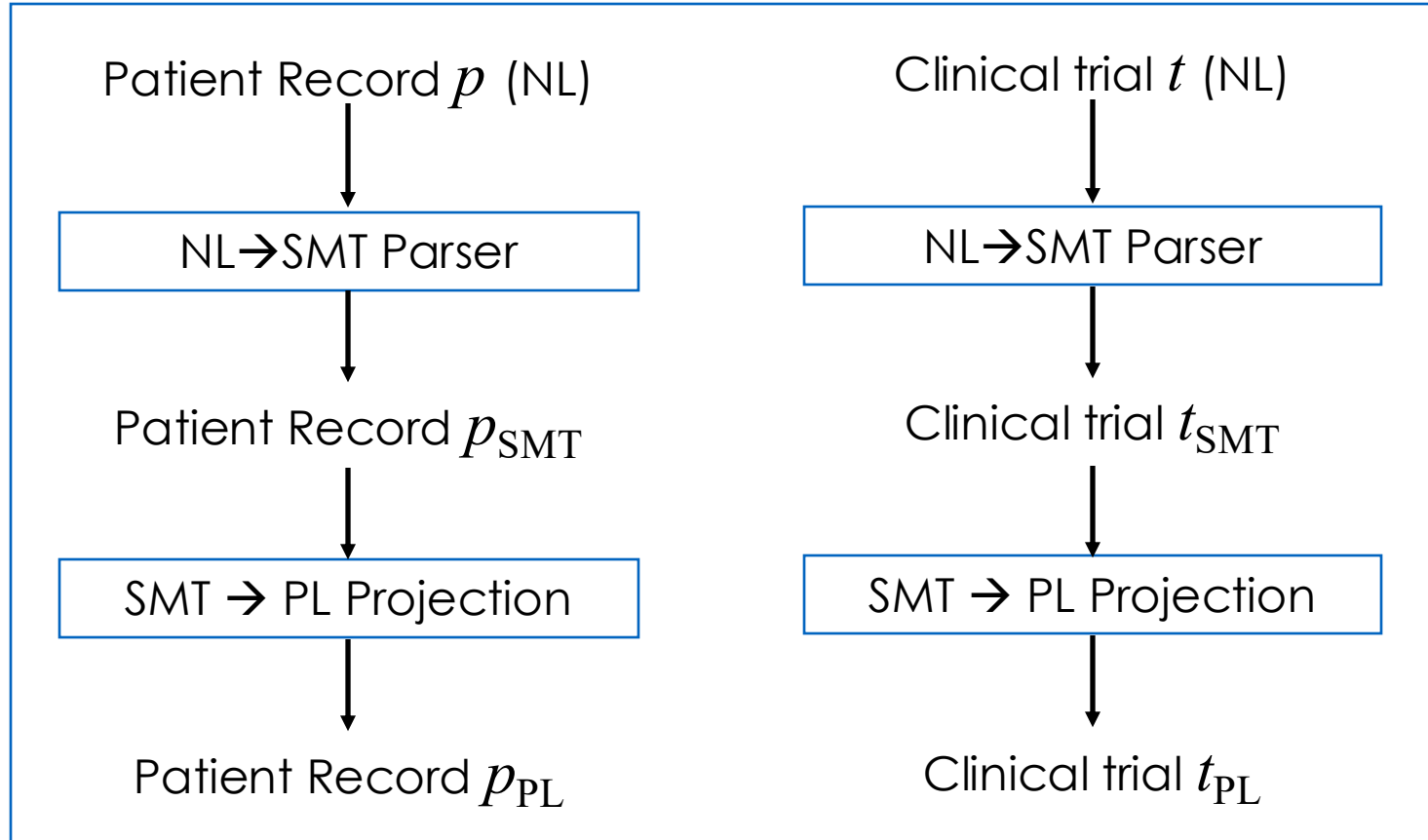
- Given
  - (A) Trial Requirements in SMT
  - (B) Patient Fact Value Assertions in SMT
- There is a match if and only if (A) + (B) is satisfiable
- Use SMT Solver (Z3)

Z3 theorem prover, Microsoft, <https://github.com/Z3Prover/z3>

# How Do We Retrieve the Potential Trials?

- Dense retrieval has inadequate recall and precision
- Use Propositional Logic (PL)
- Represent PL constraints and assertions in databases
  - Match all records quickly!

# Clinical Trail Matching using SMT



NL: Natural language  
SMT: Satisfiability Modulo Theory  
PL: Propositional Logic

