Lecture 1: Introduction to RL

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CS234 RL

Winter 2026

- Today the 3rd part of the lecture includes some slides from David Silver's introduction to RL slides or modifications of those slides

RL in 2025

- "We introduce ...DeepSeek-R1-Zero and DeepSeek-R1. DeepSeek-R1-Zero, a model trained via large-scale reinforcement learning (RL) without supervised fine-tuning (SFT) as a preliminary step, demonstrates remarkable reasoning capabilities" – DeekSeek-AI
- "We can confirm that Google DeepMind has reached the much-desired milestone, earning 35 out of a possible 42 points — a gold medal score. Their solutions were astonishing in many respects." – International Mathematical Olympiad President Dr. Dolinar
- `https://website.pi-asset.com/pi06star/PiFinal.mp4` Pi's coffee-making robot

Today's Plan

- Overview of reinforcement learning
- Course logistics
- Introduction to sequential decision making under uncertainty

Reinforcement Learning

Learning through experience/data to make good decisions under uncertainty

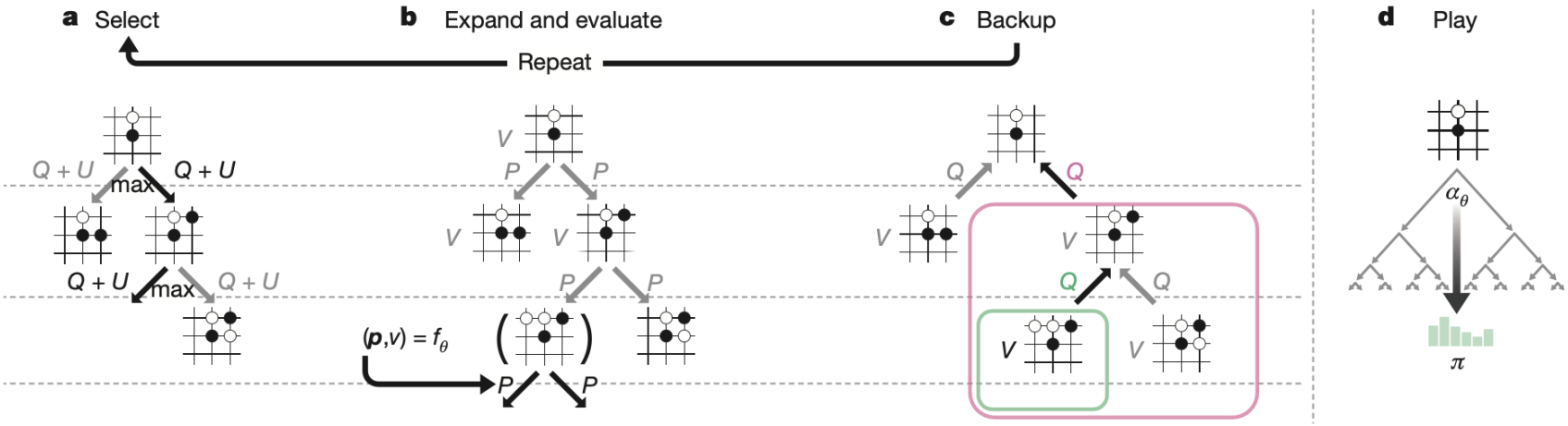
Reinforcement Learning

- Learning through experience/data to make good decisions under uncertainty
- Essential part of intelligence
- Builds strongly from theory and ideas starting in the 1950s with Richard Bellman

Reinforcement Learning

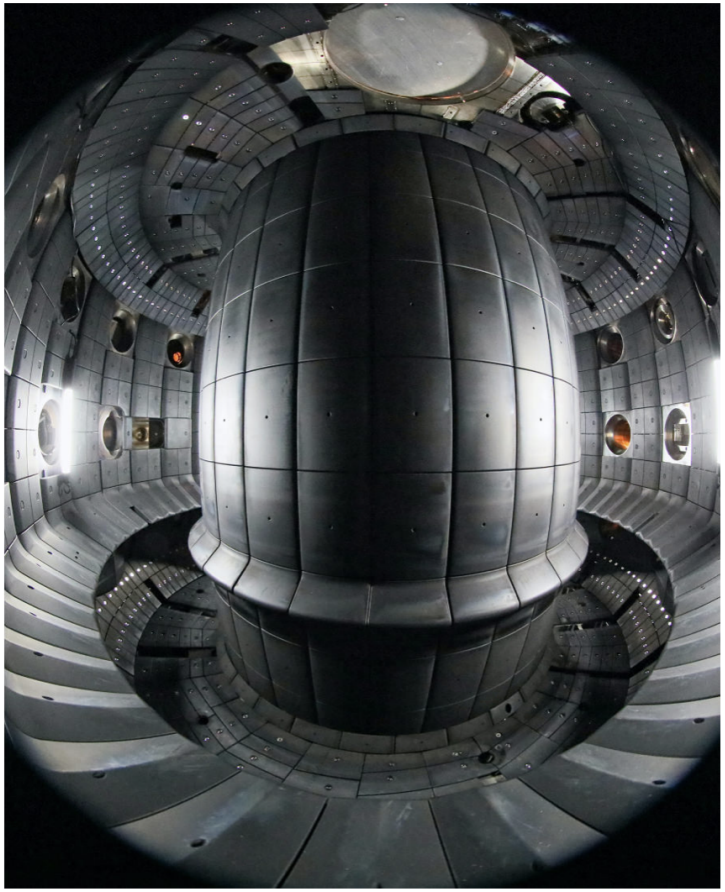
- Learning through experience/data to make good decisions under uncertainty
- Essential part of intelligence
- Builds strongly from theory and ideas starting in the 1950s with Richard Bellman
- A number of impressive successes in the last decade

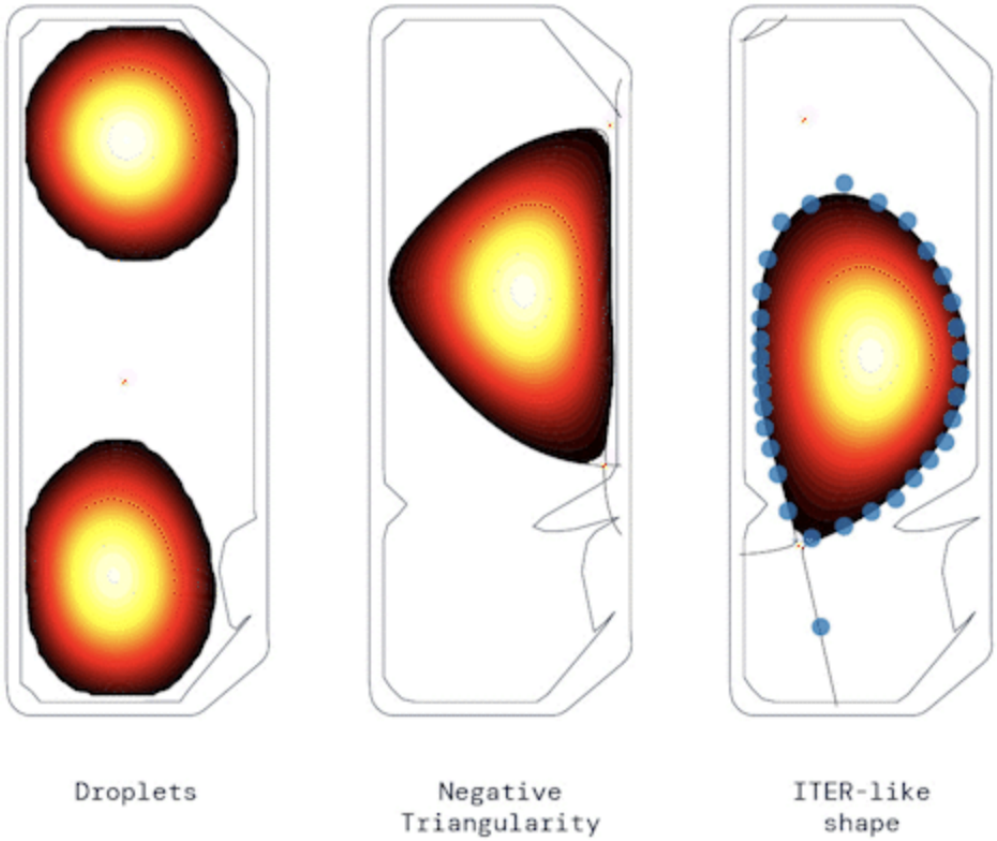
Beyond Human Performance on the Board Game Go[1]



[1]Image credits: Silver et al. Nature 2017
https://www.nature.com/articles/nature24270

