



Conditional random fields



Conditional random fields

- **Definition**

$$P(\pi \mid x) = \frac{\exp(\sum_{i=1}^{|x|} w^T F(\pi_i, \pi_{i-1}, x, i))}{\sum_{\pi'} \exp(\sum_{i=1}^{|x|} w^T F(\pi'_i, \pi'_{i-1}, x, i))}$$

where

$F : (\text{state}, \text{state}, \text{observations}, \text{index}) \rightarrow \mathbf{R}^n$ “local feature mapping”

$w \in$

\mathbf{R}^n

“parameter vector”

partition coefficient

“parameter vector”

- Summation over all possible state sequences $\pi'_1 \dots \pi'_{|x|}$
- $a^T b$ for vectors $a, b \in \mathbf{R}^n$ denotes inner product, $\sum_{i=1}^n a_i b_i$



Relationship with HMMs

$$\log P(x, \pi) = \log a_0(\pi_0) + \sum_{i=1 \dots |x|} [\log a(\pi_{i-1}, \pi_i) + \log e_{\pi_i}(x_i)]$$

- (*) For each component w_j , define F_j to be a 0/1 indicator variable of whether the j^{th} parameter should be included in scoring x, π at position i :

$$w = \begin{bmatrix} \log a_0(1) \\ \dots \\ \log a_0(K) \\ \log a_{11} \\ \dots \\ \log a_{KK} \\ \log e_1(b_1) \\ \dots \\ \log e_K(b_M) \end{bmatrix} \in \mathbf{R}^n \quad F(\pi_i, \pi_{i-1}, x, i) = \begin{bmatrix} 1\{i = 1 \wedge \pi_{i-1} = 1\} \\ \dots \\ 1\{i = 1 \wedge \pi_{i-1} = K\} \\ 1\{\pi_{i-1} = 1 \wedge \pi_i = 1\} \\ \dots \\ 1\{\pi_{i-1} = K \wedge \pi_i = K\} \\ 1\{x_i = b_1 \wedge \pi_i = 1\} \\ \dots \\ 1\{x_i = b_M \wedge \pi_i = K\} \end{bmatrix} \in \mathbf{R}^n$$

- Then, $\log P(x, \pi) = \sum_{i=1 \dots |x|} w^T F(\pi_i, \pi_{i-1}, x, i)$



Relationship with HMMs

$$\log P(x, \pi) = \sum_{i=1}^{|x|} w^T F(\pi_i, \pi_{i-1}, x, i)$$

- Equivalently,

$$P(\pi | x) = \frac{P(x, \pi)}{\sum_{\pi} P(x, \pi)} = \frac{\exp(\sum_{i=1}^{|x|} w^T F(\pi_i, \pi_{i-1}, x, i))}{\sum_{\pi} \exp(\sum_{i=1}^{|x|} w^T F(\pi_i, \pi_{i-1}, x, i))}$$

- Therefore, an HMM can be converted to an equivalent CRF



CRFS \geq HMMs (continued)

- In an HMM, our features were of the form

$$F(\pi_i, \pi_{i-1}, x, i) = F(\pi_i, \pi_{i-1}, x_i, i)$$

- I.e., when scoring position i in the sequence, feature only considered the emission x_i at position i .
 - Cannot look at other positions (e.g., x_{i-1} , x_{i+1}) since that would involve “emitting” a character more than once - double-counting of probability
- CRFs don't have this restriction
 - Why? Because CRFs don't attempt to model the observations x !



Examples of non-local features for CRFs

- Casino:

- Dealer looks at previous 100 positions, and determines whether at least 50 over them had 6's

$$F_j(\text{LOADED}, \text{FAIR}, x, i) = 1\{ x_{i-100} \dots x_i \text{ has } > 50 \text{ 6s} \}$$

- CpG islands:

- Gene occurs near a CpG island

$$F_j(*, \text{EXON}, x, i) = 1\{ x_{i-1000} \dots x_{i+1000} \text{ has } > 1/16 \text{ CpGs} \}$$



3 basic questions for CRFs

- **Evaluation:** Given a sequence of observations x and a sequence of states π , compute $P(\pi \mid x)$
- **Decoding:** Given a sequence of observations x , compute the maximum probability sequence of states $\pi_{ML} = \arg \max_{\pi} P(\pi \mid x)$
- **Learning:** Given a CRF with unspecified parameters w , compute the parameters that maximize the likelihood of π given x , i.e., $w_{ML} = \arg \max_w P(\pi \mid x, w)$



Viterbi for CRFs

- Note that:

$$\begin{aligned} \operatorname{argmax}_{\pi} P(\pi \mid x) &= \operatorname{argmax}_{\pi} \frac{\exp(\sum_{i=1}^{|x|} w^T F(\pi_i, \pi_{i-1}, x, i))}{\sum_{\pi'} \exp(\sum_{i=1}^{|x|} w^T F(\pi'_i, \pi'_{i-1}, x, i))} \\ &= \operatorname{arg max}_{\pi} \exp(\sum_{i=1}^{|x|} w^T F(\pi_i, \pi_{i-1}, x, i)) \\ &= \operatorname{arg max}_{\pi} \sum_{i=1}^{|x|} w^T F(\pi_i, \pi_{i-1}, x, i) \end{aligned}$$

- We can derive the following recurrence:

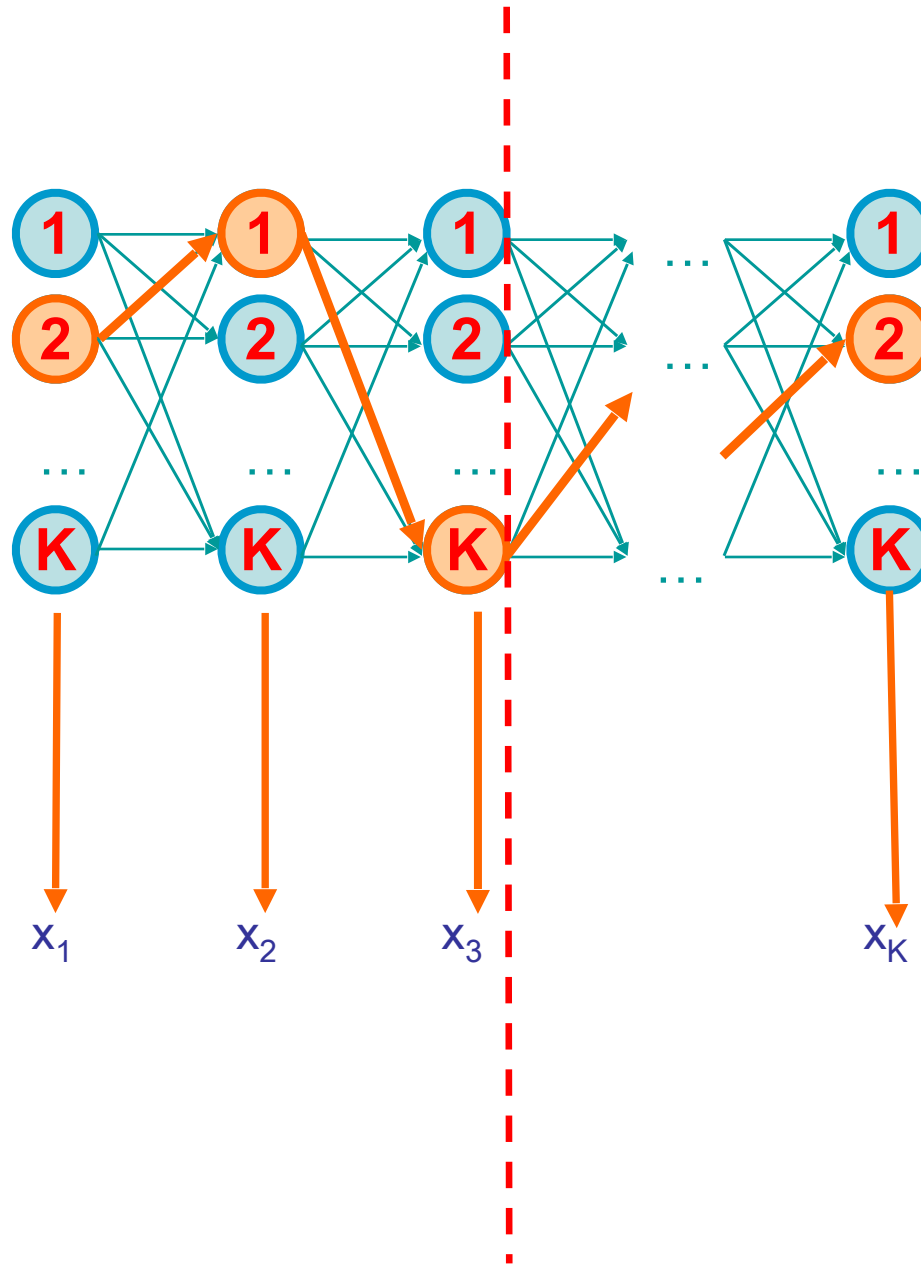
$$V_k(i) = \max_j [w^T F(k, j, x, i) + V_j(i-1)]$$

- **Notes:**

- Even though the features may depend on arbitrary positions in x , x is constant. DP depends only on knowing the previous state
- Computing the partition function (denominator) can be done by a similar adaptation of the forward/backward algorithms



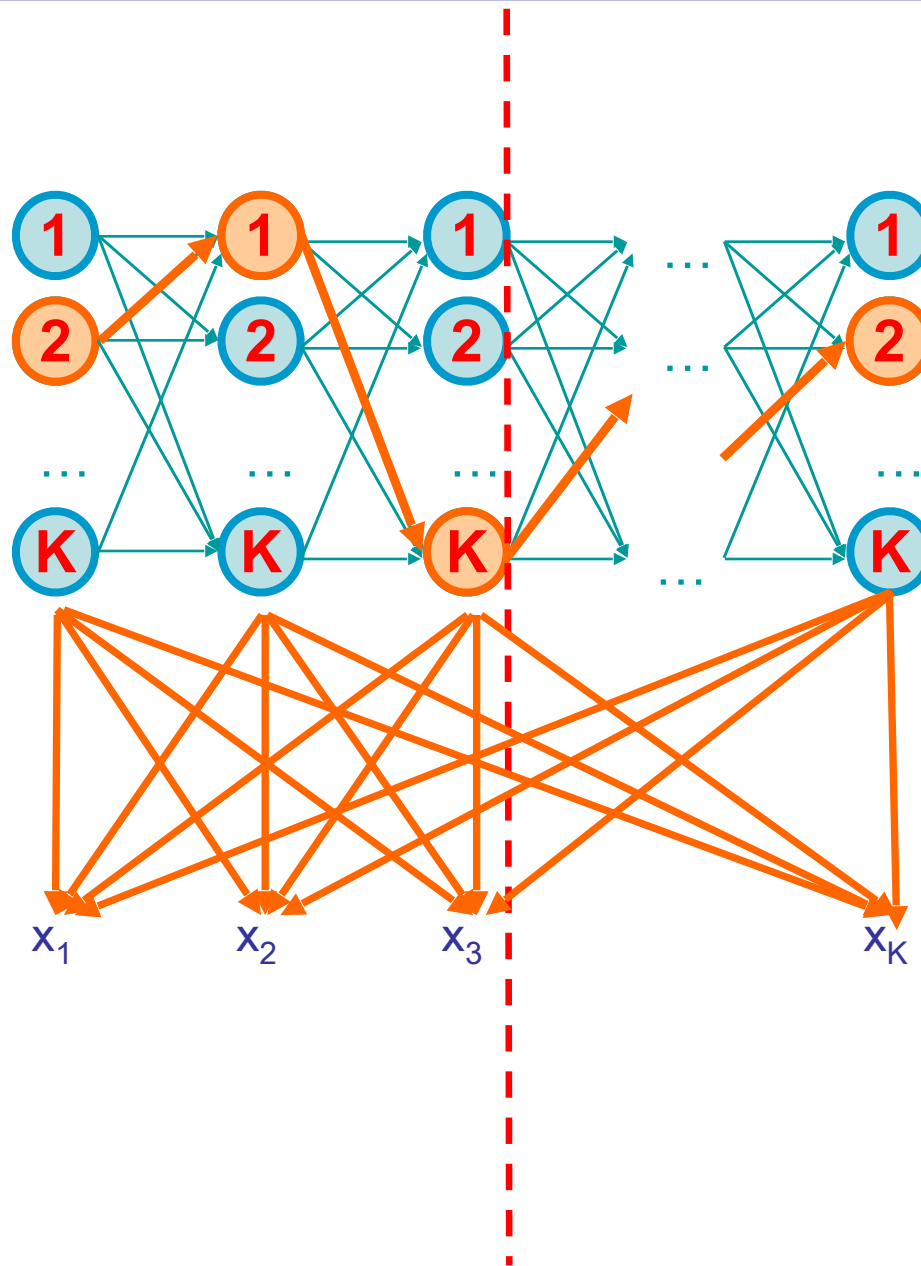
Viterbi for CRFs



Given that we end up in state k at step i , maximize score to the left and right



Viterbi for CRFs



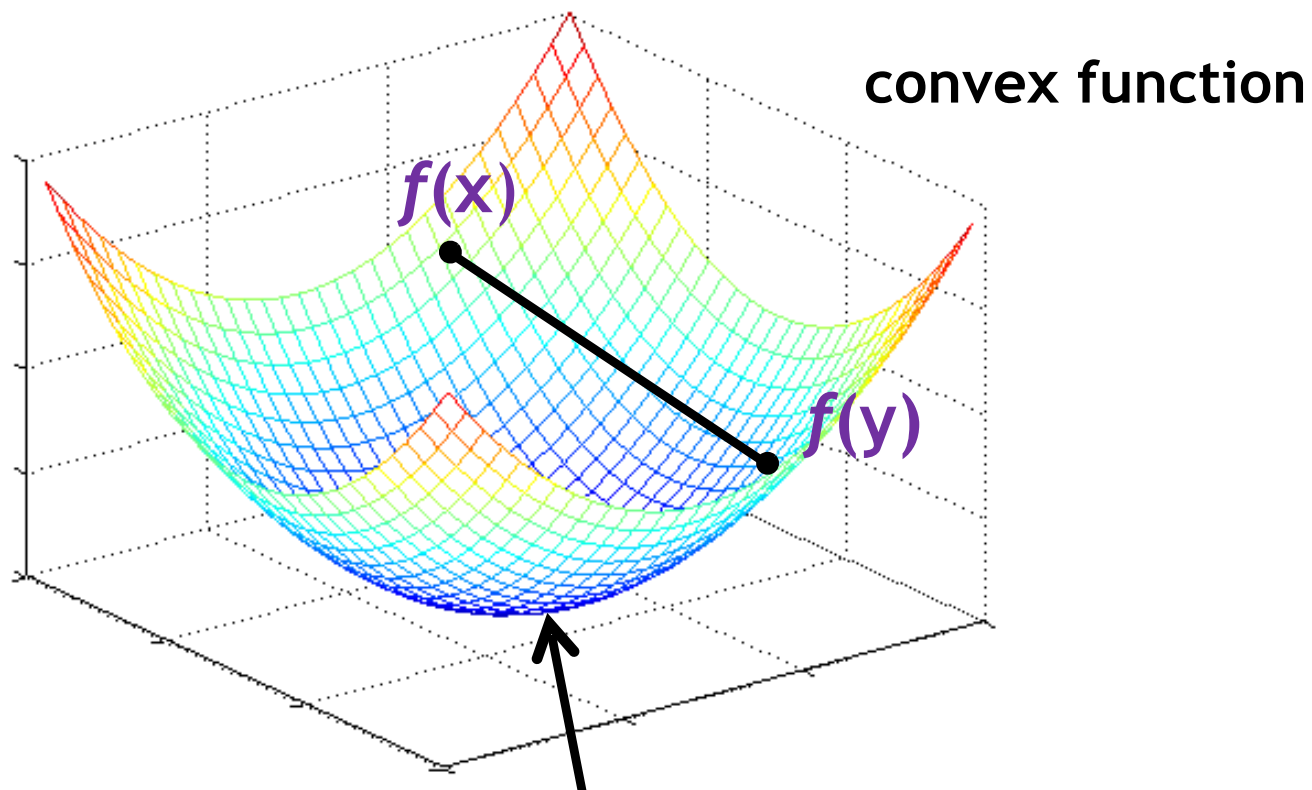
Given that we end up in state k at step i , maximize score to the left and right