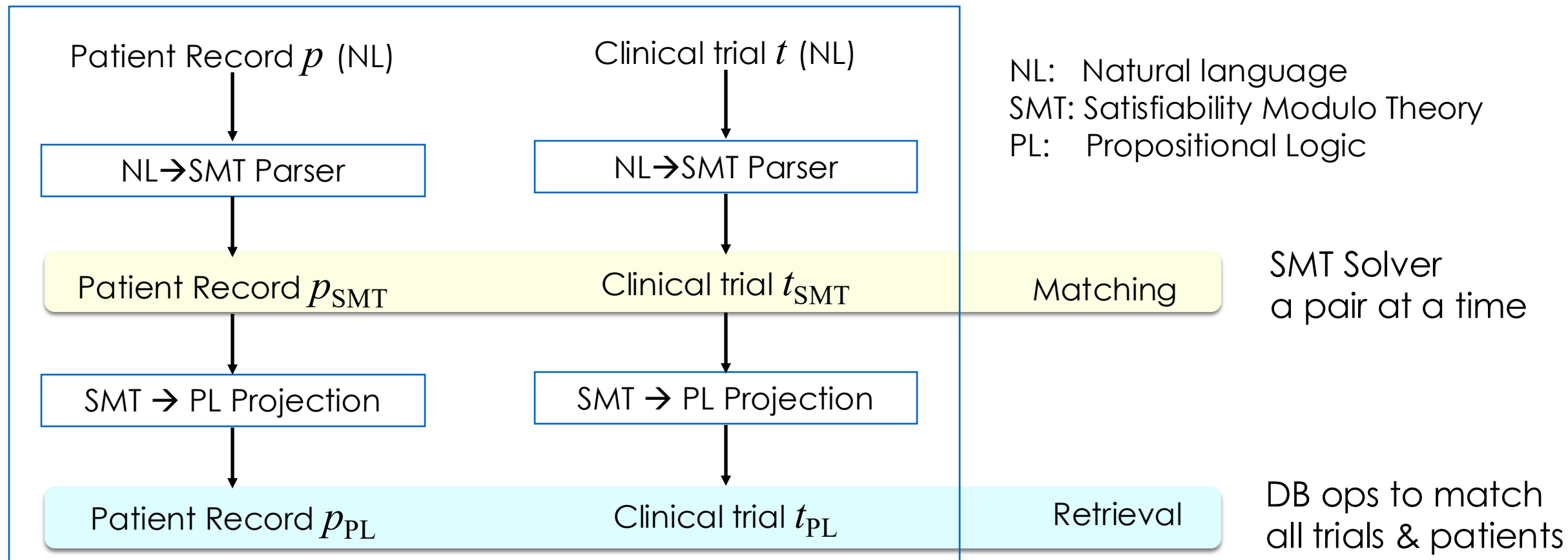
$A \models B$  means  
if  $A$  is satisfiable,  $B$  is satisfiable

$$p_{\text{SMT}} \models p_{\text{PL}}$$

$$t_{\text{SMT}} \models t_{\text{PL}}$$

$$p_{\text{SMT}} \wedge t_{\text{SMT}} \models p_{\text{PL}} \wedge t_{\text{PL}}$$

# Clinical Trail Matching using SMT



$$p_{\text{SMT}} \models p_{\text{PL}}$$

$$t_{\text{SMT}} \models t_{\text{PL}}$$

$$p_{\text{SMT}} \wedge t_{\text{SMT}} \models p_{\text{PL}} \wedge t_{\text{PL}}$$

SAT AND SMT SOLVERS ARE PROVABLY CORRECT

IF SEMANTICS OF THE PATIENT RECORDS AND TRIALS  
IS CAPTURED PERFECTLY IN SMT

➔ 100% PRECISION AND RECALL

But errors may be introduced in the NL→SMT parse

# Advantages of SMT-PL-Approach

- LLM is used to **parse** text to SMT **once** per document
  - **Accuracy:**  
Parsing is easier than reasoning
    - Defer reasoning to SMT solvers
    - Easier to improve
    - Contexts are smaller
  - **Efficiency:**  $O(n)$ , not  $O(n^2)$
  - **Interpretability**
    - Can be checked & corrected
- **Retrieval** of satisfiable trials from **millions** of trials/patients in PL is **efficient & accurate**
  - Expressiveness is close to SMT for trials  
small % that sat  $t_{PL}$  and not sat  $t_{SMT}$   
→ **few false positives**
- **Matching** of satisfiable trials in **SMT** on **hundreds** of pairwise trials/patients is **efficient and accurate**

# Disadvantages of SMT Approach

- SMT solvers operate on variables – they have no meaning
- All information useful for SMT solvers must be included formally
  - **Taxonomy**: same concepts  $\rightarrow$  same variable name **Canonicalization**
  - **Implication relations** between concepts  
e.g.  $A \Rightarrow B$  means  $A$  satisfies  $B$  **Include implication relations**
  - **Judgment**: Distinguish between hard and soft constraints  
e.g. “Patient must have an Xray” can always be satisfied **Requirement classification**
  - **Interpretation**: Require common knowledge  
e.g. “A heavy smoker”  $\Rightarrow$  daily-cigarettes  $\geq 10$  **Not handled yet**

SMT  
POWERFUL REASONING

LLMs  
WORLD KNOWLEDGE



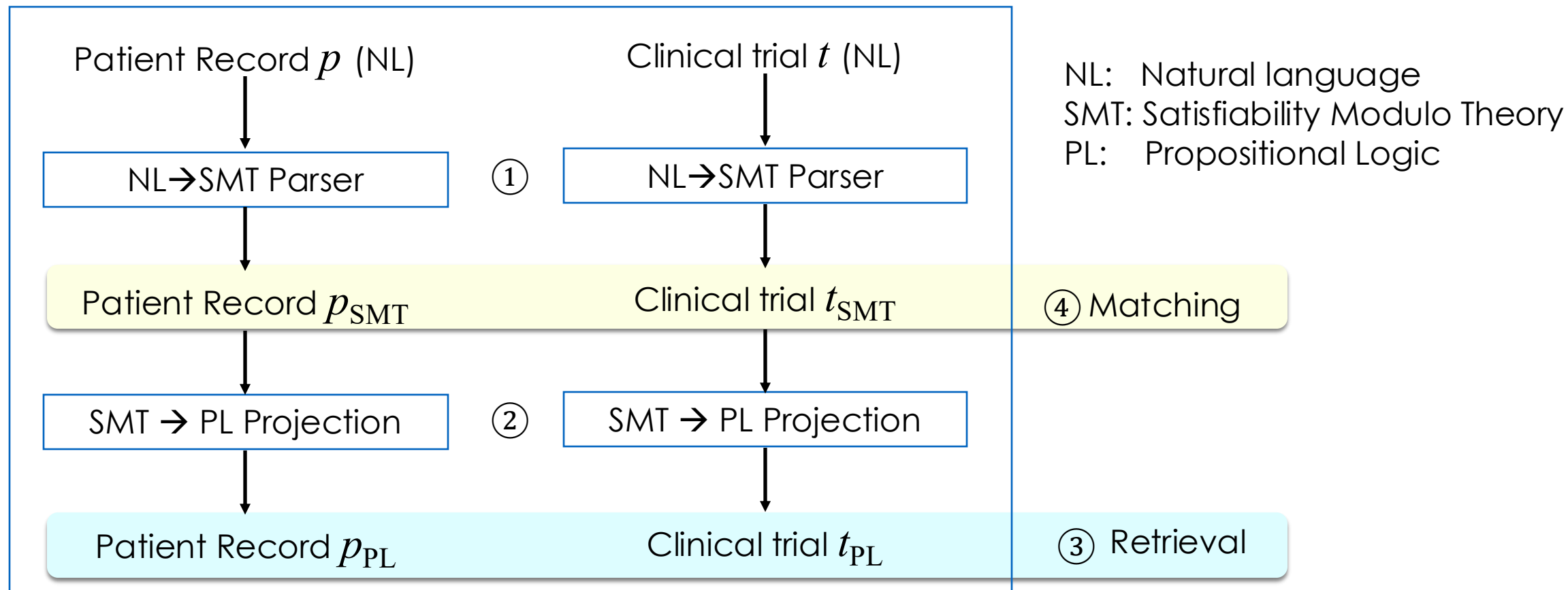
CAN WE COMBINE THE BEST OF BOTH?