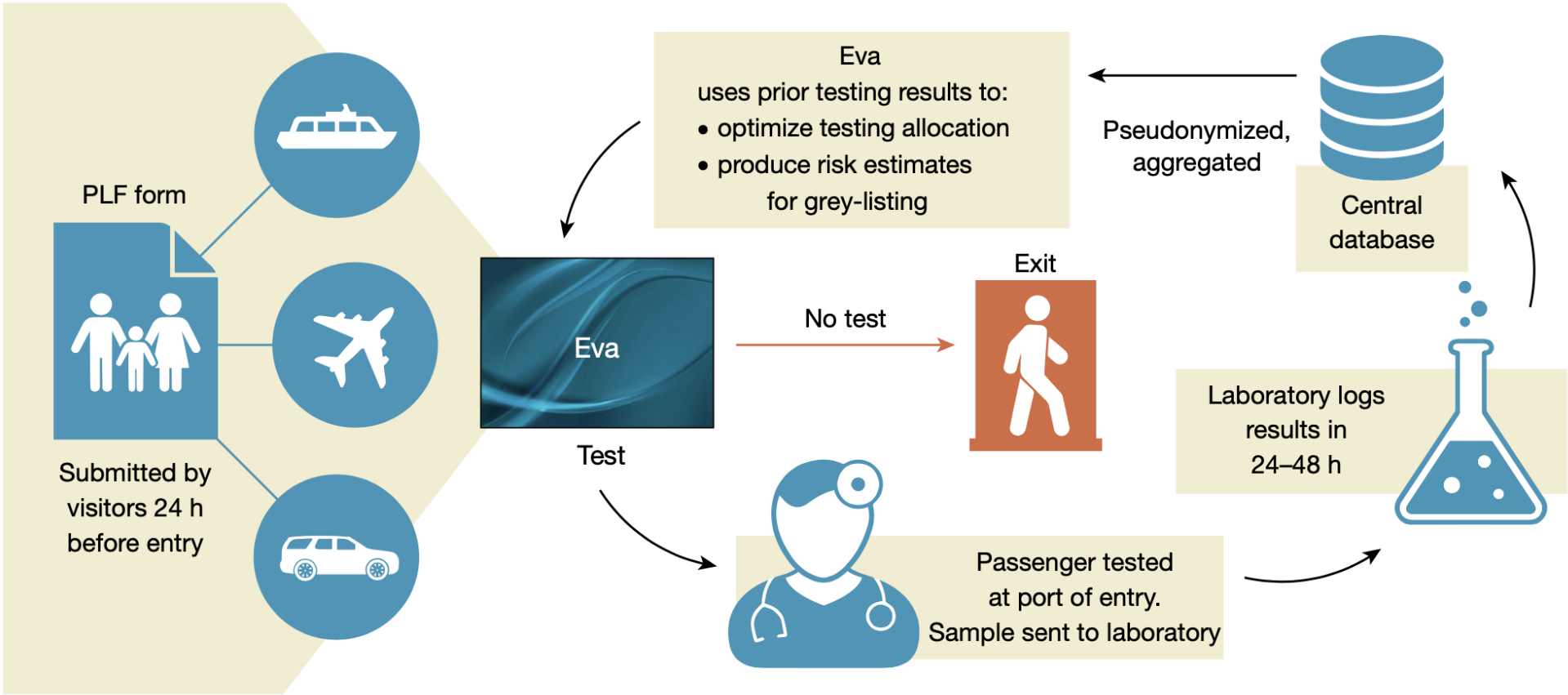
Learning Plasma Control for Fusion Science[2]





[2]Image credits: left Alain Herzog / EPFL, right DeepMind & SPC/EPFL. Degrave et al. Nature 2022 https://www.nature.com/articles/s41586-021-04301-9

Efficient and targeted COVID-19 border testing via RL [3]



[3] Bastani et al. Nature 2021
https://www.nature.com/articles/s41586-021-04014-z

ChatGPT (https://openai.com/blog/chatgpt/)

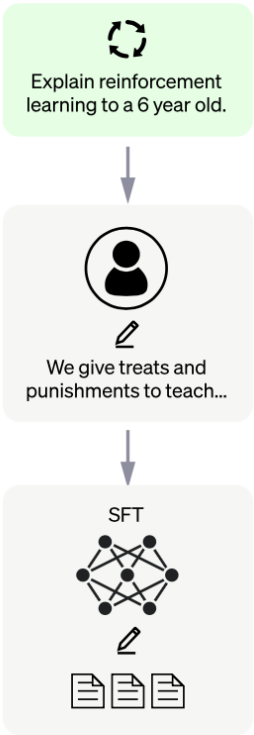
Step 1

Collect demonstration data and train a supervised policy.

A prompt is sampled from our prompt dataset.

A labeler demonstrates the desired output behavior.

This data is used to fine-tune GPT-3.5 with supervised learning.



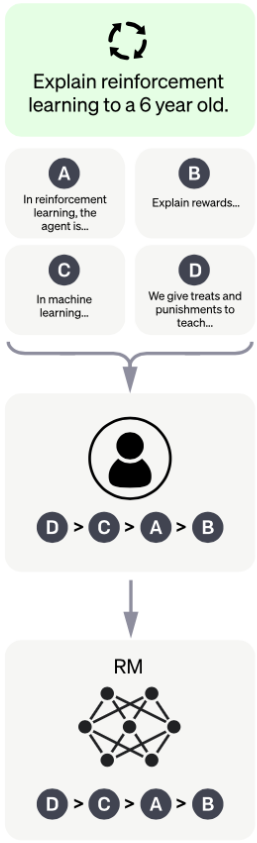
Step 2

Collect comparison data and train a reward model.

A prompt and several model outputs are sampled.

A labeler ranks the outputs from best to worst.

This data is used to train our reward model.



Step 3

Optimize a policy against the reward model using the PPO reinforcement learning algorithm.

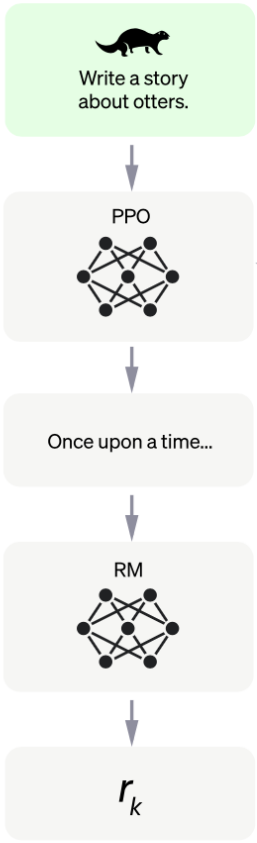
A new prompt is sampled from the dataset.

The PPO model is initialized from the supervised policy.

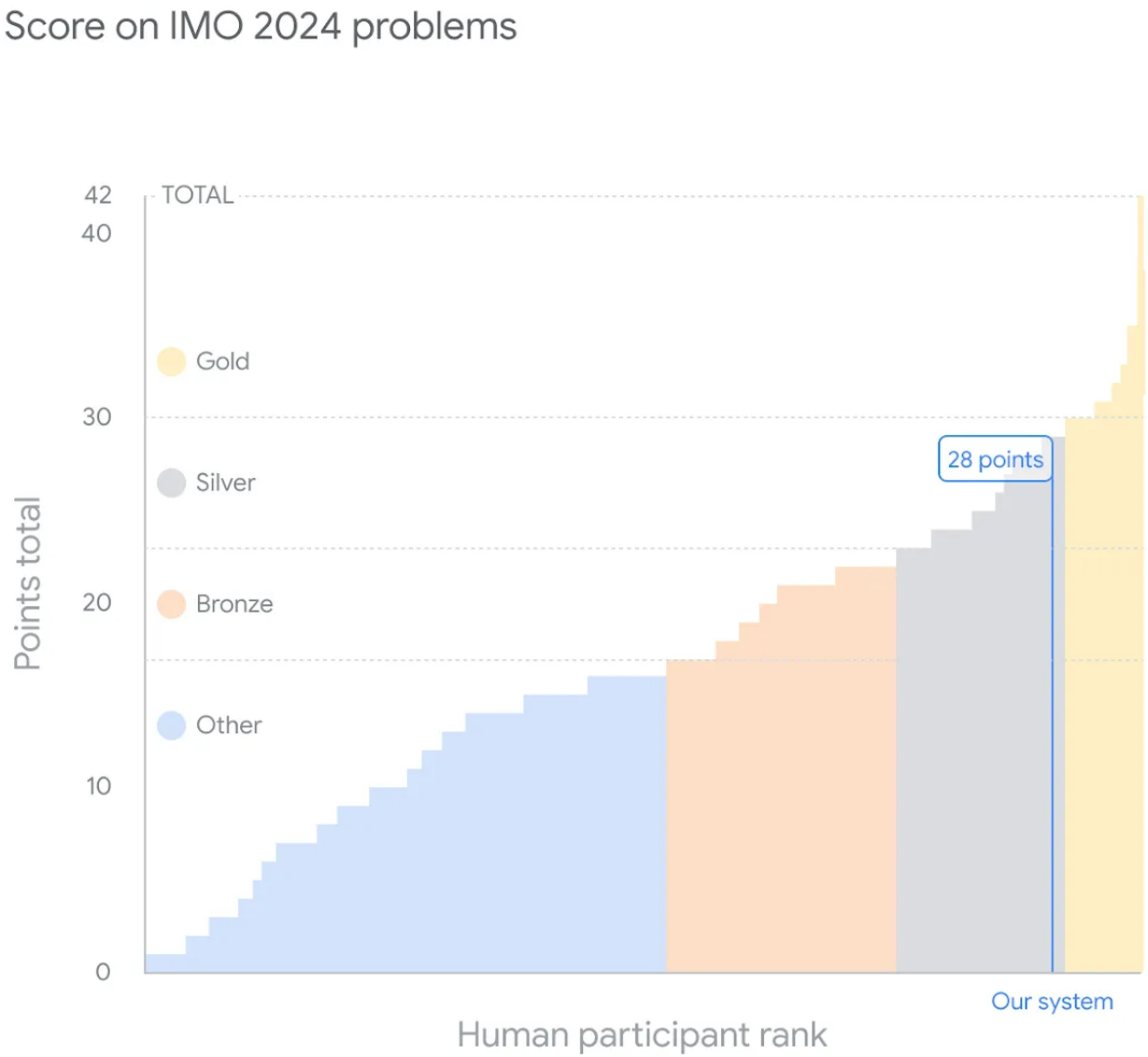
The policy generates an output.

