Homework 2

I. LAX-RICHTMYER THEOREM

Recall the standard wave equation,

\[ u_t + au_x = 0, \]

where \( a = \text{constant} \). Prove that stability and consistency are sufficient for convergence for approximations to the wave equation. See Chapter 8 of *Finite Volume Methods for Hyperbolic Problems*, by Randall J. LeVeque (there is a link to an electronic copy of the book on the class website).

II. TVD

Define the 'total variation' of \( v \) as

\[ TV(v) = \sum_{j=1}^{N} |v_{j+1} - v_j|. \]

Prove that second order Runge-Kutta is total variation diminishing in the sense that \( TV(v^{n+1}) \leq TV(v^n) \). You may assume that the forward Euler step is TVD.