# Outline

- Motivation
- TrialGPT Dataset and Prior Work
- SMT Approach
- **SMT-Based Matching Algorithm**
- Preliminary Results

# Clinical Trail Matching using SMT



$$p_{\text{SMT}} \models p_{\text{PL}}$$

$$t_{\text{SMT}} \models t_{\text{PL}}$$

$$p_{\text{SMT}} \wedge t_{\text{SMT}} \models p_{\text{PL}} \wedge t_{\text{PL}}$$

# ALGORITHM

1. NL  $\rightarrow$  SMT

# Challenge 1: Canonicalization

Theorem provers operate on variables – they have no meaning

- Same concepts must be represented by the same variables

**Example**

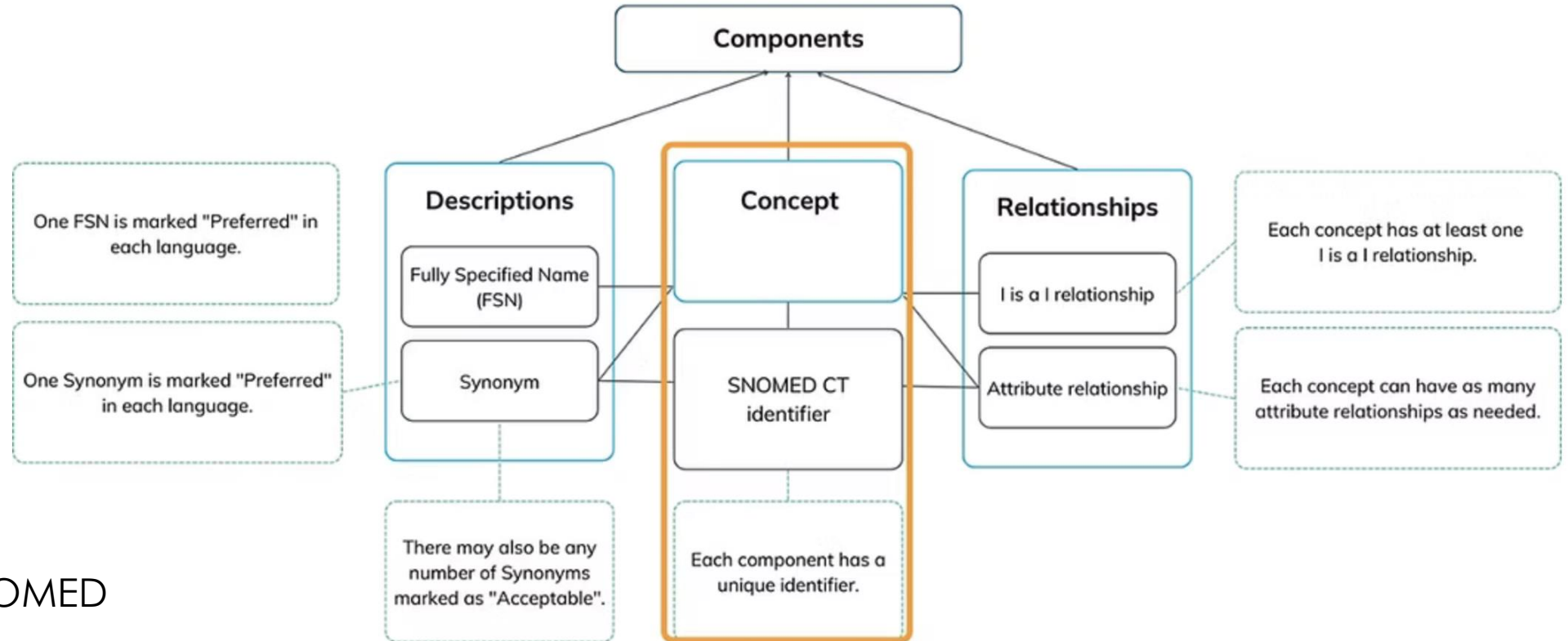
	Patient $p$	Trial $t$	$p \wedge t$
English	Has dizzy spells	Has syncope	SAT
Semantic parse	Has-dizzy-spell	Has-syncope	UNSAT
Canonicalized	Has-syncope	Has-syncope	SAT

Canonicalization is complex! So many clinical terms!