The reward model calculates a reward for the output.

The reward is used to update the policy using PPO.



AI achieves silver-medal standard solving International Mathematical Olympiad problems



OpenAI o1

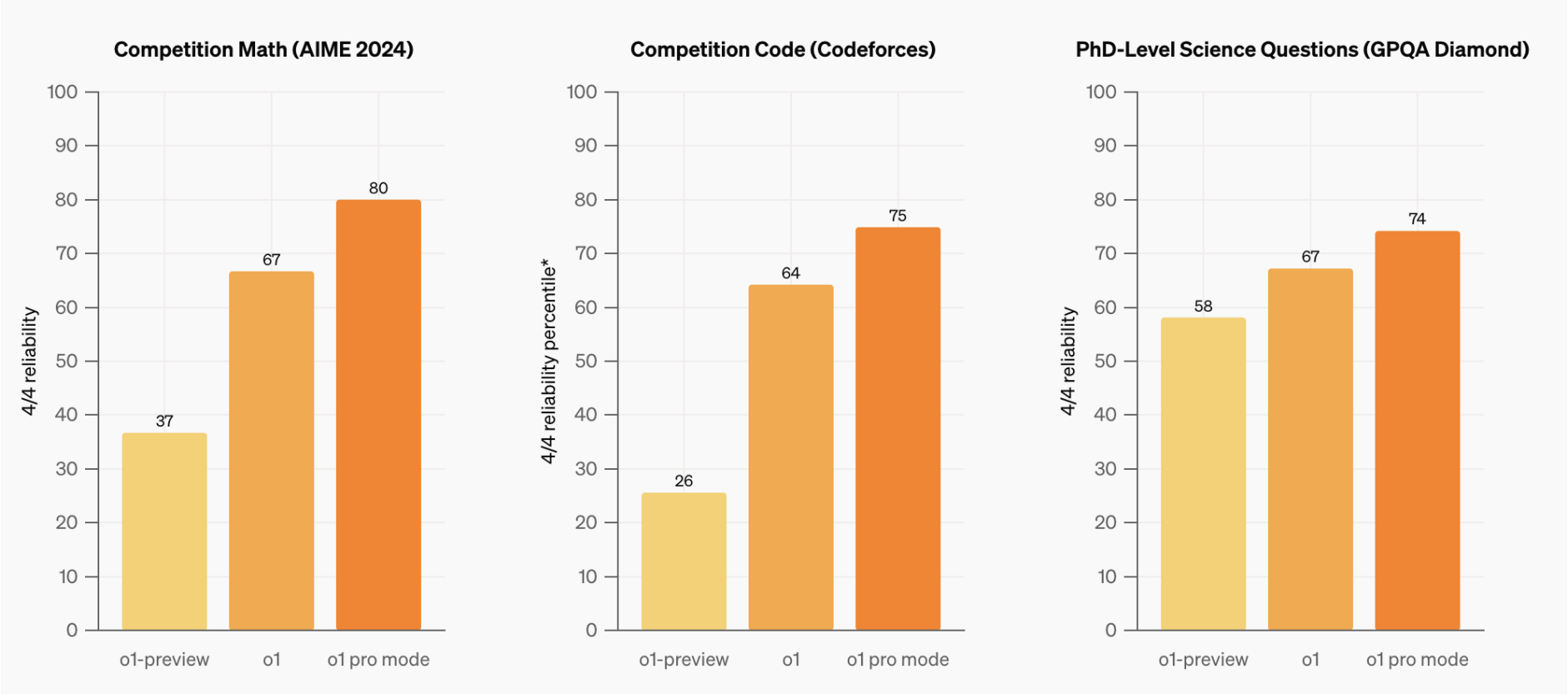


Figure: "Our large-scale reinforcement learning algorithm teaches the model how to think productively using its chain of thought in a highly data-efficient training process." Text/Image from: https://openai.com/index/introducing-chatgpt-pro/

Reinforcement Learning (Generally) Involves

- Optimization
- Delayed consequences
- Exploration
- Generalization

Optimization

- Goal is to find an optimal way to make decisions
  - Yielding best outcomes or at least very good outcomes
- Explicit notion of decision utility
- Example: finding minimum distance route between two cities given network of roads

Optimization and Intelligent Behavior

- Goal is to find an optimal way to make decisions
- "hypothesis: that the generic objective of maximising reward is enough to drive behaviour that exhibits most if not all abilities that are studied in natural and artificial intelligence." – "Reward is enough" Silver, Singh, Precup, Sutton

Delayed Consequences

- Decisions now can impact things much later...
  - Saving for retirement
  - Finding a key in video game Montezuma's revenge
- Introduces two challenges
  - When planning: decisions involve reasoning about not just immediate benefit of a decision but also its longer term ramifications
  - When learning: temporal credit assignment is hard (what caused later high or low rewards?)

Exploration

- Learning about the world by making decisions
  - Agent as scientist
  - Learn to ride a bike by trying (and failing)
- Decisions impact what we learn about
  - Only get a reward for decision made
  - Don't know what would have happened for other decision
  - If we choose to go to Stanford instead of MIT, we will have different later experiences...

Generalization

situations → a actions

- Policy is mapping from past experience to action
- Why not just pre-program a policy?

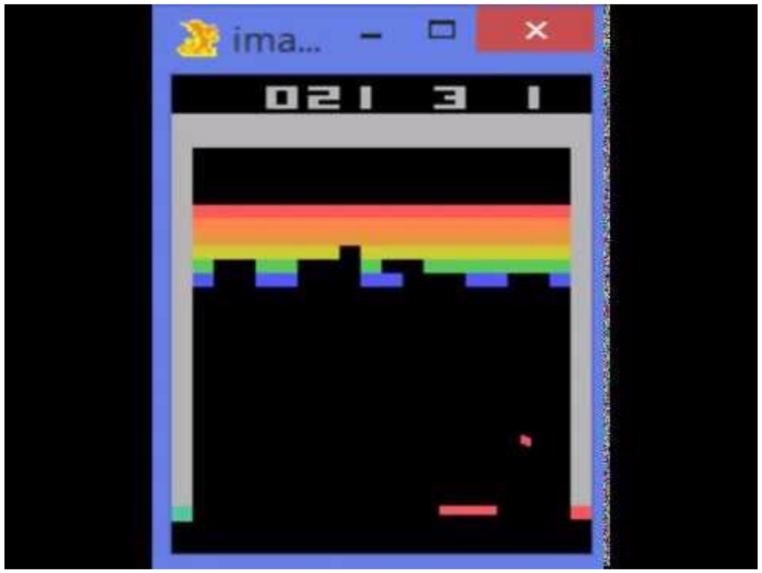


Figure: DeepMind Nature, 2015

Two of the Problem Categories Where RL is Particularly Powerful

1. No examples of desired behavior: e.g. because the goal is to go beyond human performance or there is no existing data for a task.
2. Enormous search or optimization problem with delayed outcomes:

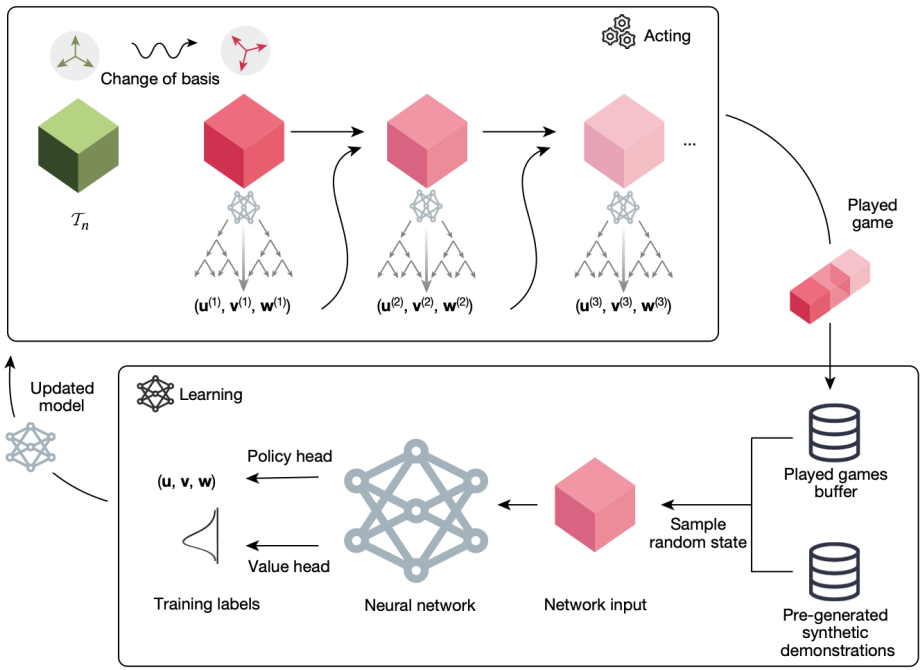


Figure: AlphaTensor. Fawzi et al. 2022

Today's Plan

- Overview of reinforcement learning
- **Course logistics**
- Introduction to sequential decision making under uncertainty

Course Outline

- Markov decision processes & planning
- Model-free policy evaluation
- Model-free control
- Policy Search
- Offline RL **including RL from Human Feedback and Direct Preference Optimization**
- Exploration
- Advanced Topics

High Level Learning Goals[4]

- Define the key features of RL
- Given an application problem know how (and whether) to use RL for it
- Implement (in code) common RL algorithms
- Understand theoretical and empirical approaches for evaluating the quality of a RL algorithm

[4] For more detailed descriptions, see website

Course Structure Overview

- Live lectures
- Tutorials
- Three homeworks
- 1 exam
- 1 multiple choice quiz
- Final project
- Check/Refresh your understanding exercises (Access through your Stanford poll everywhere account)

Tutorials

- Education in the age of AI

Tutorials: Goals

- Explaining your ideas is a key skill (needed for many jobs, and the interviews to obtain such jobs)
- Tutorials are a forum to practice this skill and further solidify your understanding of course concepts
- Weekly 30 minute session with 4 students and a CA
- Sign up will be sent out tonight for scheduling
- First tutorial will be week 2

Basic Info

- Course webpage: http://cs234.stanford.edu
- Schedule, Canvas forum (fastest way to get help, will be opened today), lecture slides
- Prerequisites, grading details, late policy, see webpage
- Office hour schedule will be announced by the end of Wednesday

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Refresher Exercise: AI Tutor as a Decision Process

- Student initially does not know either addition (easier) nor subtraction (harder)
- AI tutor agent can provide practice problems about addition or subtraction
- AI agent gets rewarded $+1$ if student gets problem right, -1 if get problem wrong
- Model this as a Decision Process. Define state space, action space, and reward model. What does the dynamics model represent? What would a policy to optimize the expected discounted sum of rewards yield?
- Which items will agent learn to give to max expected reward? Is this the best way to optimize for learning? If not, what other reward might one give to encourage student learning?
- Write down your own answers (5 min) and then discuss in small breakout groups..

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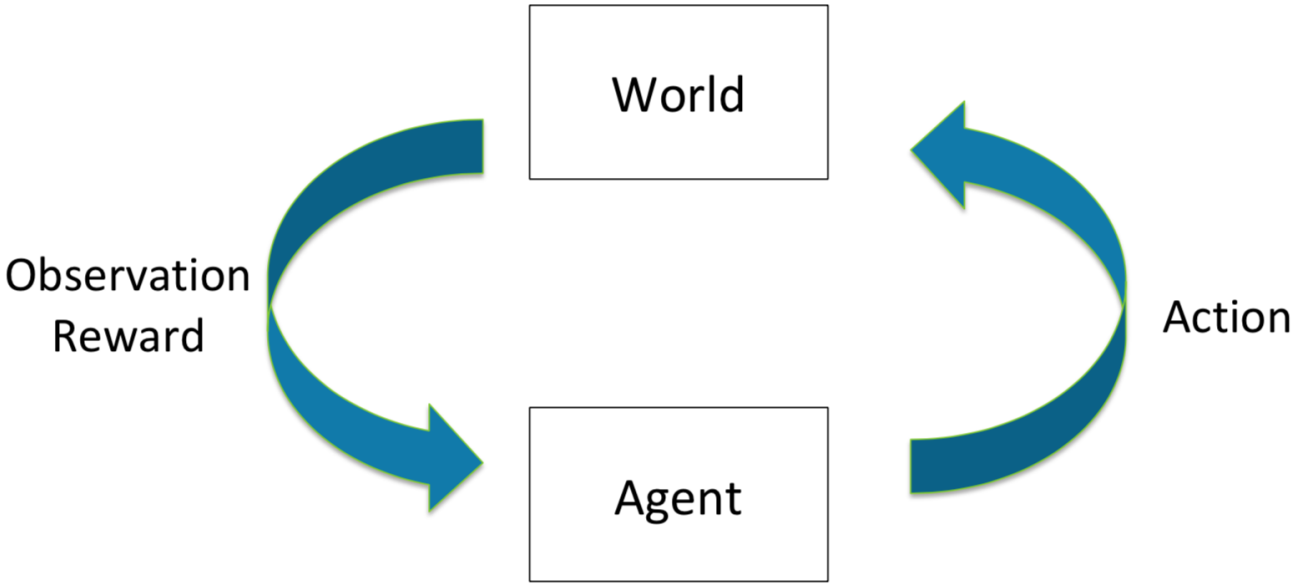
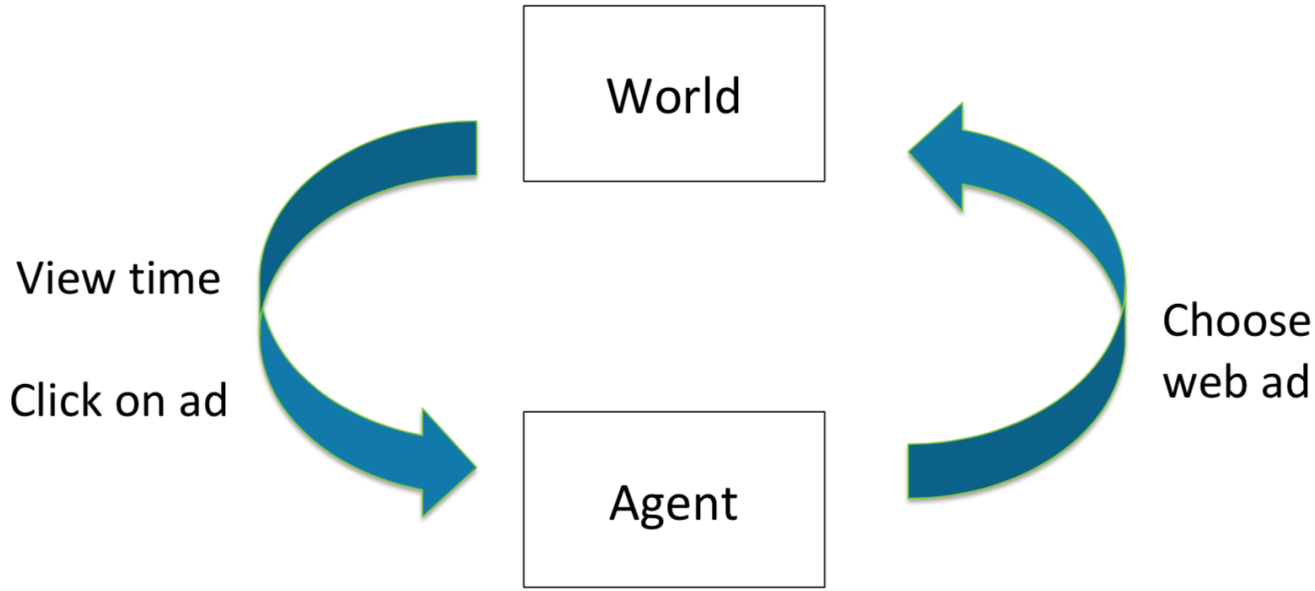
Sequential Decision Making