**X is fixed:
=> parse to the left of step i , given we end in state k , does not affect parse to the right of step i**



Learning CRFs

- Key observation: $-\log P(\pi \mid x, w)$ is a differentiable, **convex** function of w



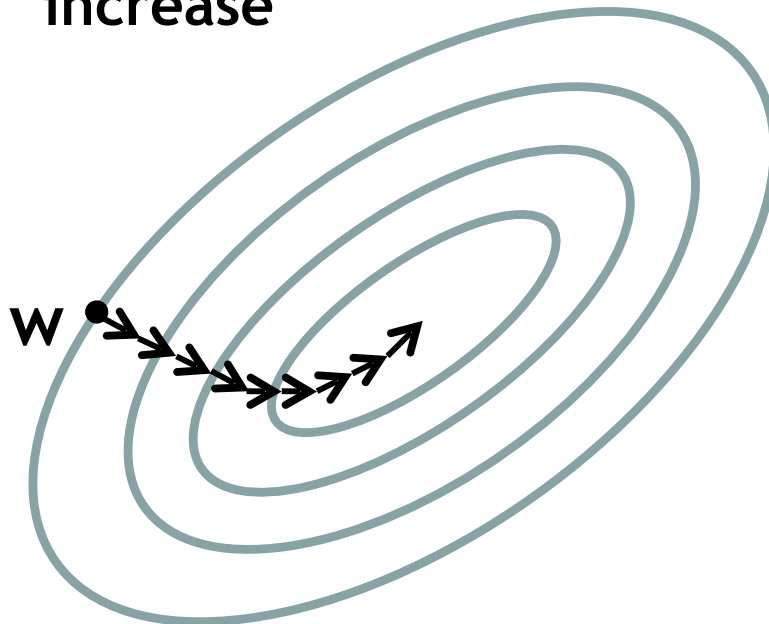
Any local minimum is a global minimum.



Learning CRFs (continued)

- Compute partial derivative of $\log P(\pi | x, w)$ with respect to each parameter w_j , and use the gradient ascent learning rule:

Gradient points in the direction of greatest function increase





The CRF gradient

- It turns out that

$$(\partial / \partial w_j) \log P(\pi \mid x, w) = F_j(x, \pi) - E_{\pi' \sim P(\pi' \mid x, w)} [F_j(x, \pi')]$$

correct value for
jth feature

expected value for
jth feature (given
the current
parameters)

- This has a very nice interpretation:
 - We increase parameters for which the correct feature values are greater than the predicted feature values
 - We decrease parameters for which the correct feature values are less than the predicted feature values
- This moves probability mass from incorrect parses to correct parses

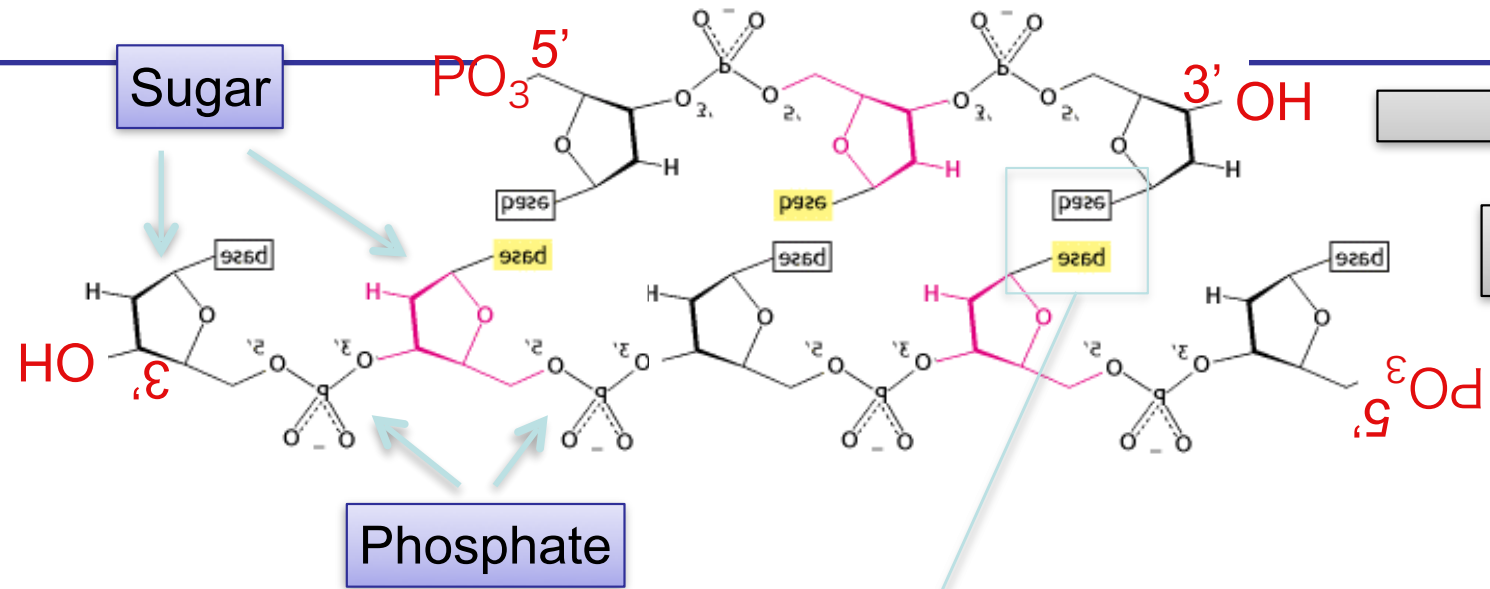


DNA Structure



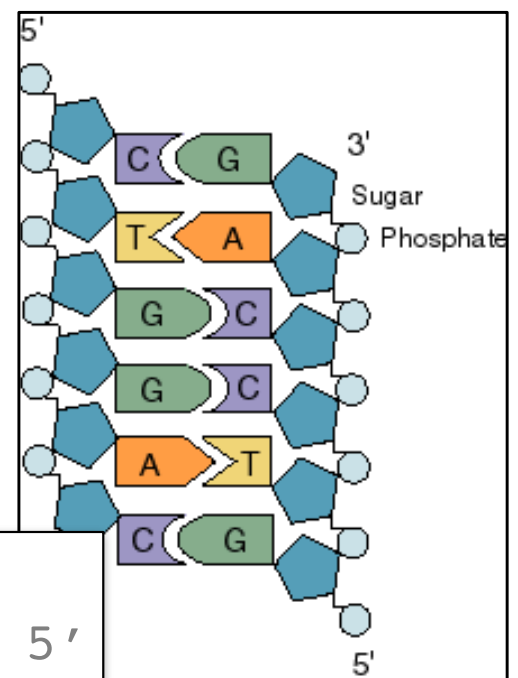
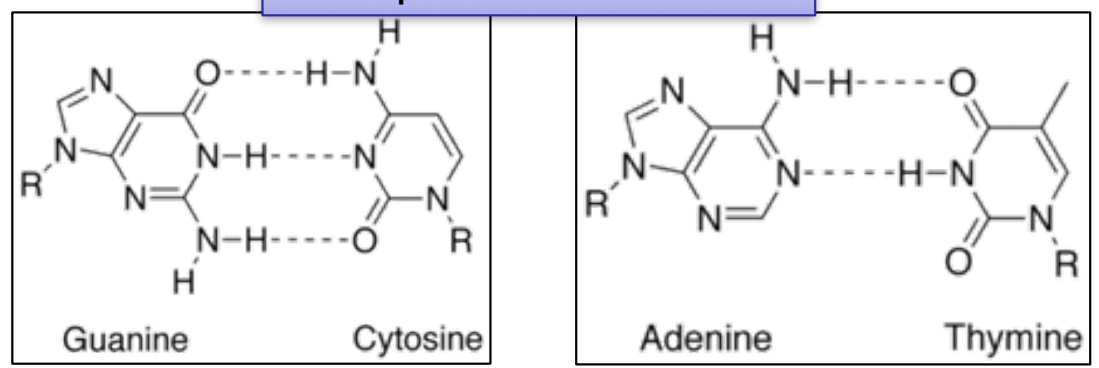


DNA structure



Direction of synthesis: nucleotides are always added to the 3' end.

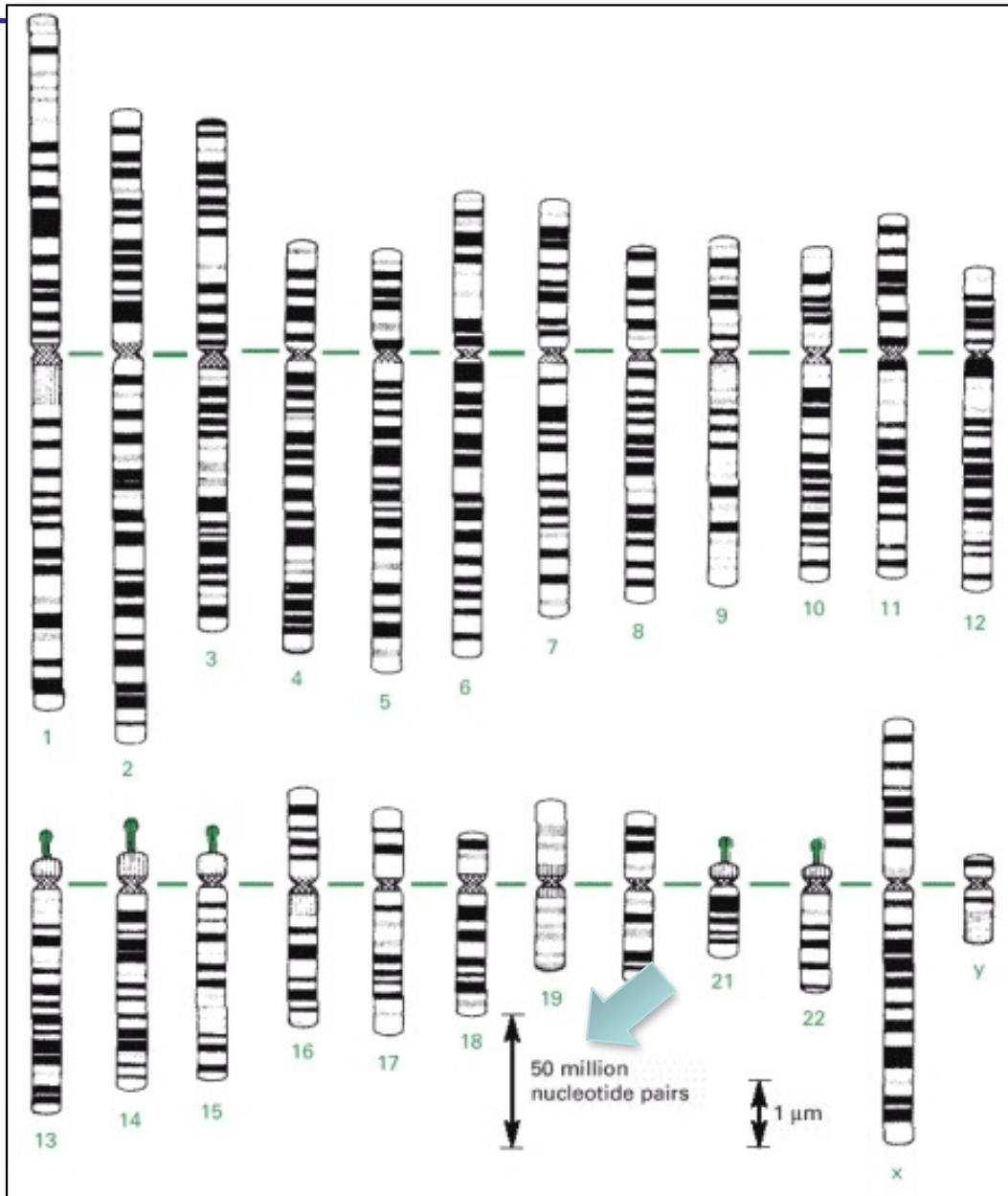
Base pairs: G-C and A-T



We only write out Watson
 Watson 5' C T G G A C 3'
 Crick 3' G A C C T G 5'



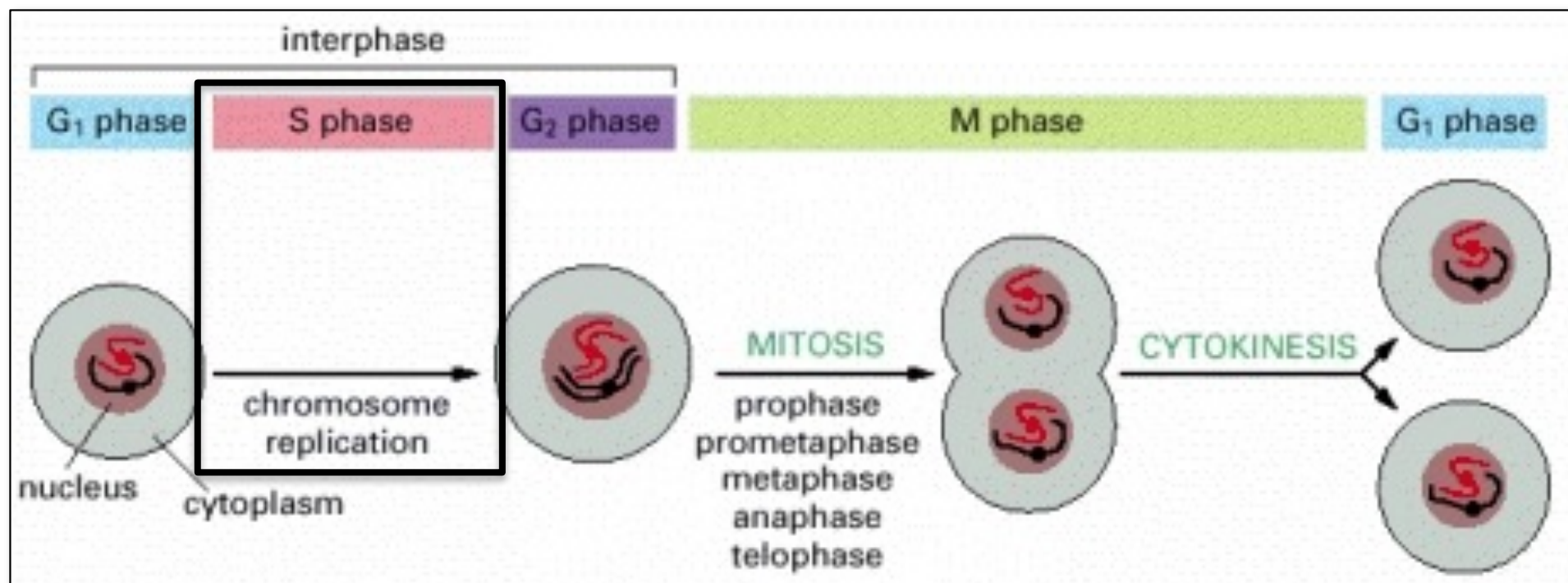
Human chromosomes



- 3,000 million base pairs total
- One replication origin every ~50 kb
- Replication happens only during a short specific period



Cell cycle



- DNA replication happens during a short time period
- Except in very early nonmammalian embryos, most time is spent in G₁ doing useful stuff
- Even in cancer cells, most time is spent in G₁ because cells don't divide until the daughter cells have grown back to standard cell size, and that requires lots of transcription and protein synthesis.