Standardization in medicine are important → SNOMED

# SNOMED

## Systematized Nomenclature of Medicine – Clinical Terms

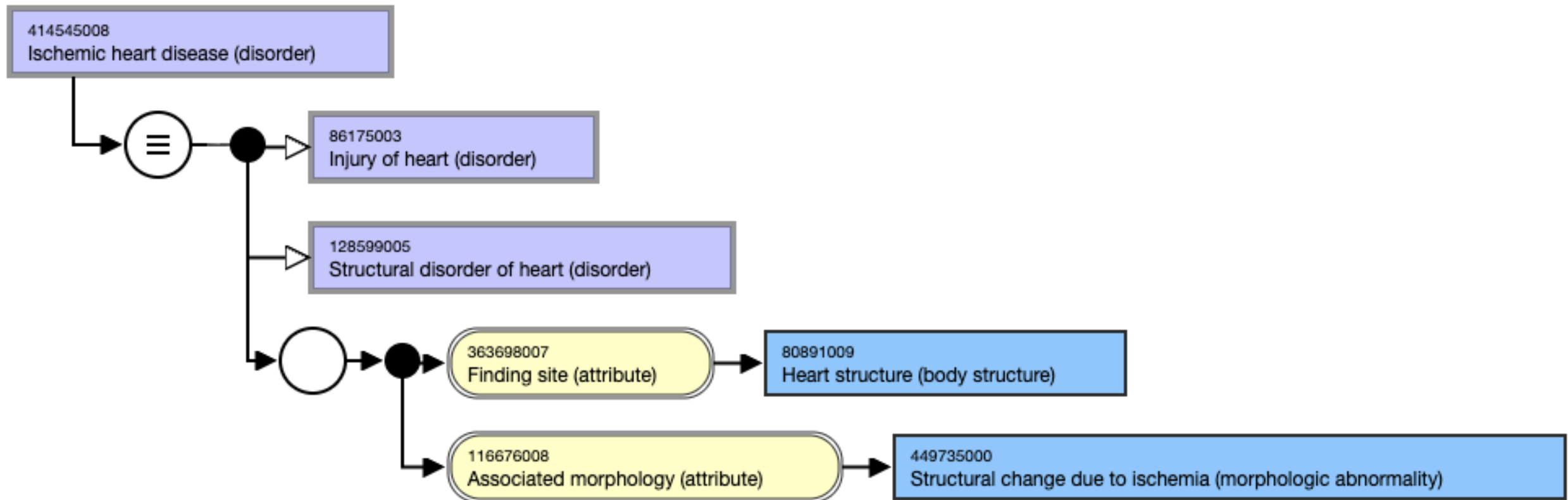


### US Edition of SNOMED

- 300,000 unique concepts
- Over 1,000,000 descriptions
- 903,000 links or semantic relationships between concepts.

# SNOMED Example

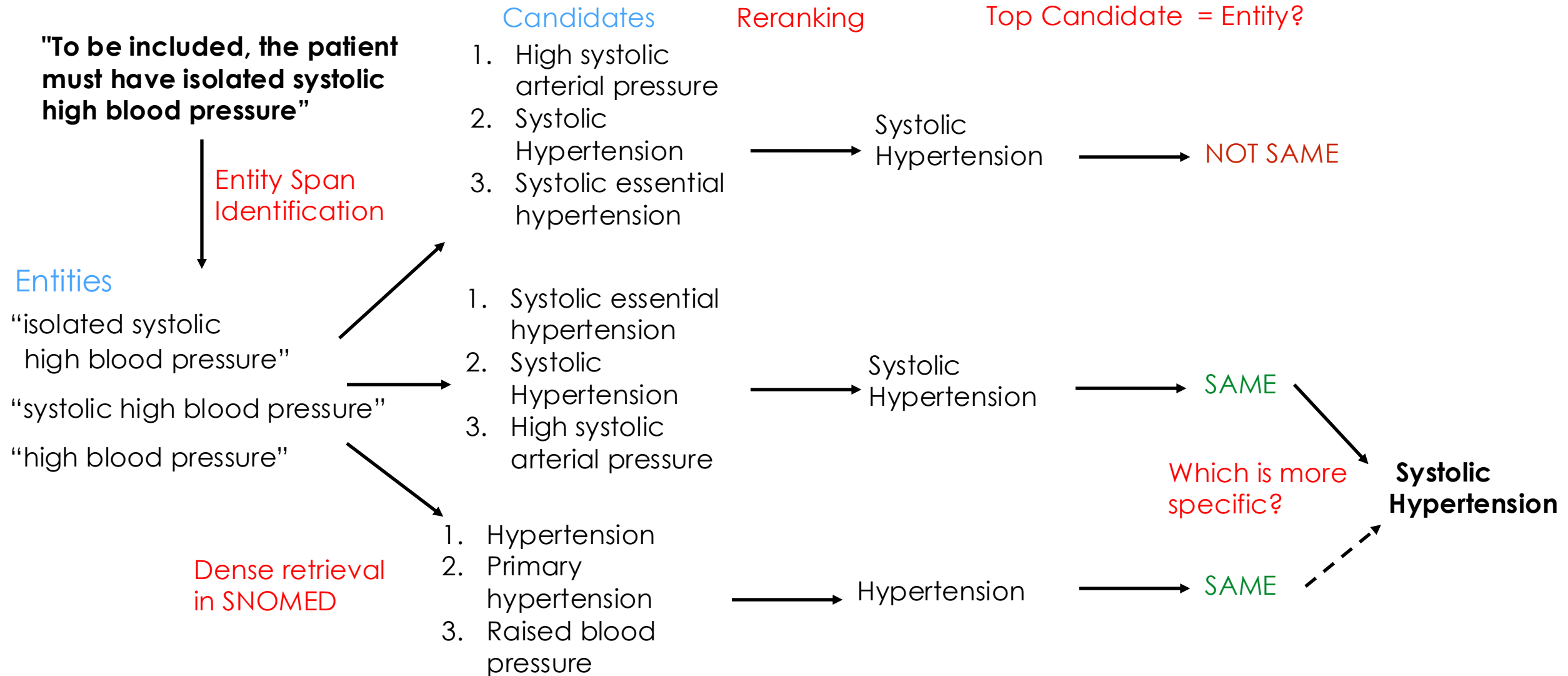
- Top-level classes: Clinical Findings, Procedures, Observable Entities, Substances...