- Goal: Select actions to maximize total expected future reward
- May require balancing immediate & long term rewards

Example: Web Advertising



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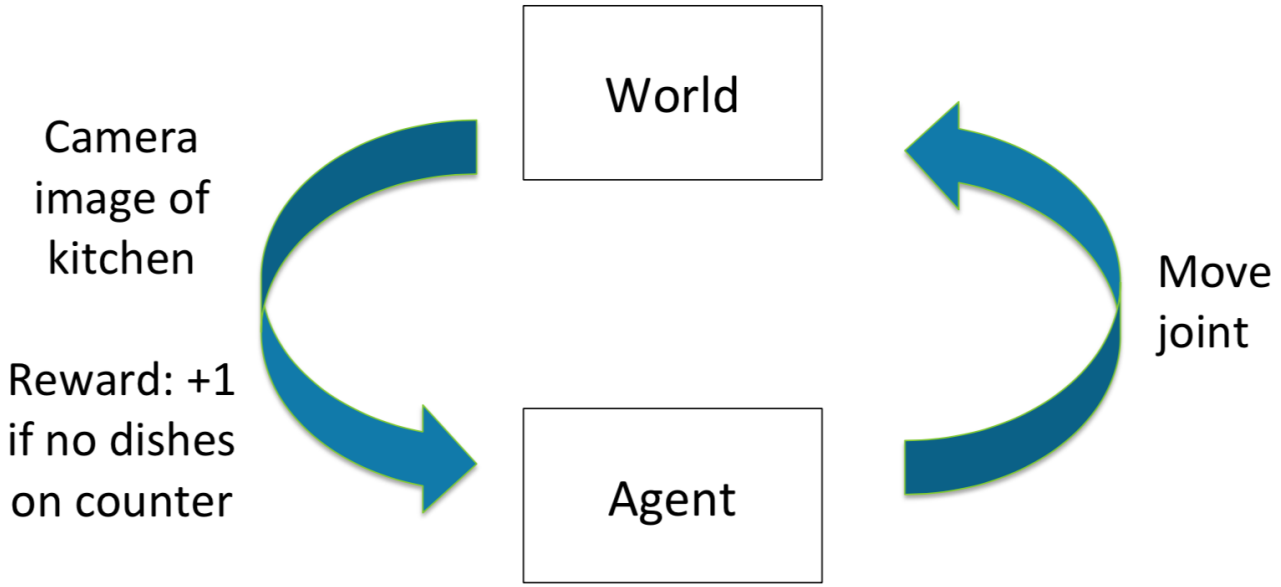
Example: Robot Unloading Dishwasher



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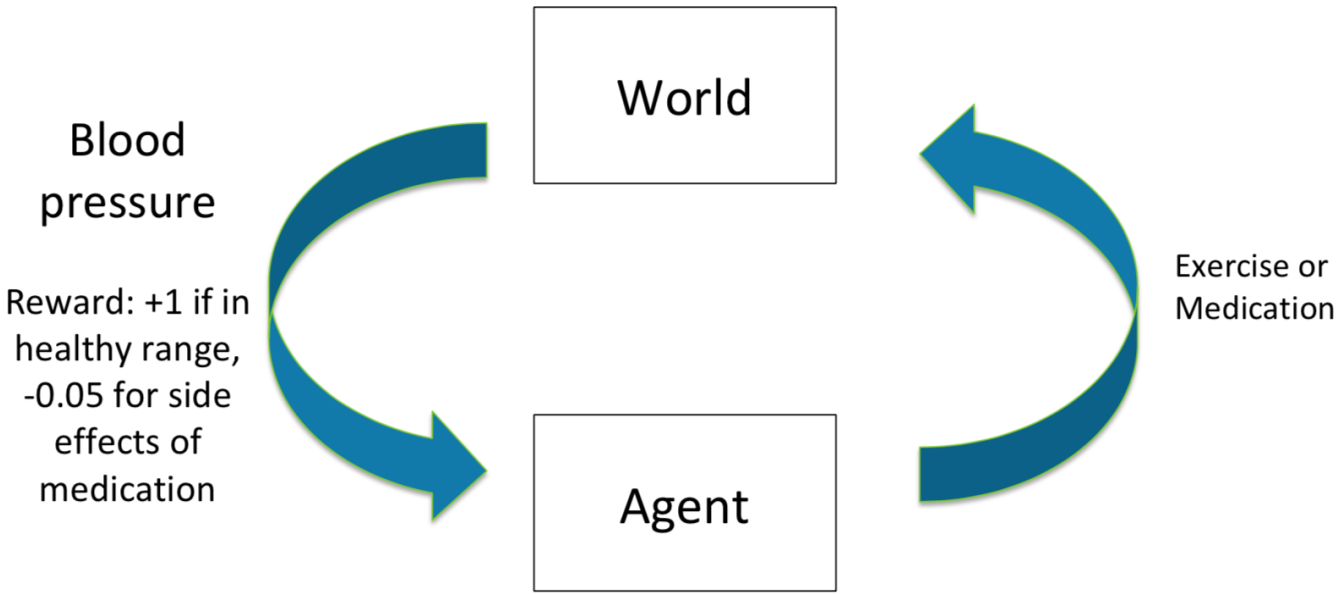
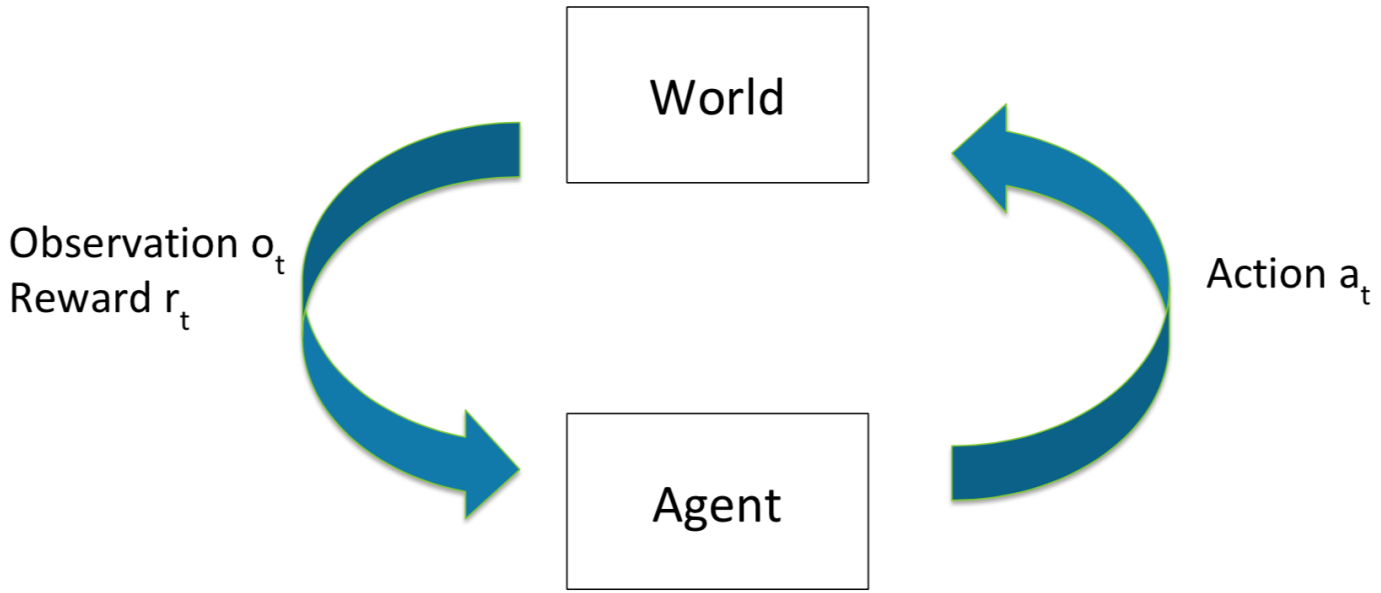
Example: Blood Pressure Control