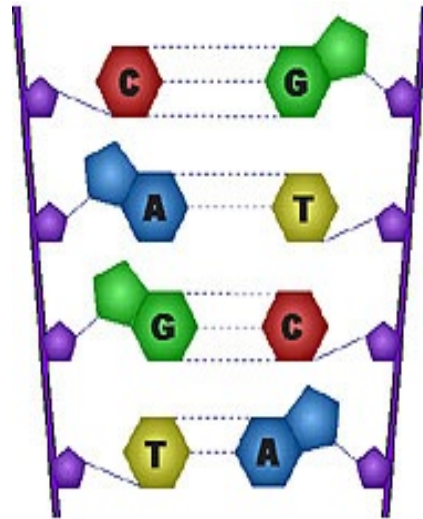
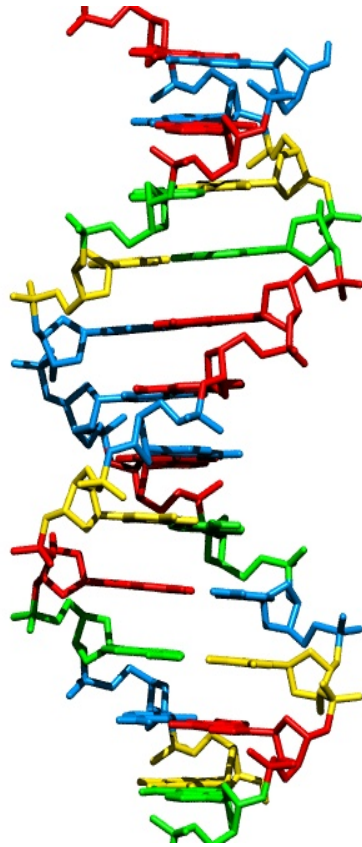
DNA Sequencing





DNA sequencing

How we obtain the sequence of nucleotides of a species



...ACGTGACTGAGGACCGTG
CGACTGAGACTGACTGGGT
CTAGCTAGACTACGTTTTA
TATATATATACGTCGTCGT
ACTGATGACTAGATTACAG
ACTGATTTAGATACCTGAC
TGATTTTAAAAAATATT...



Human Genome Project



3 billion basepairs
\$3 billion

1990: Start

2000: Bill Clinton:

2001: Draft

2003: Finished

“most important scientific discovery in the 20th century”

now what?



Which representative of the species?

Which human?

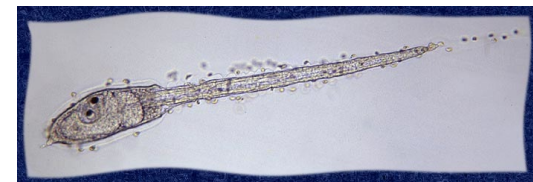
Answer one:



Answer two: it doesn't matter

Polymorphism rate: number of letter changes between two different members of a species

Humans: $\sim 1/1,000$



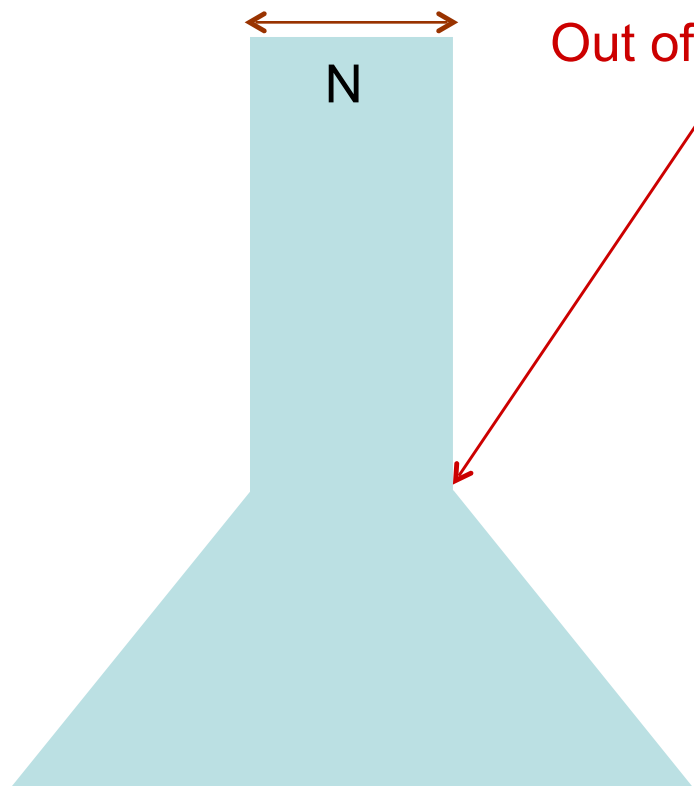
Other organisms have much higher polymorphism rates

- Population size!

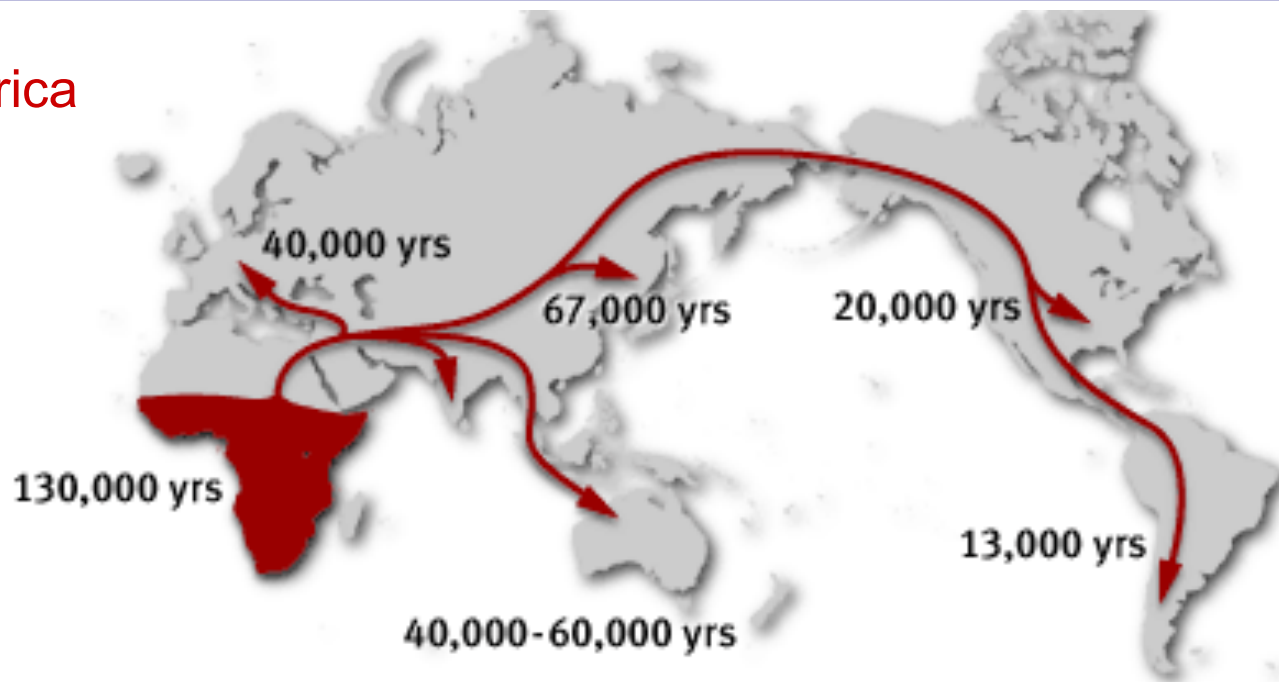




Why humans are so similar



Out of Africa

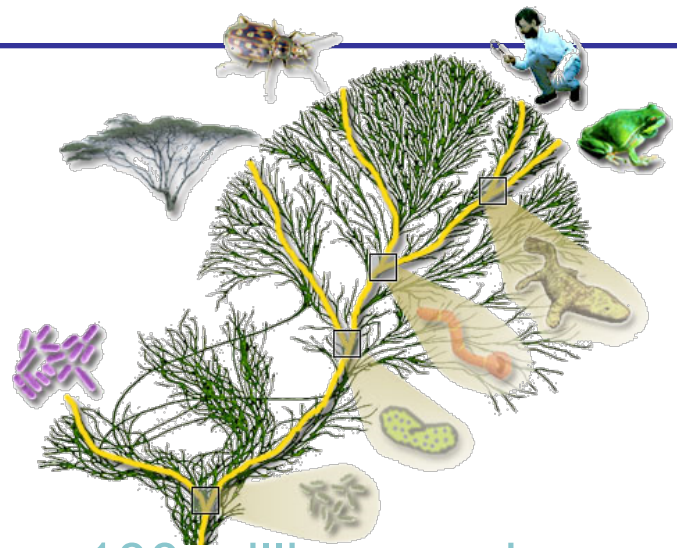


A small population that interbred reduced the genetic variation

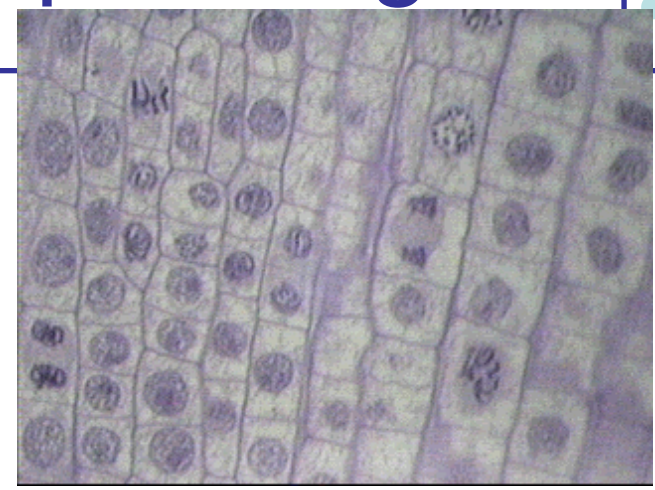
Out of Africa ~ 40,000 years ago

Heterozygosity: H
 $H = 4Nu / (1 + 4Nu)$
 $u \sim 10^{-8}$, $N \sim 10^4$
 $\Rightarrow H \sim 4 \times 10^{-4}$

There is never “enough” sequencing



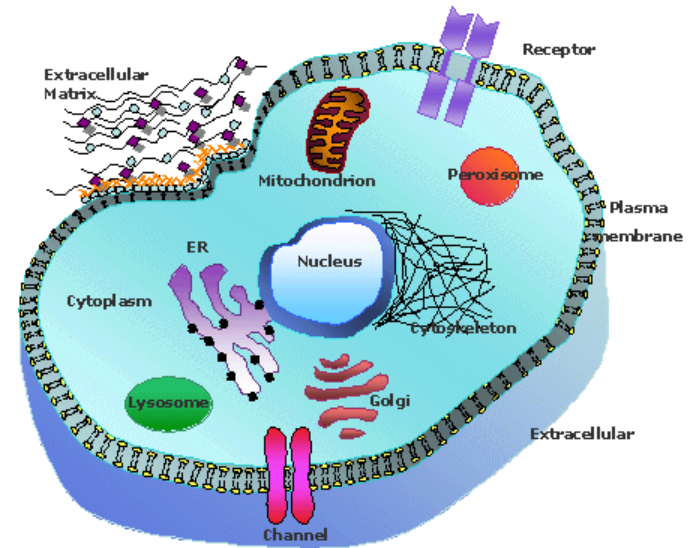
100 million species



Somatic mutations (e.g., HIV, cancer)



7 billion individuals



Sequencing is a functional assay



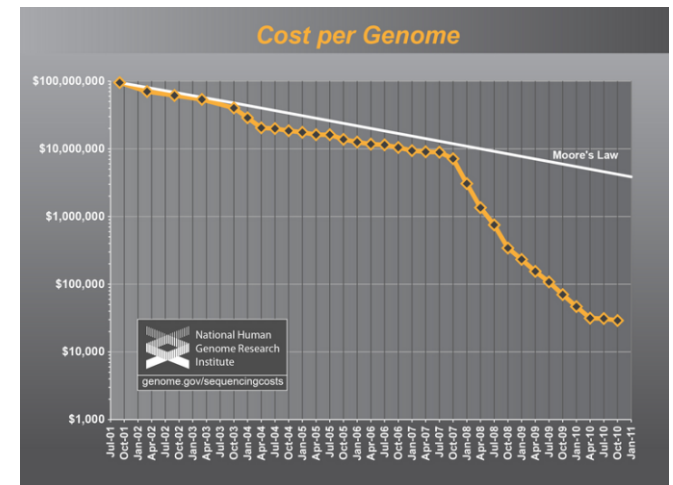
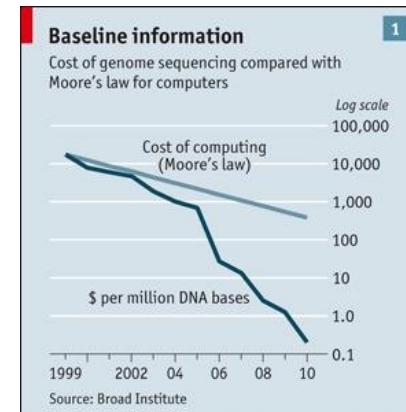
Sequencing Growth