# Canonicalization Algorithm

- Problem: NER (Named entity recognizer)
  - Turn the informal English to the closest term in SNOMED
- Challenges and solutions:
  - Many similar terms in SNOMED → Dense retrieval
  - Find the most precise entity in SNOMED →  
Retrieve, rerank, ensure equivalence, pick the most precise

# How to Canonicalize?





# Challenge 2: Relationships Between Terms

## Example

	Patient $p$	Trial $t$	$p \wedge t$
Canonicalized	Has-acute-chest-pain	Has-chest-pain	UNSAT
With implications	Has-acute-chest-pain (Has-chest-pain)	Has-chest-pain	SAT

- **SNOMED includes implication relations between terms:**

$\exists$  an edge  $(x,y)$  if  $y \Rightarrow x$

Has-chest-pain  
↓  
Has-acute-chest-pain

Has-acute-chest-pain  $\Rightarrow$  Has-chest-pain  
 $\neg$  Has-chest-pain  $\Rightarrow$   $\neg$  Has-acute-chest-pain

- Codifying the patient
  - If patient has  $x$ , includes ancestors of  $x$
  - If patient has  $\neg x$ , includes the  $\neg$  of all descendants of  $x$

# Challenge 3: How to Handle Missing Information?

- Properties that can be satisfied (can be ignored)

**Example**

	Patient $p$	Trial $t$	$p \wedge t$
Semantic parse		Has-a-chest-Xray	SAT

- Use LLM for classification
- Required properties, Expected to be reported: Default to “False”

**Example**

	Patient $p$	Trial $t$	$p \wedge t$
Semantic parse		Has-chest-pain	UNSAT

## ALGORITHM

1. NL  $\rightarrow$  SMT

**2. SMT  $\rightarrow$  PL**

SMT IS MORE EXPRESSIVE THAN PL

# Example:

At least three of the following disorders: A, B, C, D, E, F, G

## SMT

```
(assert (>= (+
    (ite A 1 0)
    (ite B 1 0)
    (ite C 1 0)
    (ite D 1 0)
    (ite E 1 0)
    (ite F 1 0)
    (ite G 1 0))
  3))
```

Ite: If This Then

## PL

```
(A $\wedge$ B $\wedge$ C)  $\vee$  (A $\wedge$ B $\wedge$ D)  $\vee$  (A $\wedge$ B $\wedge$ E)  $\vee$  (A $\wedge$ B $\wedge$ F)
 $\vee$  (A $\wedge$ B $\wedge$ G)  $\vee$  (A $\wedge$ C $\wedge$ D)  $\vee$  (A $\wedge$ C $\wedge$ E)  $\vee$  (A $\wedge$ C $\wedge$ F)
 $\vee$  (A $\wedge$ C $\wedge$ G)  $\vee$  (A $\wedge$ D $\wedge$ E)  $\vee$  (A $\wedge$ D $\wedge$ F)  $\vee$  (A $\wedge$ D $\wedge$ G)
 $\vee$  (A $\wedge$ E $\wedge$ F)  $\vee$  (A $\wedge$ E $\wedge$ G)  $\vee$  (A $\wedge$ F $\wedge$ G)  $\vee$  (B $\wedge$ C $\wedge$ D)
 $\vee$  (B $\wedge$ C $\wedge$ E)  $\vee$  (B $\wedge$ C $\wedge$ F)  $\vee$  (B $\wedge$ C $\wedge$ G)  $\vee$  (B $\wedge$ D $\wedge$ E)
 $\vee$  (B $\wedge$ D $\wedge$ F)  $\vee$  (B $\wedge$ D $\wedge$ G)  $\vee$  (B $\wedge$ E $\wedge$ F)  $\vee$  (B $\wedge$ E $\wedge$ G)
 $\vee$  (B $\wedge$ F $\wedge$ G)  $\vee$  (C $\wedge$ D $\wedge$ E)  $\vee$  (C $\wedge$ D $\wedge$ F)  $\vee$  (C $\wedge$ D $\wedge$ G)
 $\vee$  (C $\wedge$ E $\wedge$ F)  $\vee$  (C $\wedge$ E $\wedge$ G)  $\vee$  (C $\wedge$ F $\wedge$ G)  $\vee$  (D $\wedge$ E $\wedge$ F)
 $\vee$  (D $\wedge$ E $\wedge$ G)  $\vee$  (D $\wedge$ F $\wedge$ G)  $\vee$  (E $\wedge$ F $\wedge$ G)
```

SMT is direct and succinct

# Example:

At least two procedures within 30 days before index day

SMT

PL

```
; Uninterpreted sort of procedure events for a single patient
(declare-sort Proc 0)

; Attributes
(declare-fun kind (Proc) String)
(declare-fun day (Proc) Int)      ; measured in days
(declare-const index_day Int)

; The target procedure type e.g., percutaneous coronary intervention
(define-fun is_target ((p Proc)) Bool (= (kind p) "PCI"))

; "At least two distinct target procedures in [index-30, index]"
(assert (exists ((p1 Proc) (p2 Proc))
  (and (distinct p1 p2)
    (is_target p1) (is_target p2)
    (<= 0 (- index_day (day p1))) (<= (- index_day (day p1)) 30)
    (<= 0 (- index_day (day p2))) (<= (- index_day (day p2)) 30))))

(check-sat)
```

Cannot be expressed!