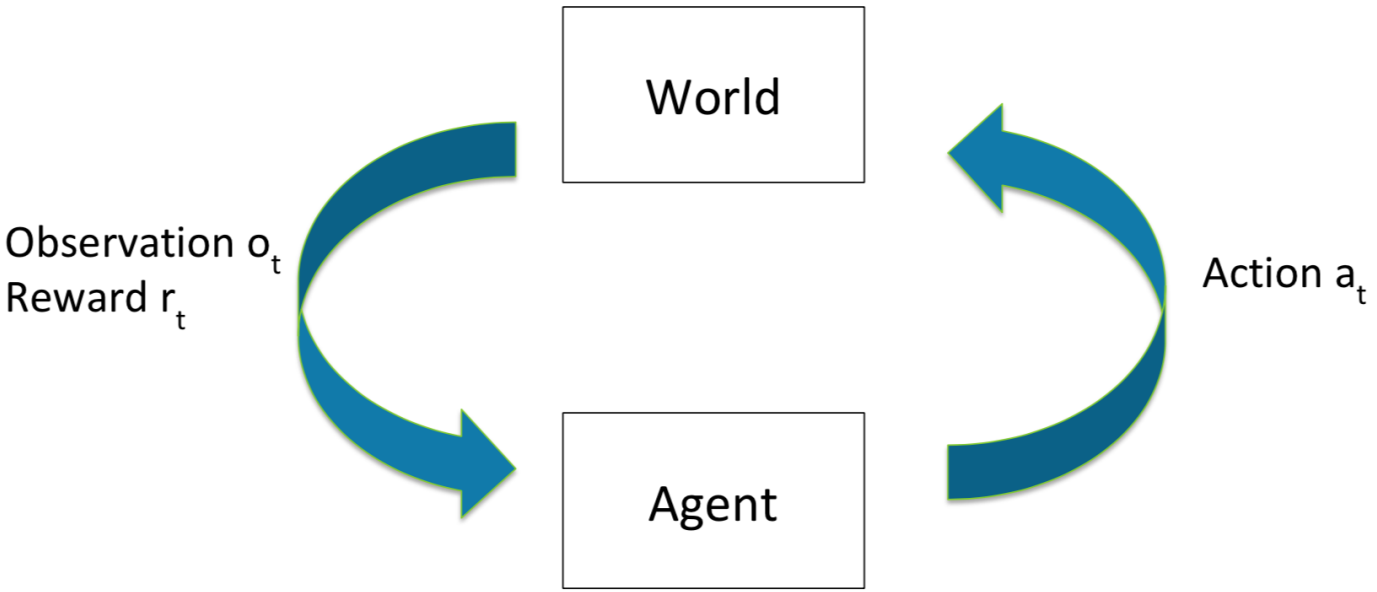
- Goal: Select actions to maximize total expected future reward
- May require balancing immediate & long term rewards

Sequential Decision Process: Agent & the World (Discrete Time)



- Each time step t:
  - Agent takes an action a_t
  - World updates given action a_t, emits observation o_t and reward r_t
  - Agent receives observation o_t and reward r_t

History: Sequence of Past Observations, Actions & Rewards



- History $h_t = (a_1, o_1, r_1, \ldots, a_t, o_t, r_t)$
- Agent chooses action based on history
- State is information assumed to determine what happens next
  - Function of history: $s_t = (h_t)$

Markov Assumption

- Information state: sufficient statistic of history
- State s_t is Markov if and only if:

$$p(s_{t+1}|s_t, a_t) = p(s_{t+1}|h_t, a_t)$$

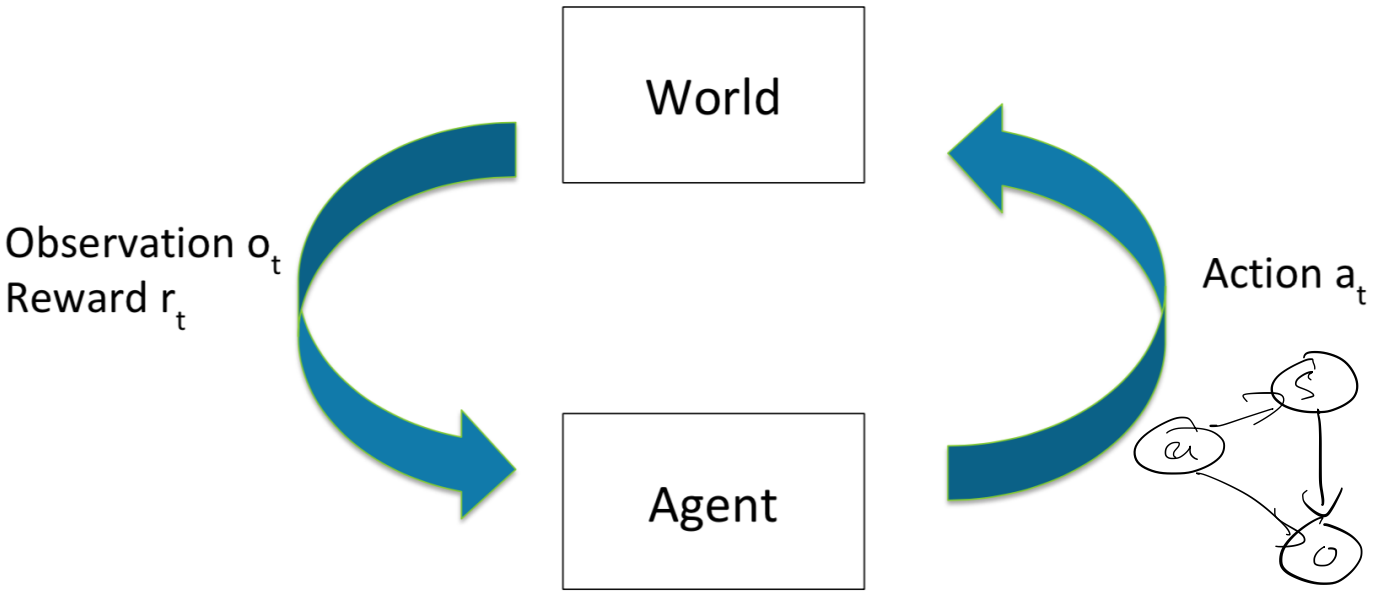
- Future is independent of past given present

state

Why is Markov Assumption Popular?

- Simple and often can be satisfied if include some history as part of the state
- In practice often assume most recent observation is sufficient statistic of history: $s_t = o_t$ Cars violate this
- State representation has big implications for:
  - Computational complexity
  - Data required
  - Resulting performance

Types of Sequential Decision Processes



- Is state Markov? Is world partially observable? (POMDP)
- Are dynamics deterministic or stochastic?
- Do actions influence only immediate reward (bandits) or reward and next state ?

Example: Mars Rover as a Markov Decision Process

tabular MDP

s_1	s_2	s_3	s_4	s_5	s_6	s_7
						

Figure: Mars rover image: NASA/JPL-Caltech

- States: Location of rover $(s_1, \ldots, s_7)$
- Actions: TryLeft or TryRight
- Rewards:
  - $+1$ in state s_1
  - $+10$ in state s_7
  - 0 in all other states

MDP Model

- Agent's representation of how world changes given agent's action
- Transition / dynamics model predicts next agent state

$$p(s_{t+1} = s' | s_t = s, a_t = a)$$

- Reward model predicts immediate reward

$$r(s_t = s, a_t = a) = \mathbb{E}[r_t | s_t = s, a_t = a]$$

Example: Mars Rover Stochastic Markov Model

s_1	s_2	s_3	s_4	s_5	s_6	s_7
$\hat{r} = 0$	$\hat{r} = 0$	$\hat{r} = 0$	$\hat{r} = 0$	$\hat{r} = 0$	$\hat{r} = 0$	$\hat{r} = 0$

- Numbers above show RL agent's reward model
- Part of agent's transition model:
  - $0.5 = P(s_1|s_1, \text{TryRight}) = P(s_2|s_1, \text{TryRight})$
  - $0.5 = P(s_2|s_2, \text{TryRight}) = P(s_3|s_2, \text{TryRight}) \cdots$
- Model may be wrong