Cost of one human genome

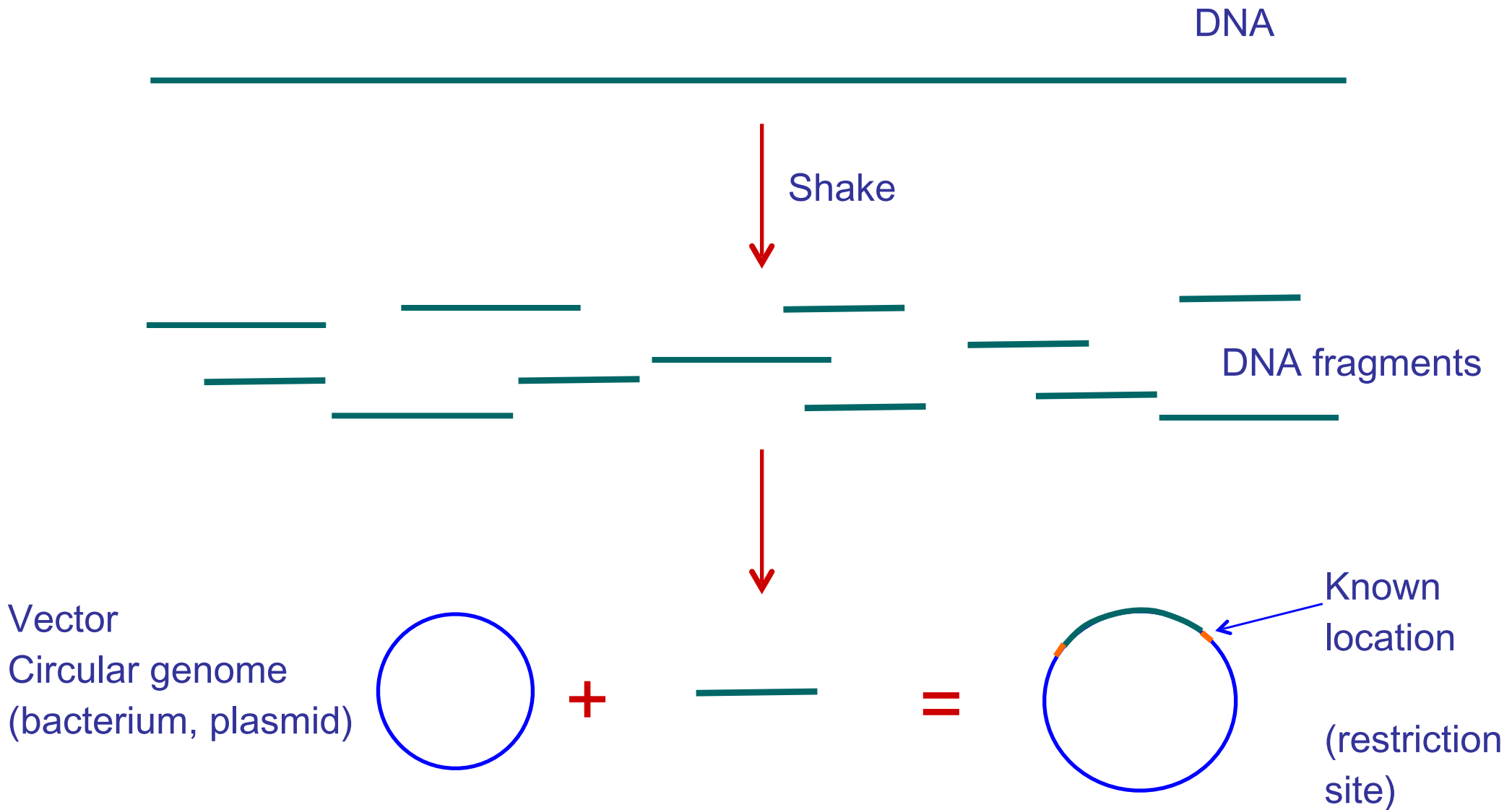
- 2004: \$30,000,000
- 2008: \$100,000
- 2010: \$10,000
- **2014: \$1,000**
- ????: \$300



How much would you pay for a smartphone?



Ancient sequencing technology – Sanger Vectors





Fluorescent Sanger sequencing trace

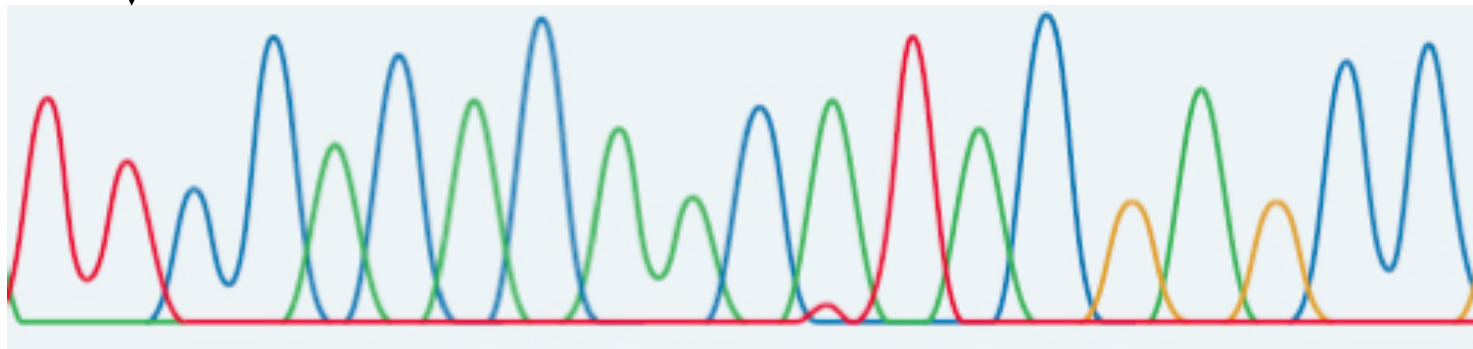
Lane signal



(Real fluorescent signals from a lane/capillary are much uglier than this).

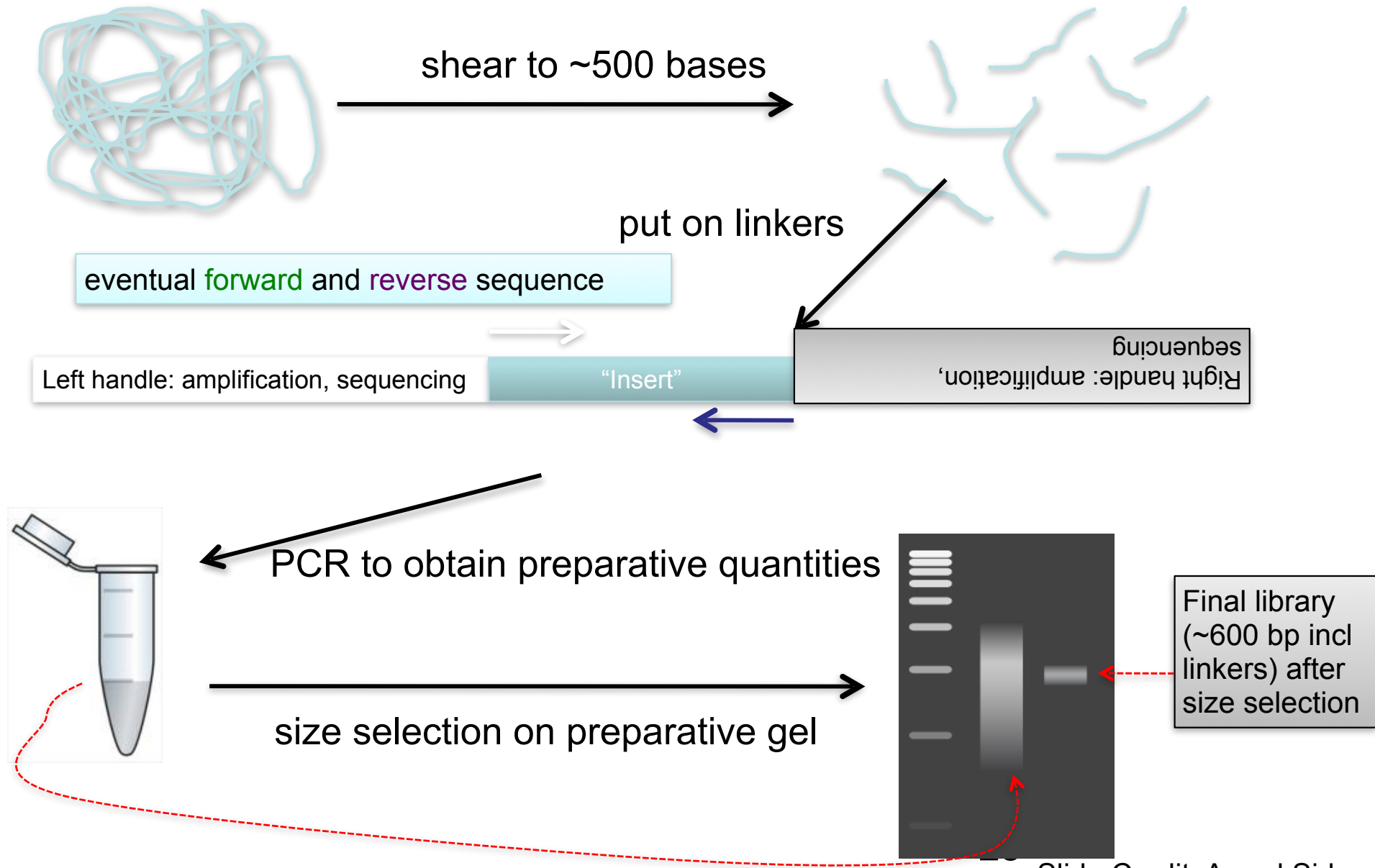
A bunch of magic to boost signal/noise, correct for dye-effects, mobility differences, etc, generates the 'final' trace (for each capillary of the run)

Trace





Making a Library (present)



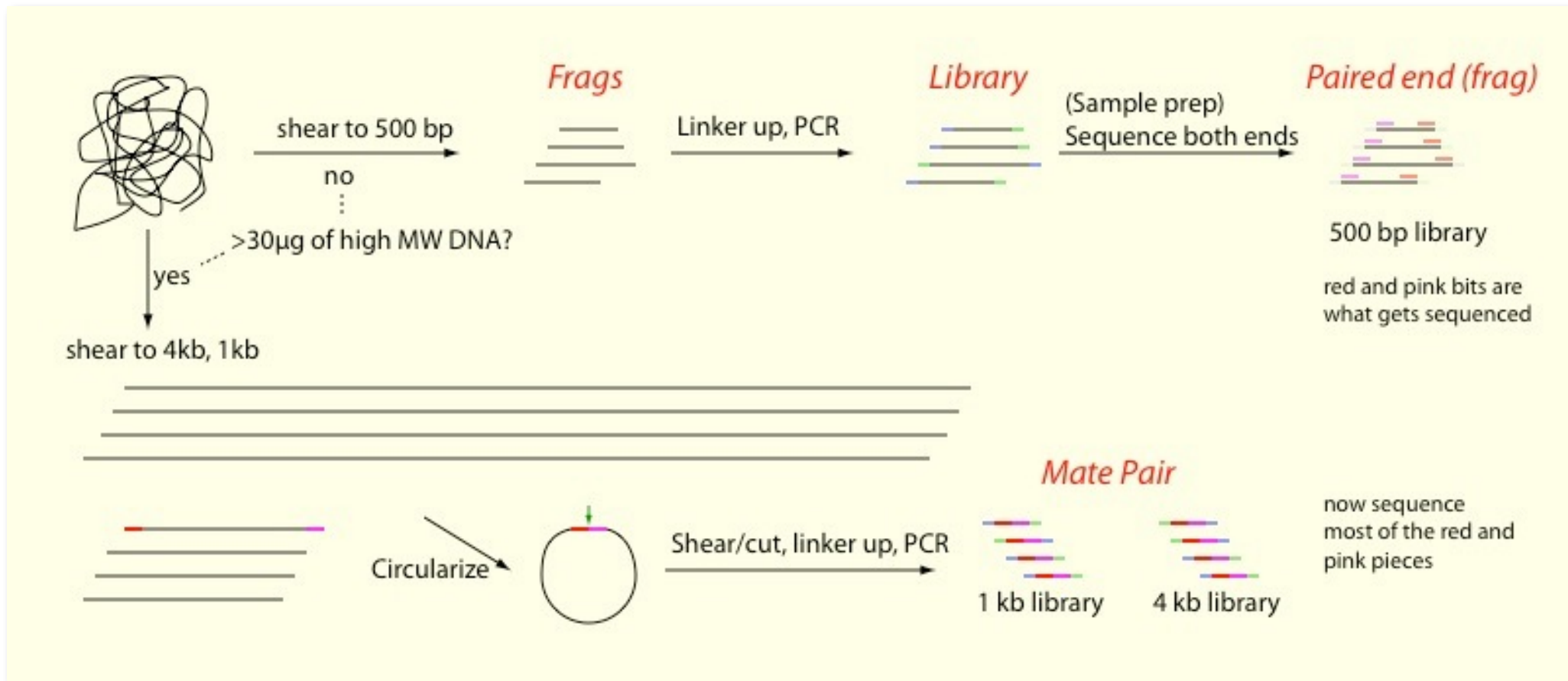
Library



- Library is a massively complex mix of -initially- individual, unique fragments
- Library amplification mildly amplifies each fragment to retain the complexity of the mix while obtaining preparative amounts
 - (how many-fold do 10 cycles of PCR amplify the sample?)



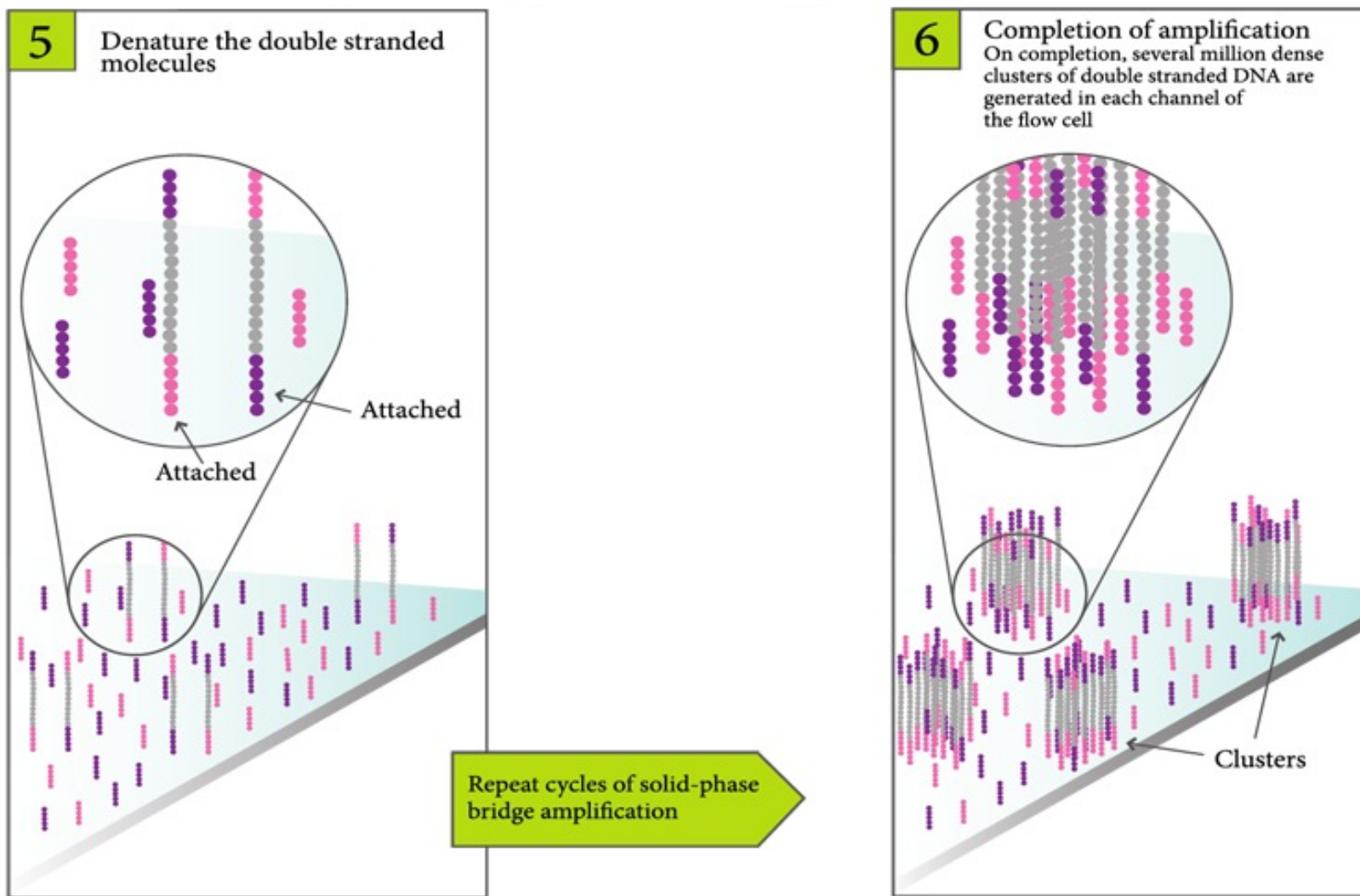
Fragment vs Mate pair ('jumping')