# SMT $\rightarrow$ PL

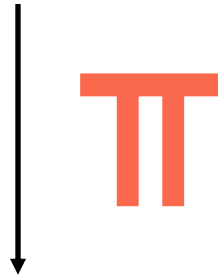
- Goal of translation from SMT  $\rightarrow$  PL:
  - $t_{PL}$  represents the tightest constraints of  $t_{SMT}$  expressible in PL
  - Remove constraints not satisfiable in PL
- Many constraints include non-canonical variables
  - Can we project away the non-canonical variables and retain the rest of the constraints?
  - Use Z3 Quantifier Elimination to handle the projection accurately

Z3 theorem prover, Microsoft, <https://github.com/Z3Prover/z3>

# Example of Projection SMT $\rightarrow$ PL

-- removing non-canonical variables

```
(assert (or has_diagnosis_of_squamous_cell_carcinoma_of_esophagus  
           (and has_diagnosis_of_adenocarcinoma_of_esophagus  
                eligible_for_potentially_curative_radiotherapy))))
```



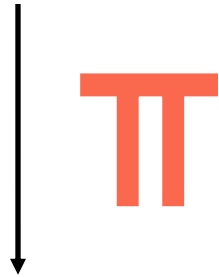
**Canonical variables are marked in bold,**  
other variables are projected away

```
(assert (or has_diagnosis_of_squamous_cell_carcinoma_of_esophagus  
           has_diagnosis_of_adenocarcinoma_of_esophagus))
```



# Examples of Projection

```
(assert (or has_diagnosis_of_squamous_cell_carcinoma_of_esophagus  
           has_diagnosis_of_adenocarcinoma_of_esophagus  
           eligible_for_potentially_curative_radiotherapy))
```



**Canonical variables are marked in bold,**  
other variables are projected away

*NOTHING!*

## ALGORITHM

1. NL  $\rightarrow$  SMT
2. SMT  $\rightarrow$  PL
- 3. RETRIEVAL (PL)**

# Retrieve Potential Trials from Corpus

1. Represent large sets of PL constraints in DB
2. Evaluate satisfaction as a DB queries

# Constraints in a Trial

- All PL formulas can be expressed in Conjunctive Normal Form
- Example of a constraint

$(\textit{has\_bipolar} \vee \textit{has\_depression} \vee \textit{has\_schizo})$   
AND (NOT  $\textit{has\_undergone\_PCI}$ )  
AND ((NOT  $\textit{is\_smoking}$ )  $\vee$  (NOT  $\textit{is\_drinking}$ ))

Disjunctive ( $\vee$ )

Conjunctive ( $\wedge$ )

Variables:  $\textit{has\_bipolar}$ ,  $\textit{has\_depression}$ ,  $\textit{has\_schizo}$ ,  
 $\textit{has\_undergone\_PCI}$ ,  $\textit{is\_smoking}$ ,  $\textit{is\_drinking}$

# Representing PL formulas in DB: Example

c1 {  $(has\_bipolar \text{ OR } has\_depression \text{ OR } has\_schizo)$  a1  
AND (NOT  $has\_undergone\_PCI$ ) a2  
AND ((NOT  $is\_smoking$ ) OR (NOT  $is\_drinking$ )) a3

C	$\wedge$		$\wedge$	V		V	Variables	Is Pos
c1	a1	↗ ↘ ↙ ↘	a1	o1	—	o1	has_bipolar	1
c1	a2	↘ ↙ ↘ ↙	a1	o2	—	o2	has_depression	1
c1	a3	↘ ↙ ↘ ↙	a1	o3	—	o3	has_schizo	1
c2	...	↘ ↙ ↘ ↙	a2	o4	—	o4	has_undergone_PCI	0
c2	...	↘ ↙ ↘ ↙	a3	o5	—	o5	is_smoking	0
c3	...	↘ ↙ ↘ ↙	a3	o6	—	o6	is_drinking	0

Constraints

$\wedge$  (and)

$\vee$  (or)

# Matching Patient 1

*has\_bipolar* AND *has\_depression*

*(has\_bipolar* OR *has\_depression* OR *has\_schizo*)  
AND (NOT *has\_undergone\_PCI*)  
AND ((NOT *is\_smoking*) OR (NOT *is\_drinking*))

pid	Variable	Val
123	has_bipolar	1
123	has_depression	1
123	has_schizo	0
123	has_undergone_PCI	0
123	is_smoking	0
123	is_drinking	0

Patient Fact Table

v	Variable	Is Pos
o1	has_bipolar	1
o2	has_depression	1
o3	has_schizo	1
o4	has_undergone_PCI	0
o5	is_smoking	0
o6	is_drinking	0

v (or)

^	v
a1	o1
a1	o2
a1	o3
a2	o4
a3	o5
a3	o6

^ (and)

C	^
c1	a1
c1	a2
c1	a3

Constraints

# Matching Patient 1

*has\_bipolar* AND *has\_depression*      (*has\_bipolar* OR *has\_depression* OR *has\_schizo*)  
AND (NOT *has\_undergone\_PCI*)  
AND ((NOT *is\_smoking*) OR (NOT *is\_drinking*))

pid	Variable	Val
123	has_bipolar	1
123	has_depression	1
123	has_schizo	0
123	has_undergone_PCI	0
123	is_smoking	0
123	is_drinking	0

Patient Fact Table



Eval v

v	Variable	Is Pos
o1	has_bipolar	1
o2	has_depression	1
o3	has_schizo	1
o4	has_undergone_PCI	0
o5	is_smoking	0
o6	is_drinking	0

v (or)

^	v
a1	o1
a1	o2
a1	o3
a2	o4
a3	o5
a3	o6

^ (and)