Policy

- Policy π determines how the agent chooses actions
- $\pi : S \rightarrow A$, mapping from states to actions
- Deterministic policy:

$$\pi(s) = a$$

- Stochastic policy:

$$\pi(a|s) = Pr(a_t = a|s_t = s)$$

Example: Mars Rover Policy

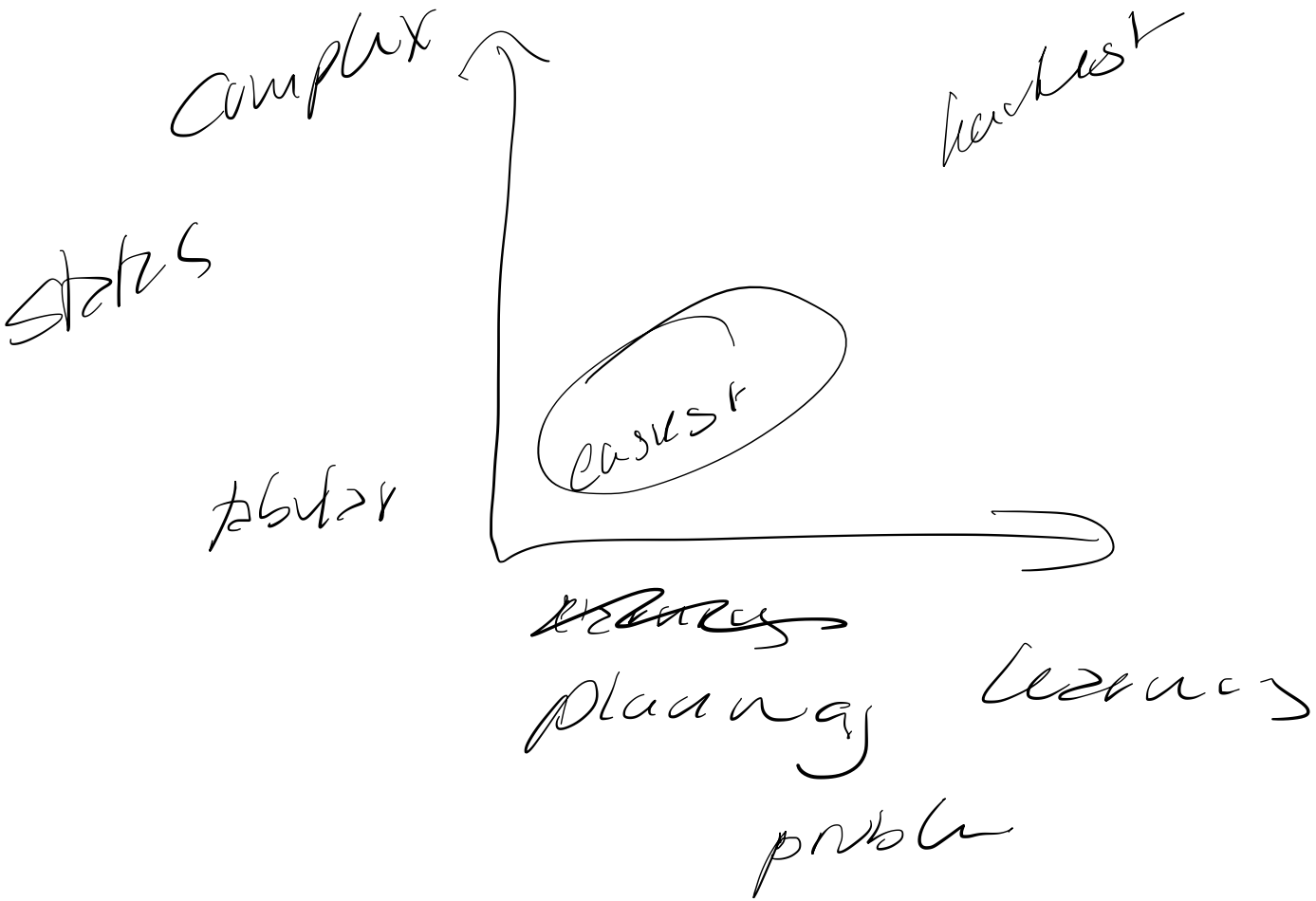
s_1	s_2	s_3	s_4	s_5	s_6	s_7
						

- $\pi(s_1) = \pi(s_2) = \cdots = \pi(s_7) =$ TryRight
- Quick check your understanding: is this a deterministic policy or a stochastic policy?

Evaluation and Control

- Evaluation
  - Estimate/predict the expected rewards from following a given policy
- Control
  - Optimization: find the best policy

Build Up in Complexity



Making Sequences of Good Decisions Given a Model of the World

- Assume finite set of states and actions
- Given models of the world (dynamics and reward)
- Evaluate the performance of a particular decision policy
- Compute the best policy
- This can be viewed as an AI planning problem

Making Sequences of Good Decisions Given a Model of the World

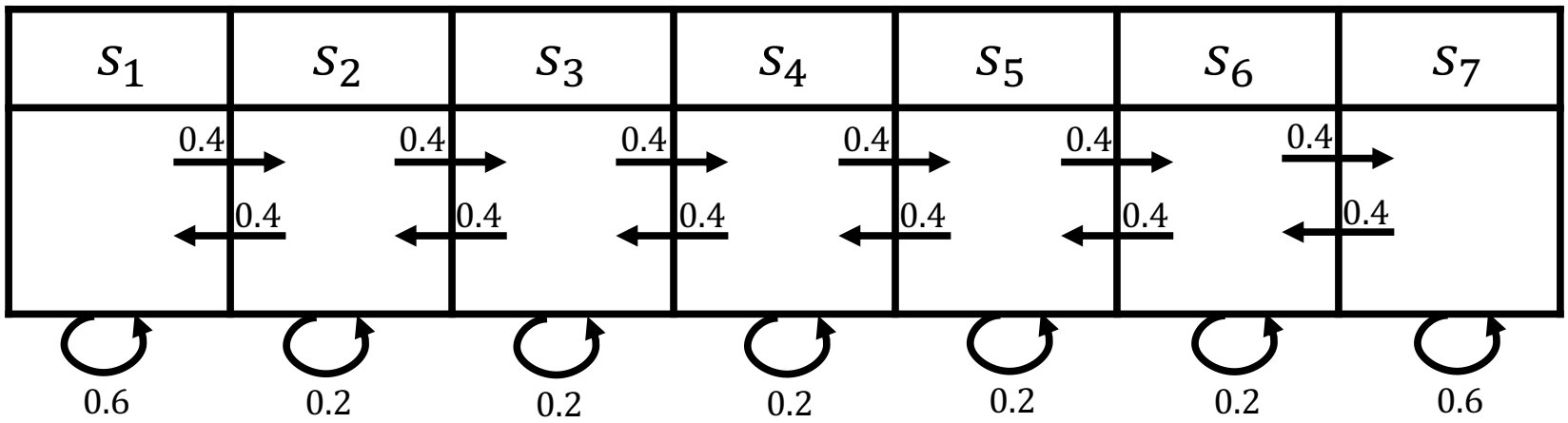
- Markov Processes
- Markov Reward Processes (MRPs)
- Markov Decision Processes (MDPs)
- Evaluation and Control in MDPs

Markov Process or Markov Chain

- Memoryless random process
  - Sequence of random states with Markov property
- Definition of Markov Process
  - S is a (finite) set of states ($s \in S$)
  - P is dynamics/transition model that specifices $p(s_{t+1} = s' | s_t = s)$
- Note: no rewards, no actions
- If finite number (N) of states, can express P as a matrix

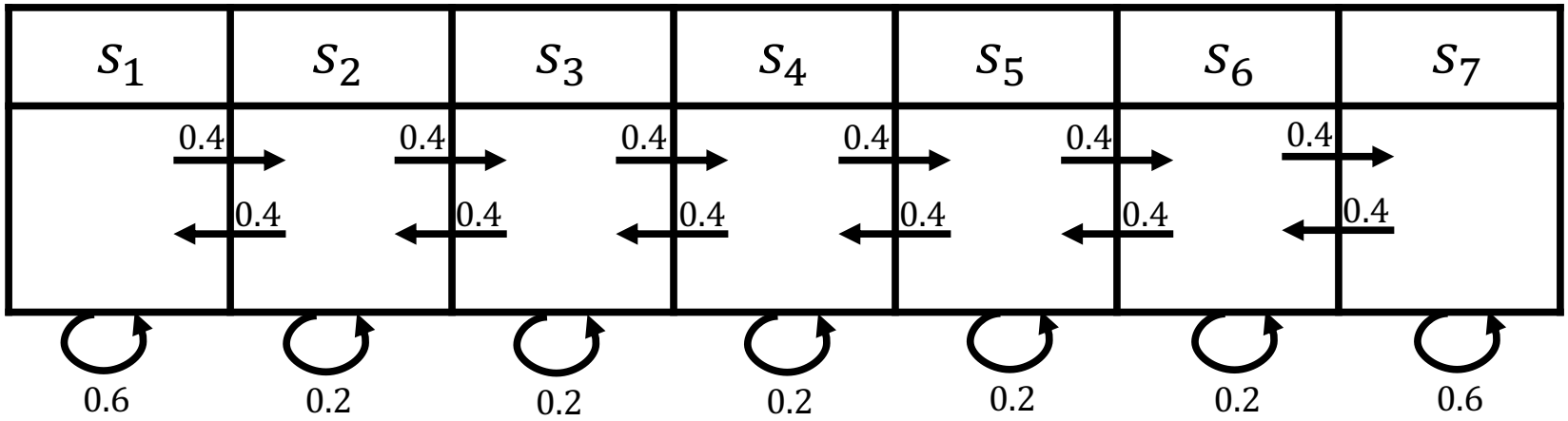
$$P = \begin{pmatrix} P(s_1|s_1) & P(s_2|s_1) & \cdots & P(s_N|s_1) \\ P(s_1|s_2) & P(s_2|s_2) & \cdots & P(s_N|s_2) \\ \vdots & \vdots & \ddots & \vdots \\ P(s_1|s_N) & P(s_2|s_N) & \cdots & P(s_N|s_N) \end{pmatrix}$$