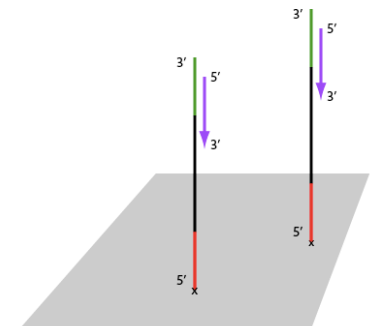
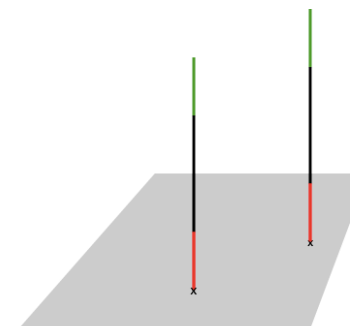
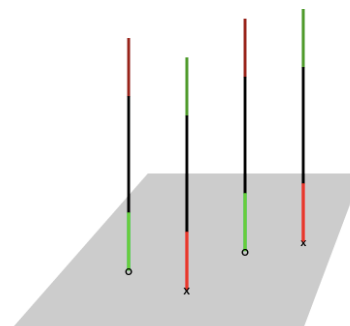
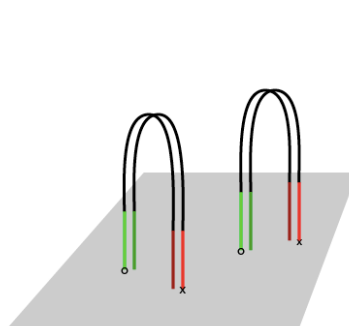
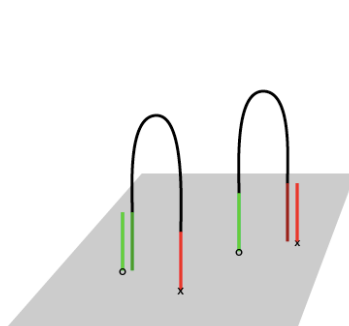
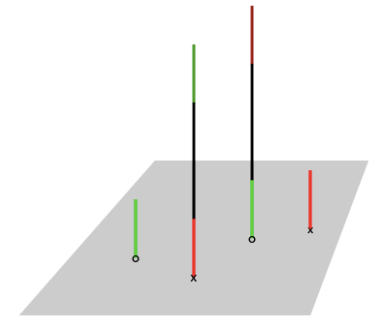
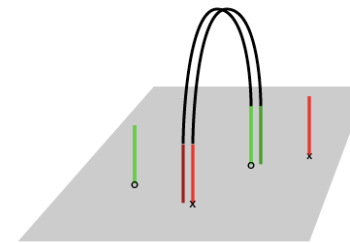
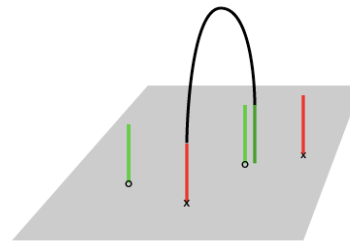
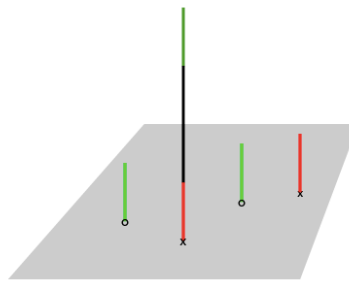
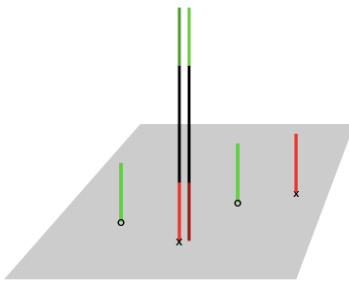
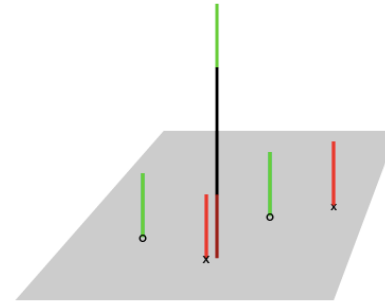
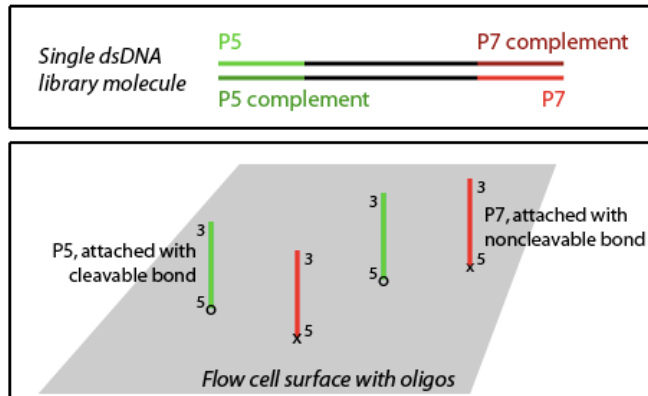
(Illumina has new kits/methods with which mate pair libraries can be built with less material)



Illumina cluster concept

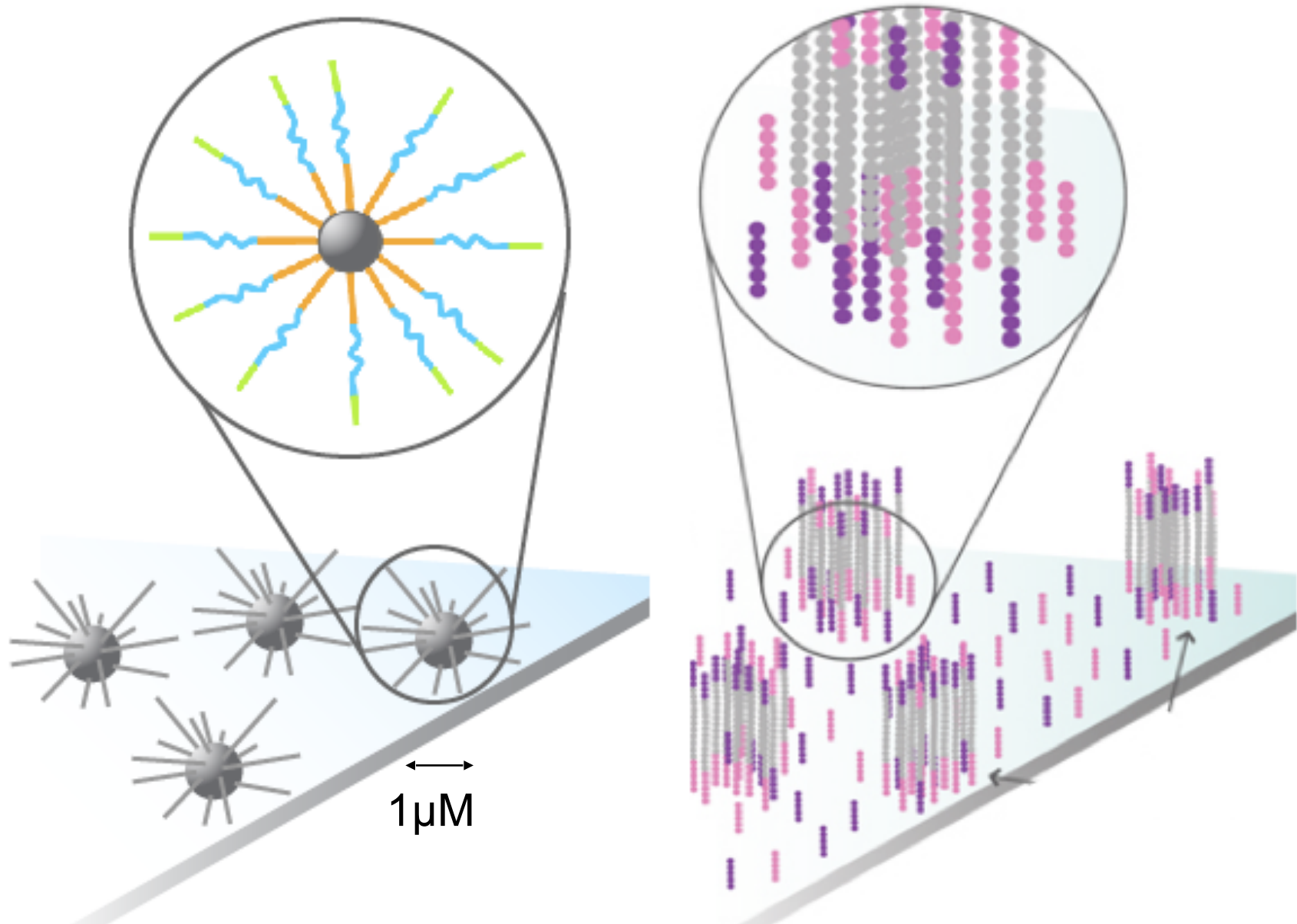


Cluster generation ('bridge amplification')



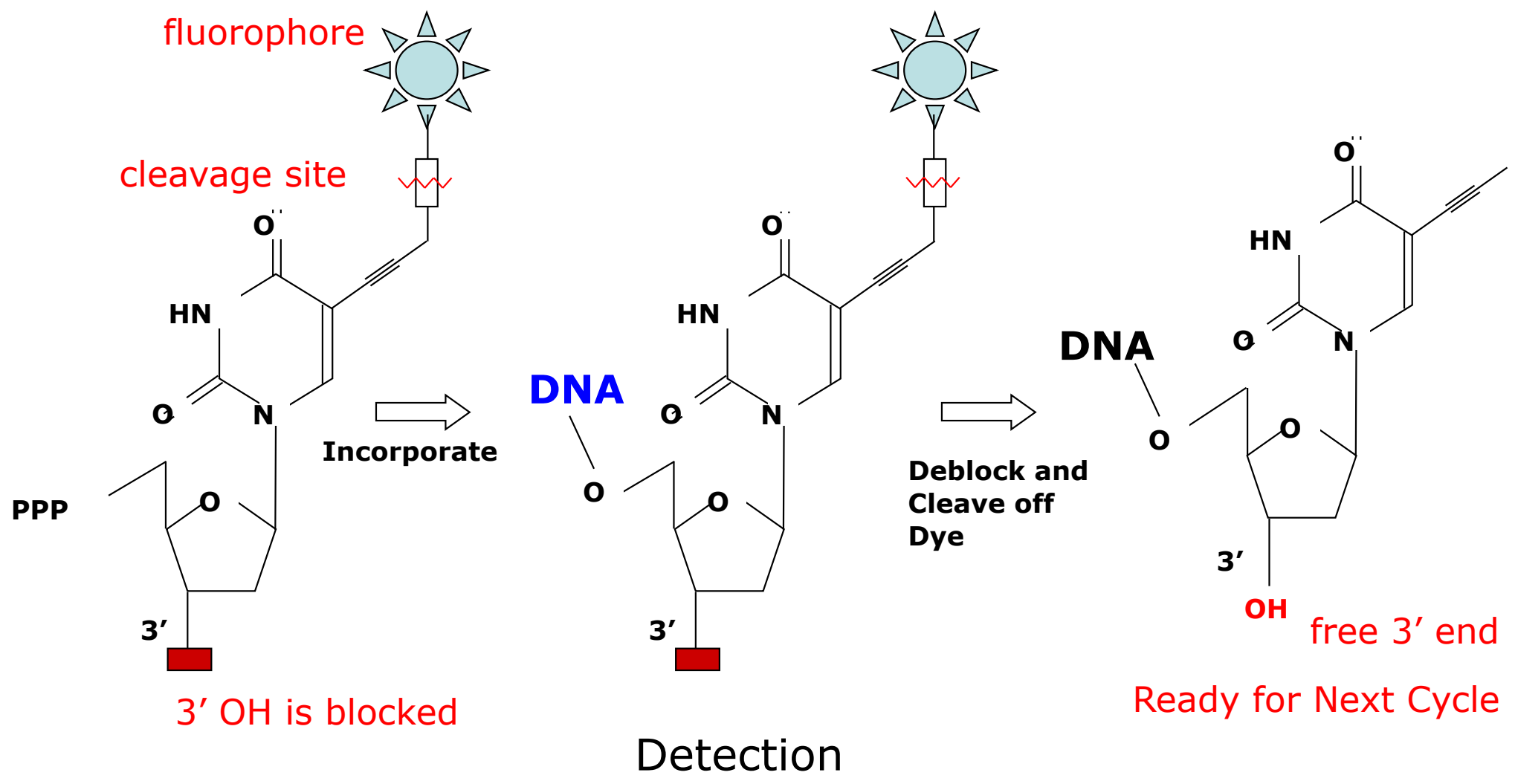


Clonally Amplified Molecules on Flow Cell



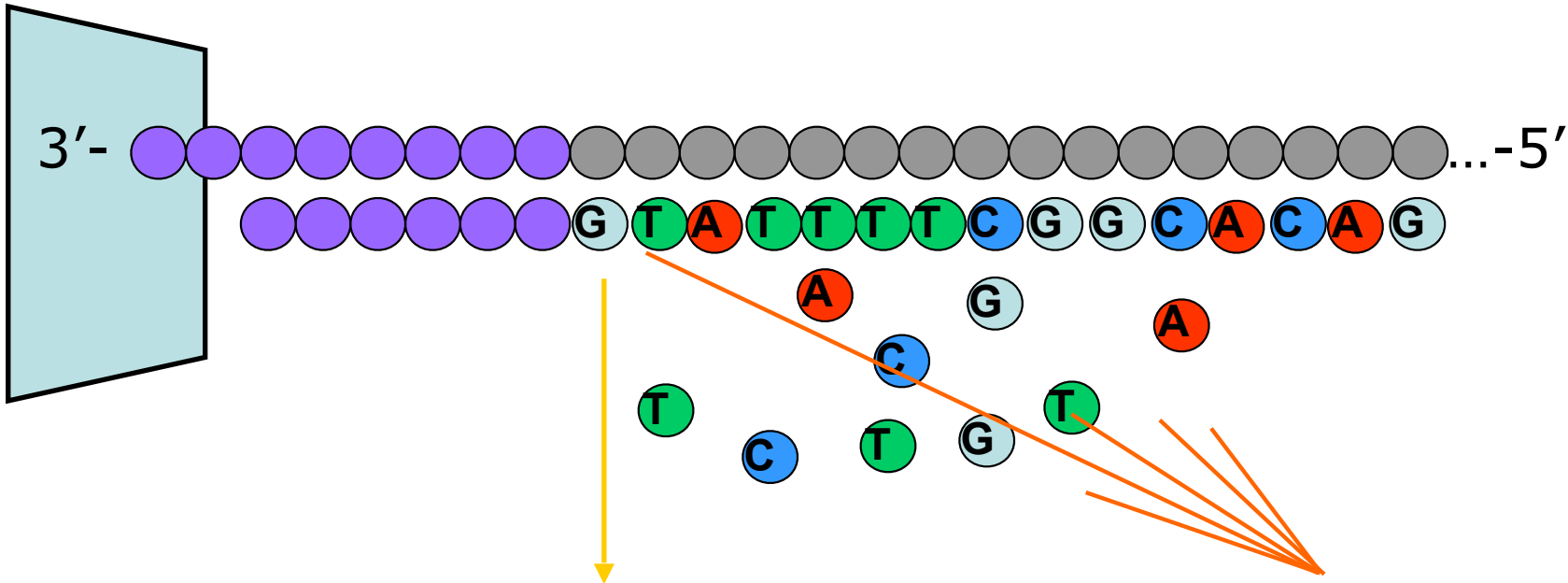


Reversible Terminators





Sequencing by Synthesis, One Base at a Time



Cycle 1: Add sequencing reagents
 First base incorporated
 Remove unincorporated bases
 Detect signal