C	^
c1	a1
c1	a2
c1	a3

Constraints

# Matching Patient 1

*has\_bipolar* AND *has\_depression*      (*has\_bipolar* OR *has\_depression* OR *has\_schizo*)  
AND (NOT *has\_undergone\_PCI*)  
AND ((NOT *is\_smoking*) OR (NOT *is\_drinking*))

pid	v	Variable	Is_pos	v-Eval
123	o1	has_bipolar	1	1
123	o2	has_depression	1	1
123	o3	has_schizo	1	0
123	o4	has_undergone_PCI	0	1
123	o5	is_smoking	0	1
123	o6	is_drinking	0	1

v-Evaluated



Eval  $\wedge$

$\wedge$	v
a1	o1
a1	o2
a1	o3
a2	o4
a3	o5
a3	o6

$\wedge$  (and)

C	$\wedge$
c1	a1
c1	a2
c1	a3

Constraints



# Matching Patient 1

*has\_bipolar* AND *has\_depression*      (*has\_bipolar* OR *has\_depression* OR *has\_schizo*)  
AND (NOT *has\_undergone\_PCI*)  
AND ((NOT *is\_smoking*) OR (NOT *is\_drinking*))

$\wedge$	$\wedge\_eval$
a1	1
a2	1
a3	1

C	$\wedge$
c1	a1
c1	a2
c1	a3



C	C_eval
C1	1

**SAT!**

$\wedge$  (and) Evaluated



Constraints

Eval Constraints

# Matching Patient 2

*has\_schizo* AND *is\_smoking*  
AND *is\_drinking*

(*has\_bipolar* OR *has\_depression* OR *has\_schizo*)  
AND (NOT *has\_undergone\_PCI*)  
AND ((NOT *is\_smoking*) OR (NOT *is\_drinking*))

pid	Variable	val
123	has_bipolar	0
123	has_depression	0
123	has_schizo	1
123	has_undergone_PCI	0
123	is_smoking	1
123	is_drinking	1

Patient Fact Table

v	Variable	Is Pos
o1	has_bipolar	1
o2	has_depression	1
o3	has_schizo	1
o4	has_undergone_PCI	0
o5	is_smoking	0
o6	is_drinking	0

v (or)

^	v
a1	o1
a1	o2
a1	o3
a2	o4
a3	o5
a3	o6

^ (and)

C	^
c1	a1
c1	a2
c1	a3

Constraints

# Matching Patient 2

*has\_schizo* AND *is\_smoking*  
AND *is\_drinking*

(*has\_bipolar* OR *has\_depression* OR *has\_schizo*)  
AND (NOT *has\_undergone\_PCI*)  
AND ((NOT *is\_smoking*) OR (NOT *is\_drinking*))

pid	Variable	val
123	has_bipolar	0
123	has_depression	0
123	has_schizo	1
123	has_undergone_PCI	0
123	is_smoking	1
123	is_drinking	1

Patient Fact Table

v	Variable	Is Pos
o1	has_bipolar	1
o2	has_depression	1
o3	has_schizo	1
o4	has_undergone_PCI	0
o5	is_smoking	0
o6	is_drinking	0

v (or)

^	v
a1	o1
a1	o2
a1	o3
a2	o4
a3	o5
a3	o6

^ (and)

C	^
c1	a1
c1	a2
c1	a3

Constraints



Eval v

# Matching Patient 2

*has\_schizo* AND *is\_smoking*  
AND *is\_drinking*

(*has\_bipolar* OR *has\_depression* OR *has\_schizo*)  
AND (NOT *has\_undergone\_PCI*)  
AND ((NOT *is\_smoking*) OR (NOT *is\_drinking*))

pid	v	Variable	Is_pos	V-Eval
123	o1	has_bipolar	1	0
123	o2	has_depression	1	0
123	o3	has_schizo	1	1
123	o4	has_undergone_PCI	0	1
123	o5	is_smoking	0	0
123	o6	is_drinking	0	0

V-Evaluated



Eval  $\wedge$

$\wedge$	V
a1	o1
a1	o2
a1	o3
a2	o4
a3	o5
a3	o6

$\wedge$  (and)

C	$\wedge$
c1	a1
c1	a2
c1	a3

Constraints

# Matching Patient (unsat)

*has\_schizo* AND *is\_smoking*  
AND *is\_drinking*

(*has\_bipolar* OR *has\_depression* OR *has\_schizo*)  
AND (NOT *has\_undergone\_PCI*)  
AND ((NOT *is\_smoking*) OR (NOT *is\_drinking*))

$\wedge$	$\wedge\_eval$
a1	1
a2	1
a3	0

C	$\wedge$
c1	a1
c1	a2
c1	a3



C	C_eval
C1	0

**UNSAT!**

$\wedge$  (and) Evaluated



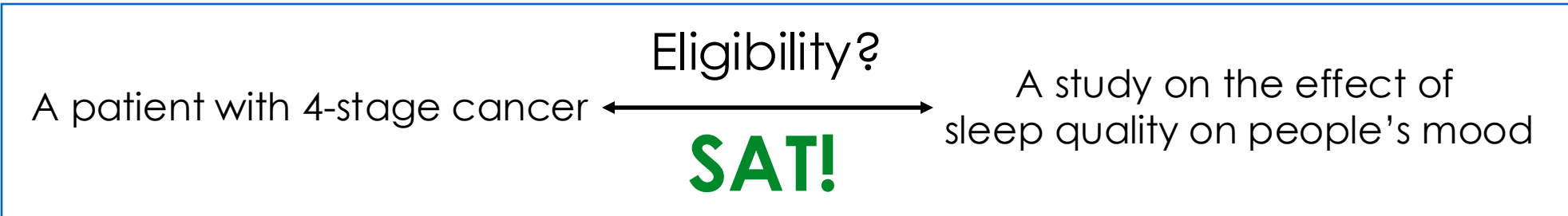
Constraints

Eval Constraints

# Speed and Accuracy of Retrieval

- The above example shows that
  - A few DB calls can match 1 patient with 1 constraint
  - The procedure **handles all constraints in all trials**
  - Adding one more level of eval handles **eligibility of all trials**
- Very fast and efficient