Example: Mars Rover Markov Chain Transition Matrix, P



$$P = \begin{pmatrix} 0.6 & 0.4 & 0 & 0 & 0 & 0 & 0 \\ 0.4 & 0.2 & 0.4 & 0 & 0 & 0 & 0 \\ 0 & 0.4 & 0.2 & 0.4 & 0 & 0 & 0 \\ 0 & 0 & 0.4 & 0.2 & 0.4 & 0 & 0 \\ 0 & 0 & 0 & 0.4 & 0.2 & 0.4 & 0 \\ 0 & 0 & 0 & 0 & 0.4 & 0.2 & 0.4 \\ 0 & 0 & 0 & 0 & 0 & 0.4 & 0.6 \end{pmatrix}$$

Example: Mars Rover Markov Chain Episodes



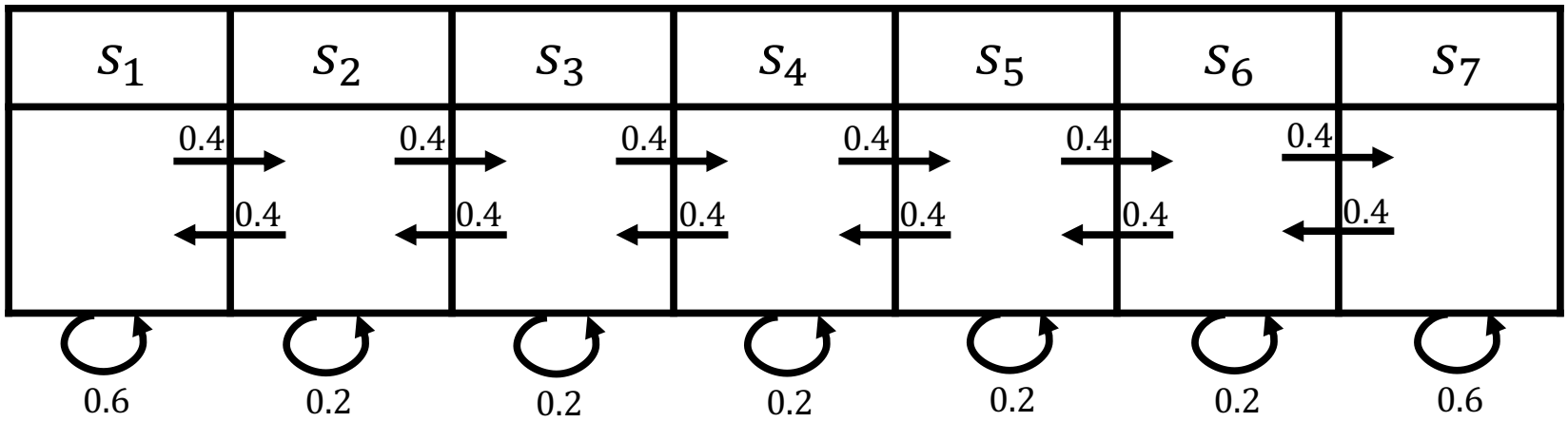
Example: Sample episodes starting from S4

- $s_4, s_5, s_6, s_7, s_7, s_7, \ldots$
- $s_4, s_4, s_5, s_4, s_5, s_6, \ldots$
- $s_4, s_3, s_2, s_1, \ldots$

Markov Reward Process (MRP)

- Markov Reward Process is a Markov Chain + rewards
- Definition of Markov Reward Process (MRP)
  - S is a (finite) set of states ($s \in S$)
  - P is dynamics/transition model that specifices $P(s_{t+1} = s' | s_t = s)$
  - R is a reward function $R(s_t = s) = \mathbb{E}[r_t | s_t = s]$
  - Discount factor $\gamma \in [0, 1]$
- Note: no actions
- If finite number (N) of states, can express R as a vector

Example: Mars Rover Markov Reward Process



- Reward: $+1$ in s_1, $+10$ in s_7, 0 in all other states

Return & Value Function

- Definition of Horizon (H)
  - Number of time steps in each episode
  - Can be infinite
  - Otherwise called **finite** Markov reward process
- Definition of Return, G_t (for a Markov Reward Process)
  - Discounted sum of rewards from time step t to horizon H

$$G_t = r_t + \gamma r_{t+1} + \gamma^2 r_{t+2} + \cdots + \gamma^{H-1} r_{t+H-1}$$

- Definition of State Value Function, $V(s)$ (for a Markov Reward Process)
  - Expected return from starting in state s

$$V(s) = \mathbb{E}[G_t | s_t = s] = \mathbb{E}[r_t + \gamma r_{t+1} + \gamma^2 r_{t+2} + \cdots + \gamma^{H-1} r_{t+H-1} | s_t = s]$$

Discount Factor

- Mathematically convenient (avoid infinite returns and values)
- Humans often act as if there's a discount factor < 1
- If episode lengths are always finite ($H < \infty$), can use $\gamma = 1$

Discount Factor

- Mathematically convenient (avoid infinite returns and values)
- Humans often act as if there's a discount factor < 1
- $\gamma = 0$: Only care about immediate reward
- $\gamma = 1$: Future reward is as beneficial as immediate reward
- If episode lengths are always finite ($H < \infty$), can use $\gamma = 1$

Computing the Value of a Markov Reward Process

- Markov property provides structure
- MRP value function satisfies

dynamics

$$V(s) = \underbrace{R(s)}_{\text{Immediate reward}} + \underbrace{\gamma \sum_{s' \in S} P(s'|s) V(s')}_{\text{Discounted sum of future rewards}}$$

vector vector matrix vector

$$V = R + \gamma P V$$

$$V - \gamma P V = R$$

$$(I - \gamma P) V = R$$

$$V = (I - \gamma P)^{-1} R$$

always invertible (prove why if interested)

Matrix Form of Bellman Equation for MRP

- For finite state MRP, we can express $V(s)$ using a matrix equation

$$\begin{pmatrix} V(s_1) \\ \vdots \\ V(s_N) \end{pmatrix} = \begin{pmatrix} R(s_1) \\ \vdots \\ R(s_N) \end{pmatrix} + \gamma \begin{pmatrix} P(s_1|s_1) & \cdots & P(s_N|s_1) \\ P(s_1|s_2) & \cdots & P(s_N|s_2) \\ \vdots & \ddots & \vdots \\ P(s_1|s_N) & \cdots & P(s_N|s_N) \end{pmatrix} \begin{pmatrix} V(s_1) \\ \vdots \\ V(s_N) \end{pmatrix}$$

$$V = R + \gamma P V$$

Analytic Solution for Value of MRP

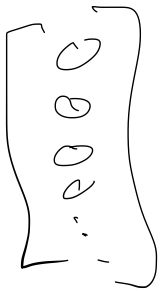
- For finite state MRP, we can express $V(s)$ using a matrix equation

$$\begin{pmatrix} V(s_1) \\ \vdots \\ V(s_N) \end{pmatrix} = \begin{pmatrix} R(s_1) \\ \vdots \\ R(s_N) \end{pmatrix} + \gamma \begin{pmatrix} P(s_1|s_1) & \cdots & P(s_N|s_1) \\ P(s_1|s_2) & \cdots & P(s_N|s_2) \\ \vdots & \ddots & \vdots \\ P(s_1|s_N) & \cdots & P(s_N|s_N) \end{pmatrix} \begin{pmatrix} V(s_1) \\ \vdots \\ V(s_N) \end{pmatrix}$$

$$\begin{aligned} V &= R + \gamma P V \\ V - \gamma P V &= R \\ (I - \gamma P) V &= R \\ V &= (I - \gamma P)^{-1} R \end{aligned}$$

- Solving directly requires taking a matrix inverse $\sim O(N^3)$
- Note that $(I - \gamma P)$ is invertible

Iterative Algorithm for Computing Value of a MRP



$k=1 \quad V(s) = R(s)$

- Dynamic programming
- Initialize $V_0(s) = 0$ for all s
- For $k = 1$ until convergence
  - For all s in S

$$V_k(s) = R(s) + \gamma \sum_{s' \in S} P(s'|s) V_{k-1}(s')$$

- Computational complexity: $O(|S|^2)$ for each iteration ($|S| = N$)

Summary of Today

- Reinforcement learning involves learning, optimization, delayed consequences, generalization and exploration
- Goal is to learn to make good decisions under uncertainty