Cycle 2-n: Add sequencing reagents and repeat



HiSeq X & NextSeq



Preliminary specs:

Run time: 3 days
 Output: 1.6 Tb
 #reads: 6×10^9
 Read length: 2x150bp



NextSeq 500 Sequencing System Performance Parameters

NEXTSEQ 500 HIGH OUTPUT KIT *

READ LENGTH	TOTAL TIME†	OUTPUT
2 × 150 bp	~29 hrs	100-120 Gb
2 × 75 bp	18 hrs	50-60 Gb
1 × 75 bp	11 hrs	25-30 Gb

NEXTSEQ 500 MID OUTPUT KIT *

READ LENGTH	TOTAL TIME†	OUTPUT
2 × 150 bp	26 hrs	32.5-39 Gb
2 × 75 bp	15 hrs	16.25-19.5 Gb

Reads Passing Filter

NEXTSEQ 500 HIGH OUTPUT KIT

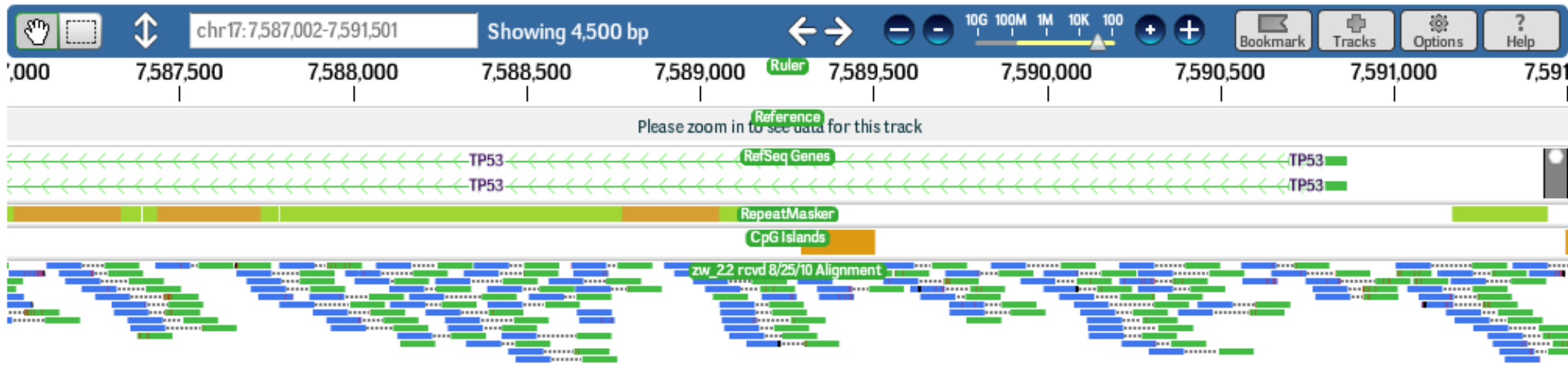
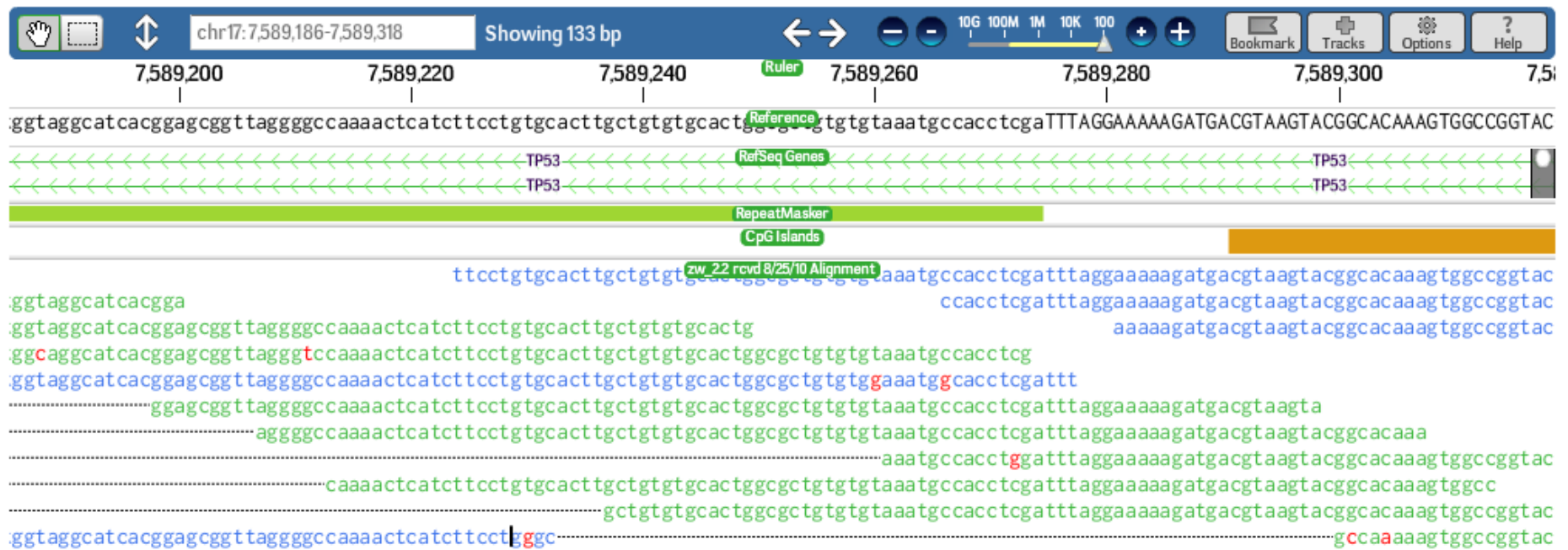
Single Reads	Up to 400 Million
Paired-End Reads	Up to 800 million

NEXTSEQ 500 MID OUTPUT KIT

Single Reads	Up to 130 Million
Paired-End Reads	Up to 260 Million



Read Mapping

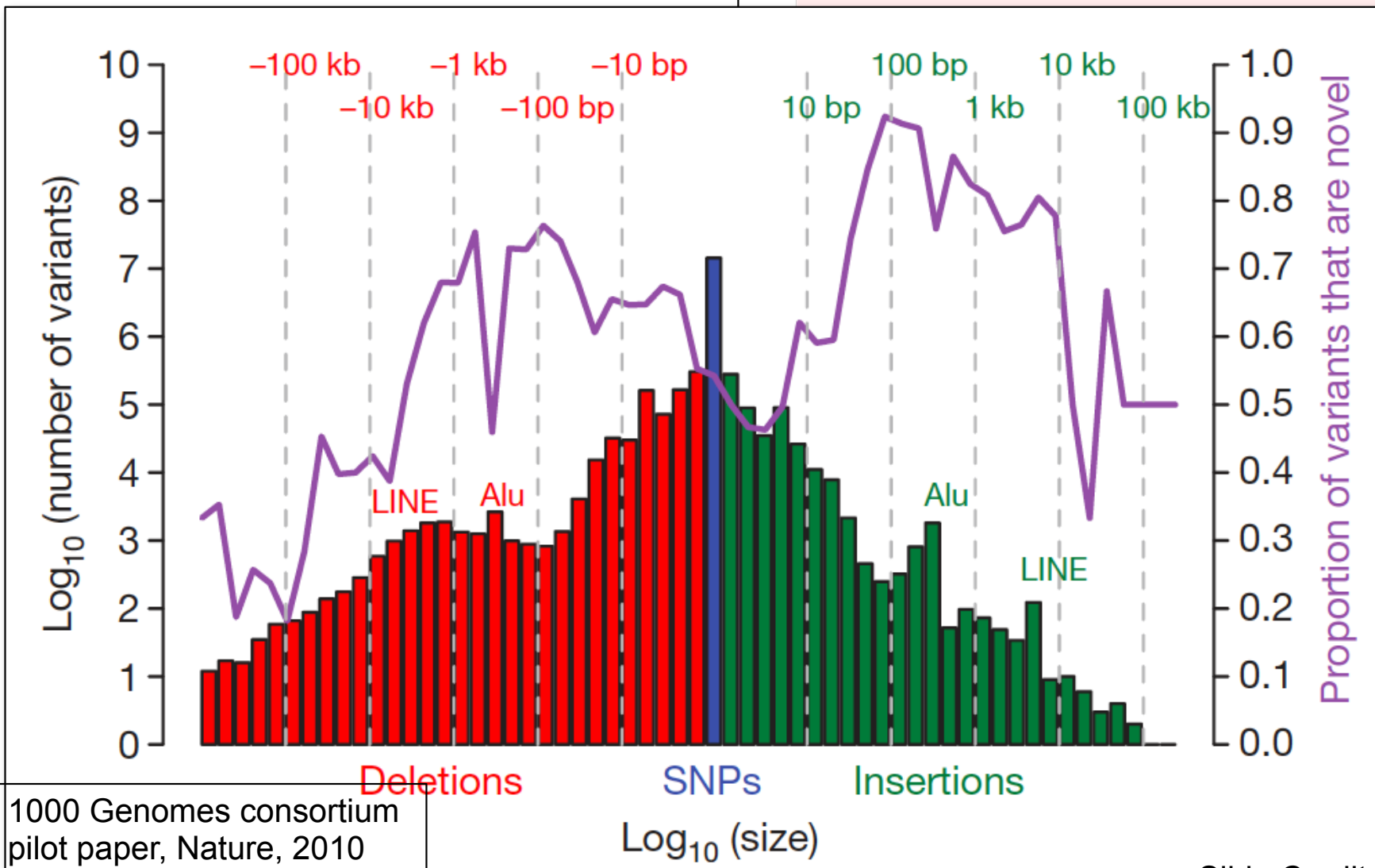




Amount of variation – types of lesions

Mutation Types

Lesion type	Typical lesion size range (bp)	Lesion cartoon	Lesion in het
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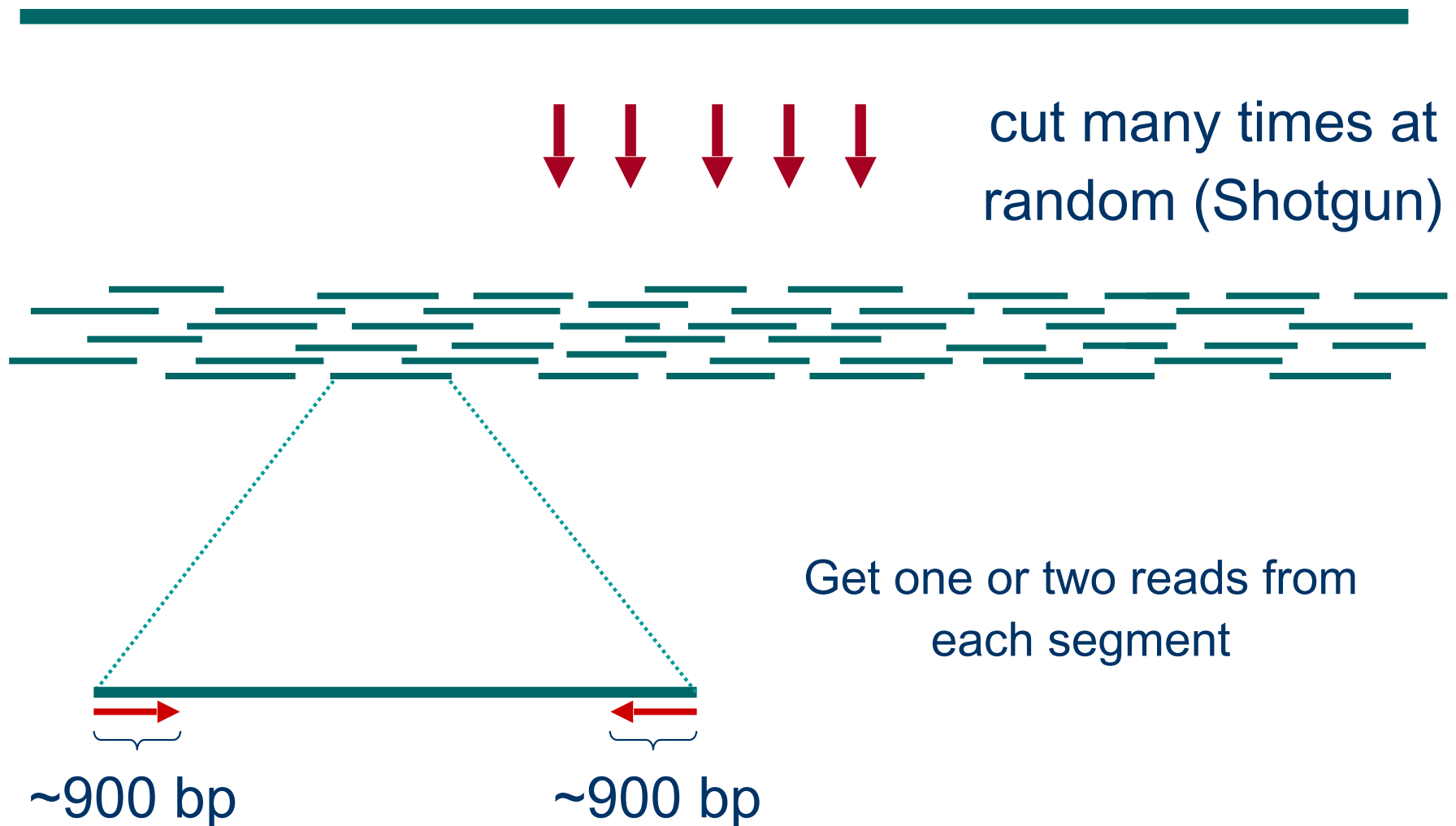
TGCTGAGA TG-----GA	
TG-----GA TGCGGAGA	
TGCTGAGA TGCGGAGA	