# Relevance



- Eligibility is inadequate
  - Patients seeking clinical trials have a medical concern
- Solution: Add a **relevance filter** to retrieval



## ALGORITHM

1. NL  $\rightarrow$  SMT
2. SMT  $\rightarrow$  PL
3. RETRIEVAL (PL)
- 4. MATCHING (SMT)**



# Trial Matching as a Satisfiability Problem

- Given
  - (A) Trial Requirements in SMT
  - (B) Patient Fact Value Assertions in SMT
- There is a match if and only if (A) + (B) is satisfiable
- Use SMT Solver (Z3)

Z3 theorem prover, Microsoft, <https://github.com/Z3Prover/z3>

# Matching with SMT

Trial Req  
(SMT) {  
  (assert (or has\_cough has\_fever))  
  (assert (>= age 18))  
  (assert (not is\_pregnant))

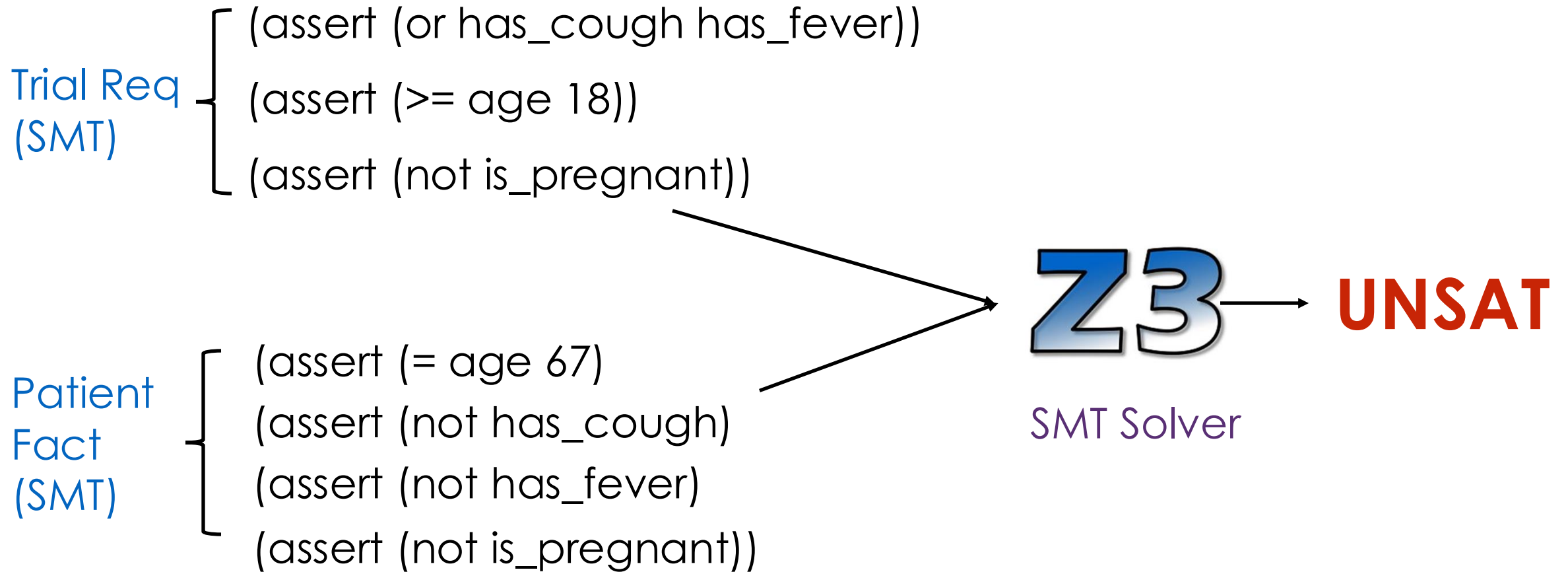
Patient  
Fact  
(SMT) {  
  (assert (= age 67))  
  (assert has\_cough)  
  (assert (not is\_pregnant))

**Z3**

SMT Solver

**SAT!**

# Matching with SMT (Using Z3)



# Outline

- Motivation
- TrialGPT Dataset and Prior Work
- SMT Approach
- SMT-Based Matching Algorithm
- **Preliminary Results**

# Experiment: Based on TrialGPT

- Definition
  - Eligible trials:  
Patient satisfies all constraints, with diseases targeted by trials
  - Relevant trials: Trials address the patient's major disease
  - Potential trials: Trials for similar diseases
- Experiment setup
  - Number of trials: 3621
  - Represented in SMT and PL

# Very Preliminary Results on Retrieval

- On 8 patients:
  - **Relevant and not excluded explicitly: 105 (average)**
  - **Our Recall: 100%**  
(Checked the disagreement with gold by hand)
- For 4 out of the 8 patients (Based on sampling 10%)
  - The gold is full of errors! Checked by hand
  - **Our Precision: 89%**
  - Use TrialGPT to retrieve the same number of trials
    - **Their Precision: 56%**

**Speed: 2.95s** per patient against 3621 trials (M2 MacBook, SQLite)

# Conclusions

- **SMT: A new approach to large-scale hard reasoning tasks**
  - Important application: clinical trials matching
- **PL DB: Retrieval with 100% recall and high precision**
- **LLMs encodes the logic of the constraints ONCE and FOR ALL**
  - Interpretable!
  - **Requires canonicalization: SNOMED**

## Future research

- Can we apply to other domains? Use of SMTs requires canonicalization
  - Medicine has SNOMED
  - **Can we use AI to canonicalize other fields?**
- Can we improve LLMs reasoning skills?
  - **By fine-tuning LLMs to learn the formal representation?**