1000 Genomes consortium
pilot paper, Nature, 2010



Method to sequence longer regions

genomic segment





Two main assembly problems

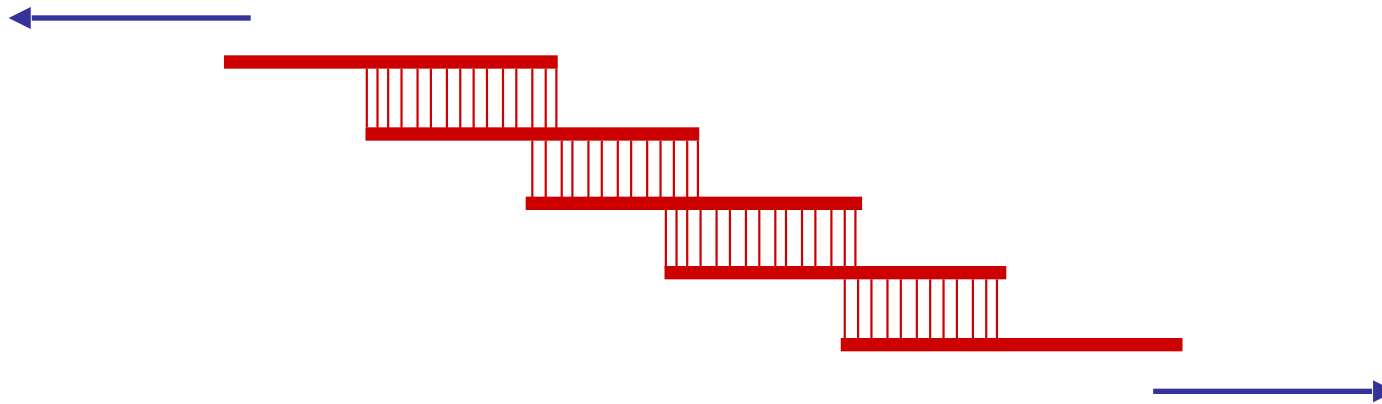
- De Novo Assembly



- Resequencing



Reconstructing the Sequence (De Novo Assembly)

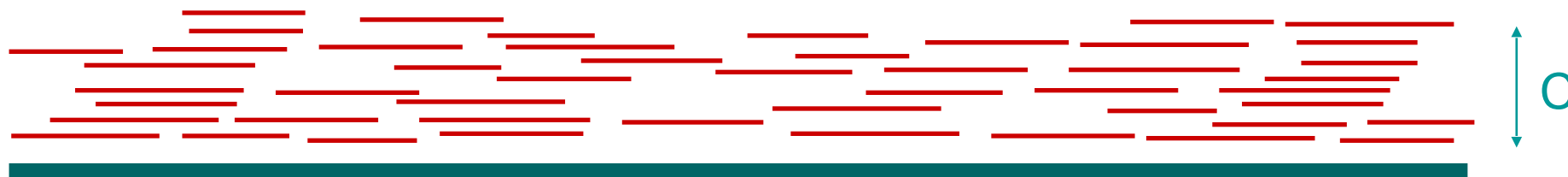


Cover region with high redundancy

Overlap & extend reads to reconstruct the original genomic region



Definition of Coverage



Length of genomic segment: **G**

Number of reads: **N**

Length of each read: **L**

Definition: Coverage **$C = N L / G$**

How much coverage is enough?

Lander-Waterman model: **$\text{Prob}[\text{not covered bp}] = e^{-C}$**

Assuming uniform distribution of reads, $C=10$ results in 1 gapped region / 1,000,000 nucleotides

Repeats



Bacterial genomes: 5%

Mammals:

50%

Repeat types:

- **Low-Complexity DNA** (e.g. ATATATATACATA...)
- **Microsatellite repeats** $(a_1 \dots a_k)^N$ where $k \sim 3-6$
(e.g. CAGCAGTAGCAGCACCAG)
- **Transposons**
 - **SINE** (Short Interspersed Nuclear Elements)
e.g., ALU: ~300-long, 10^6 copies
 - **LINE** (Long Interspersed Nuclear Elements)
~4000-long, 200,000 copies
 - **LTR retroposons** (Long Terminal Repeats (~700 bp) at each end)
cousins of HIV
- **Gene Families** genes duplicate & then diverge (paralogs)
- **Recent duplications** ~100,000-long, very similar copies



Sequencing and Fragment Assembly



3×10^9 nucleotides

50% of human DNA is composed



Error!

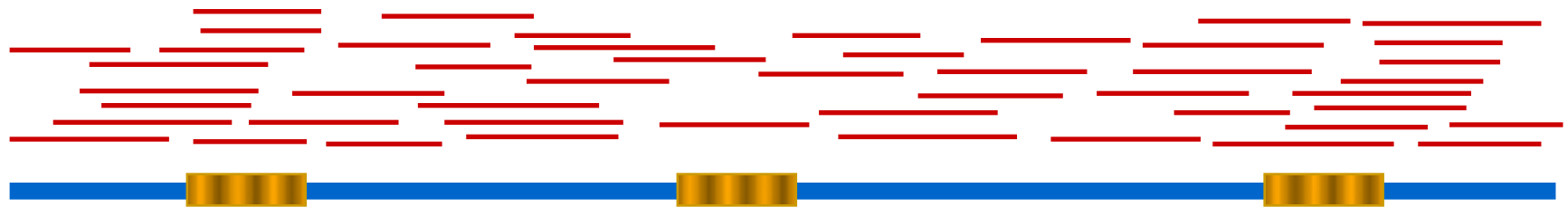
Glued together two distant regions



What can we do about repeats?

Two main approaches:

- Cluster the reads



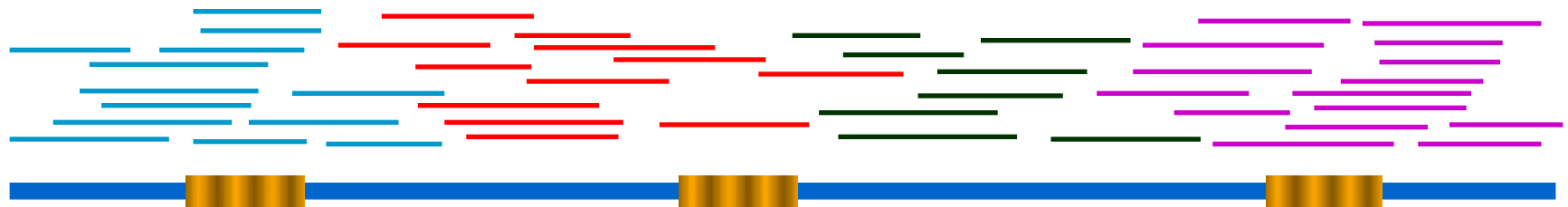
- Link the reads



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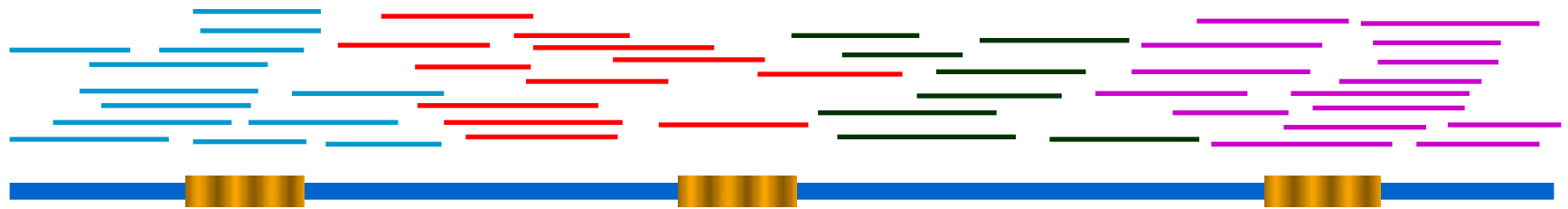
- Link the reads



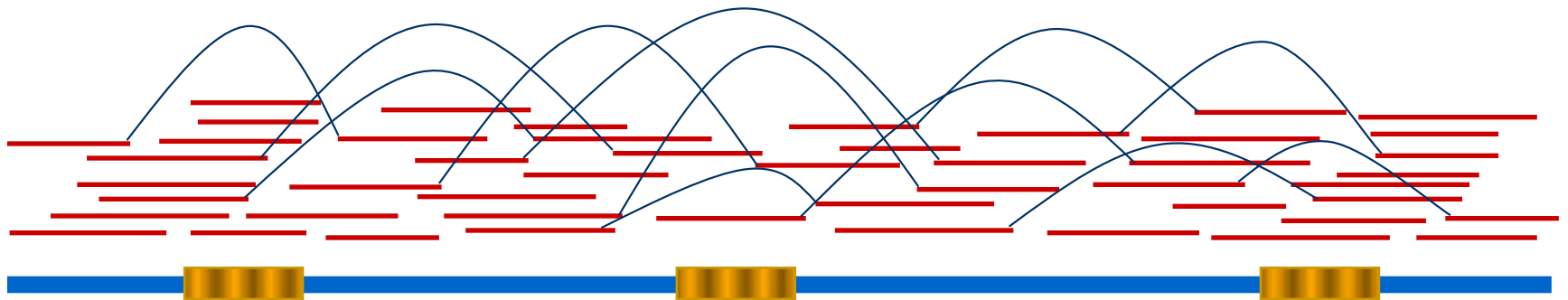
What can we do about repeats?

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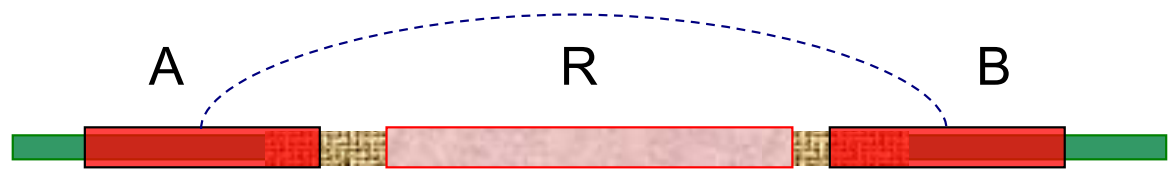




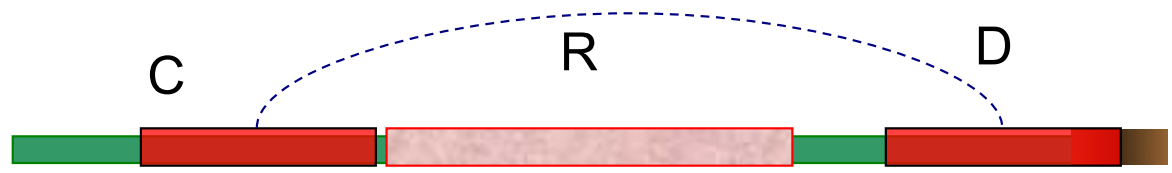
Sequencing and Fragment Assembly



3×10^9 nucleotides



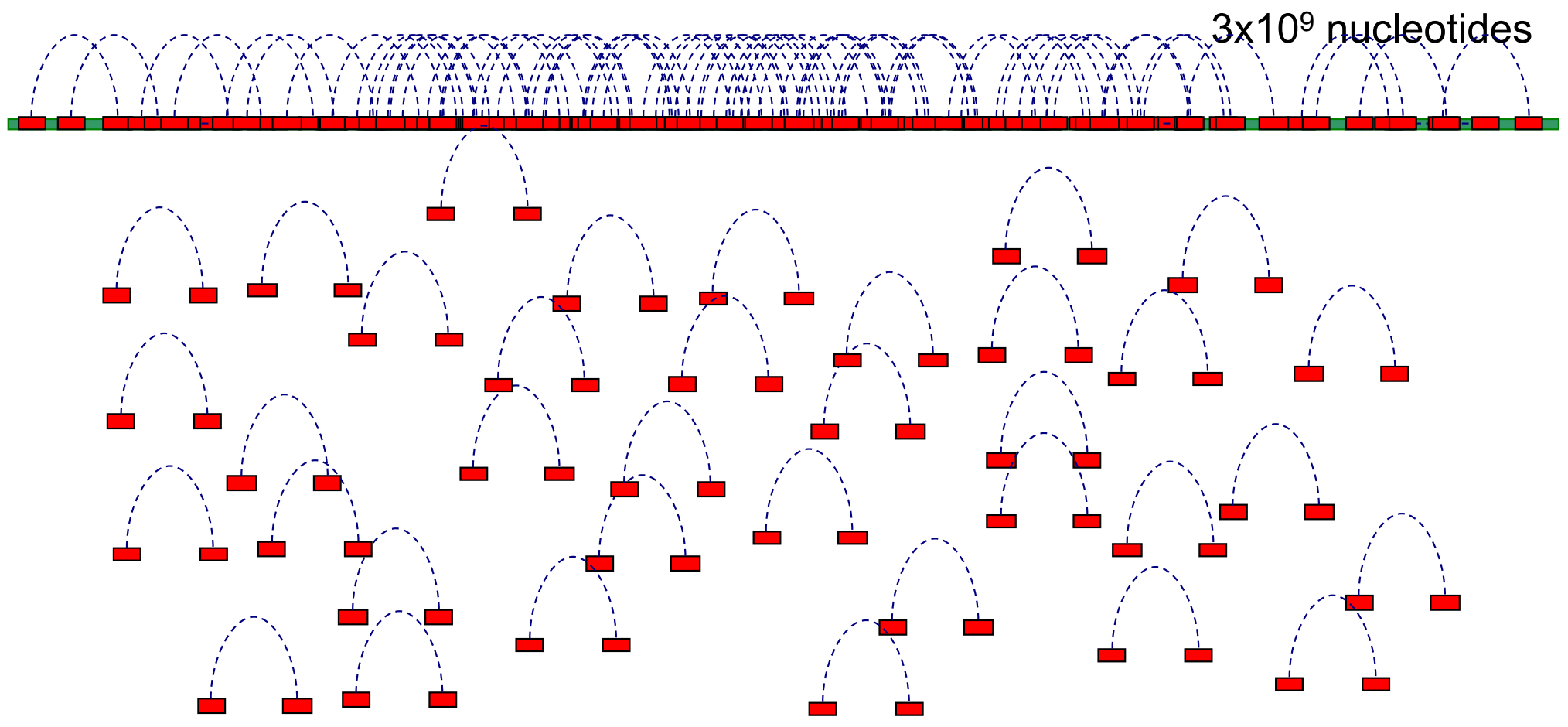
ARB, CRD



or
~~ARD, CRB ?~~



Sequencing and Fragment Assembly





Fragment Assembly

(in whole-genome shotgun sequencing)



Fragment Assembly



Given N reads...
Where $N \sim 30$
million...

We need to use a
linear-time
algorithm

Steps to Assemble a Genome



Some Terminology

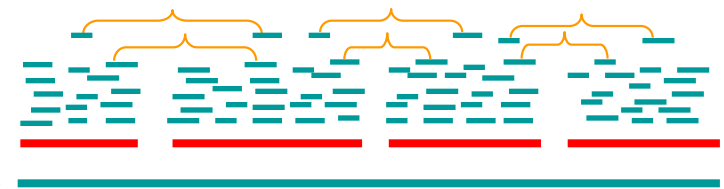
read a 500-900 long word that comes out of sequencer

mate pair a pair of reads from two ends of the same insert fragment

contig a contiguous sequence formed by several overlapping reads with no gaps

supercontig (scaffold) an ordered and oriented set of contigs, usually by mate pairs

consensus sequence sequence derived from the multiple alignment of reads in a contig



..ACGATTACAATAGGTT..



1. Find Overlapping Reads

aaactgcagtacggatct
aaactgcag
 aactgcagt

...

gtacggatct
 tacggatct

gggcccaactgcagtac
gggcccaaa
 ggcccaaac

...

actgcagta
 ctgcagtac

gtacggatctactacaca
gtacggatc
 tacggatct

...

ctactacac
 tactacaca

(read, pos., word, orient.)

aaactgcag
aactgcagt
actgcagta

...

gtacggatc
tacggatct

gggcccaaa
ggcccaaac
gcccaaac

...

actgcagta
ctgcagtac

gtacggatc
tacggatct
acggatcta

...

ctactacac
tactacaca

(word, read, orient., pos.)

aaactgcag
aactgcagt
acggatcta

actgcagta
actgcagta

cccaactg
cggatctac
ctactacac

ctgcagtac

ctgcagtac
gcccaaac

ggcccaaac
gggcccaaa
gtacggatc

gtacggatc
tacggatct

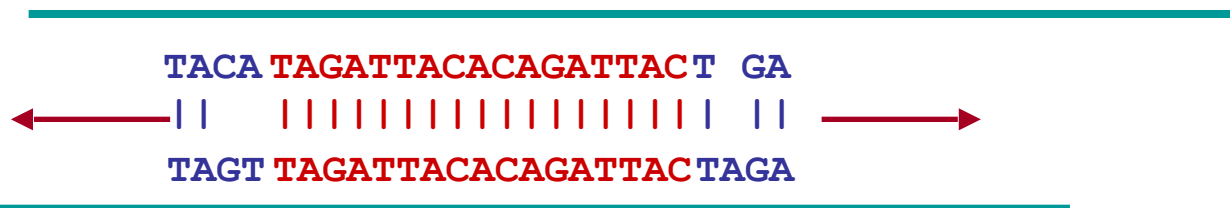
tacggatct
tactacaca

tactacaca



1. Find Overlapping Reads

- Find pairs of reads sharing a k-mer, $k \sim 24$
- Extend to full alignment – throw away if not $>98\%$ similar



- Caveat: repeats
 - A k-mer that occurs N times, causes $O(N^2)$ read/read comparisons
 - ALU k-mers could cause up to $1,000,000^2$ comparisons
- Solution:
 - Discard all k-mers that occur “too often”
 - Set cutoff to balance sensitivity/speed tradeoff, according to genome at hand and computing resources available



1. Find Overlapping Reads

Create local multiple alignments from the overlapping reads

```
TAGATTACACAGATTACTGA  
TAGATTACACAGATTACTGA  
TAG TTACACAGATTATTGA  
TAGATTACACAGATTACTGA  
TAGATTACACAGATTACTGA  
TAGATTACACAGATTACTGA  
TAG TTACACAGATTATTGA  
TAGATTACACAGATTACTGA
```




1. Find Overlapping Reads

- Correct errors using multiple alignment

```
TAGATTACACAGATTACTGA
TAGATTACACAGATTACTGA
TAGATTACACAGATTACTGA
TAGATTACACAGATTACTGA
TAGATTACACAGATTACTGA
```

insert A

replace T with C

```
TAGATTACACAGATTACTGA
TAGATTACACAGATTACTGA
TAGATTACACAGATTACTGA
TAGATTACACAGATTACTGA
TAGATTACACAGATTACTGA
```

correlated errors—
probably caused by repeats
⇒ disentangle overlaps

```
TAGATTACACAGATTACTGA
TAGATTACACAGATTACTGA
TAGATTACACAGATTACTGA
```

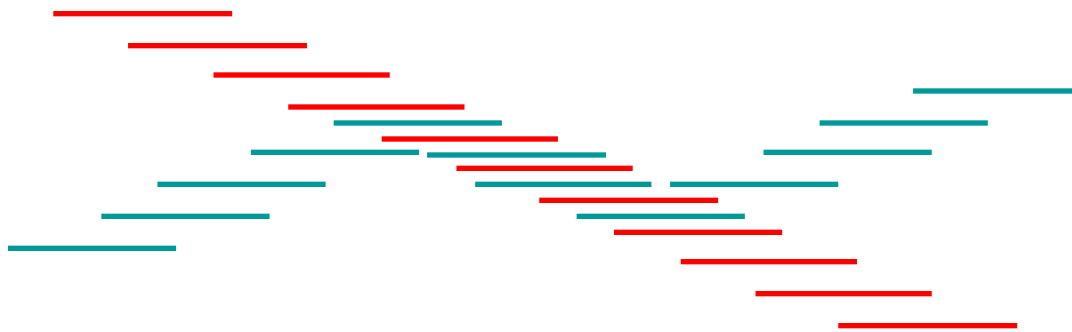
In practice, error correction removes up to 98% of the errors

```
TAG-TTACACAGATTATTGA
TAG-TTACACAGATTATTGA
```

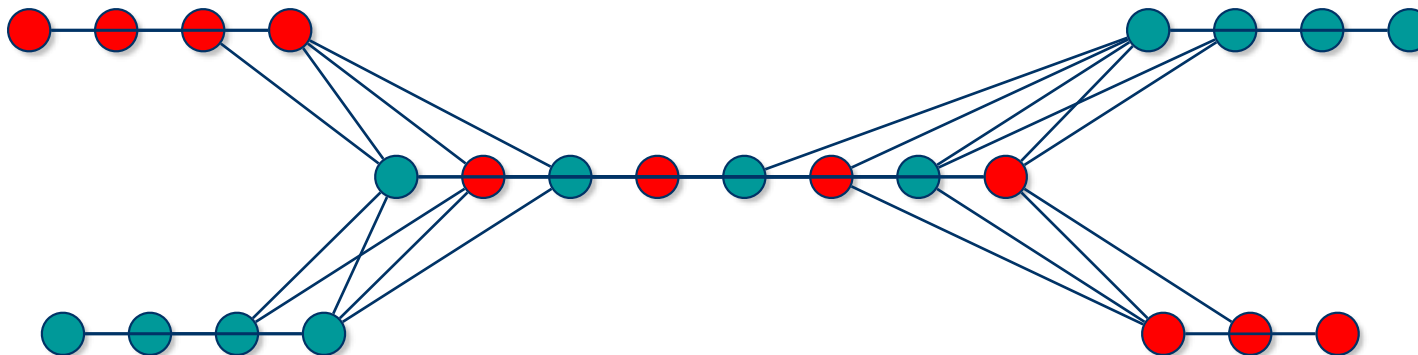


2. Merge Reads into Contigs

- Overlap graph:
 - Nodes: reads $r_1 \dots r_n$
 - Edges: overlaps (r_i, r_j , shift, orientation, score)



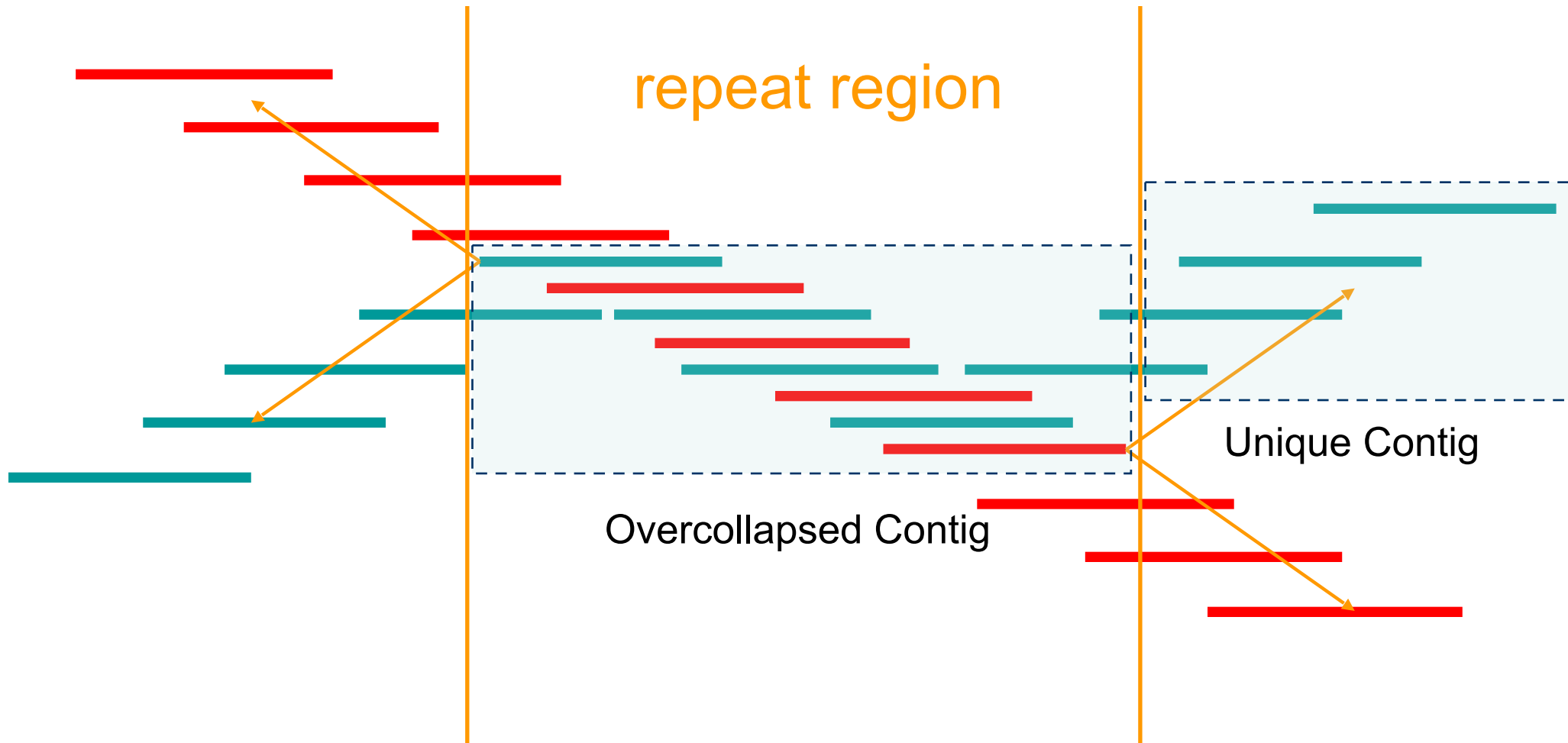
Reads that come from two regions of the genome (blue and red) that contain the same repeat



Note:
of course, we don't know the "color" of these nodes



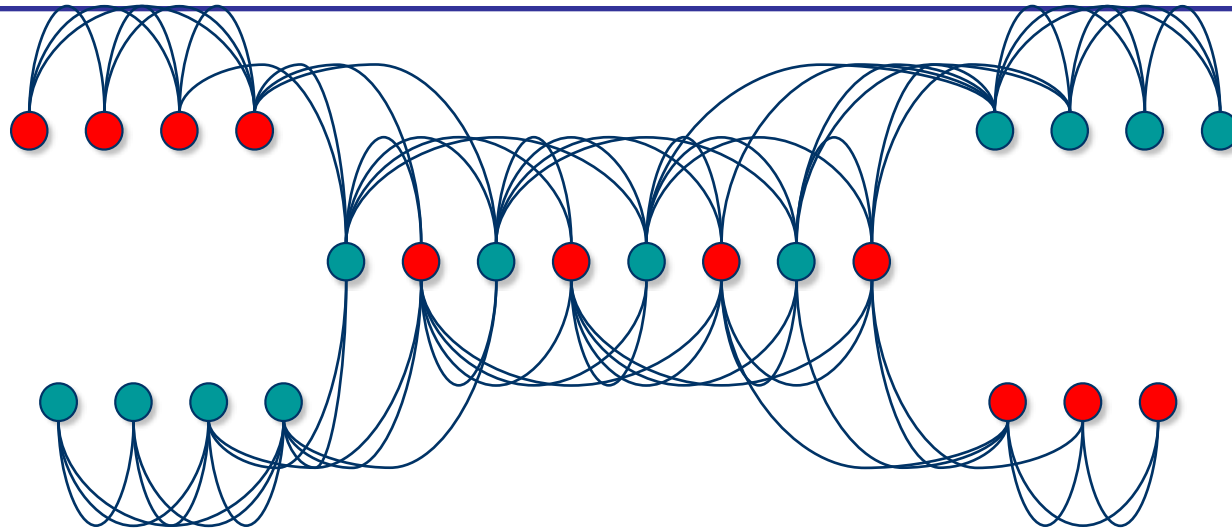
2. Merge Reads into Contigs



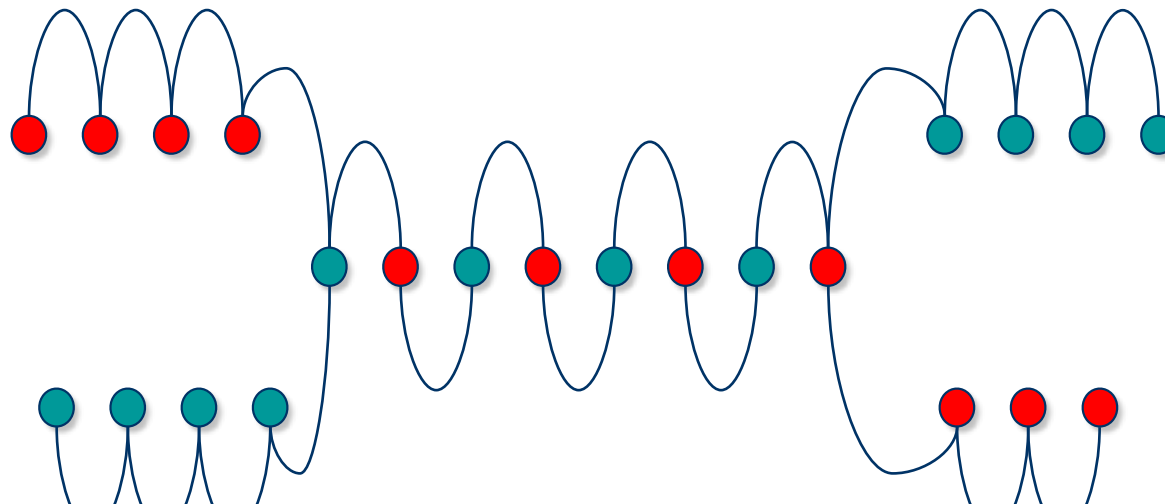
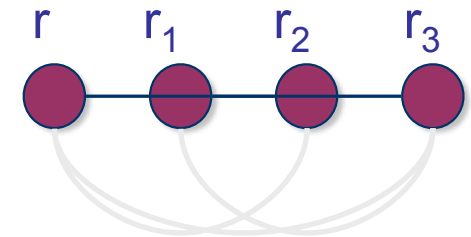
We want to merge reads up to potential repeat boundaries



2. Merge Reads into Contigs

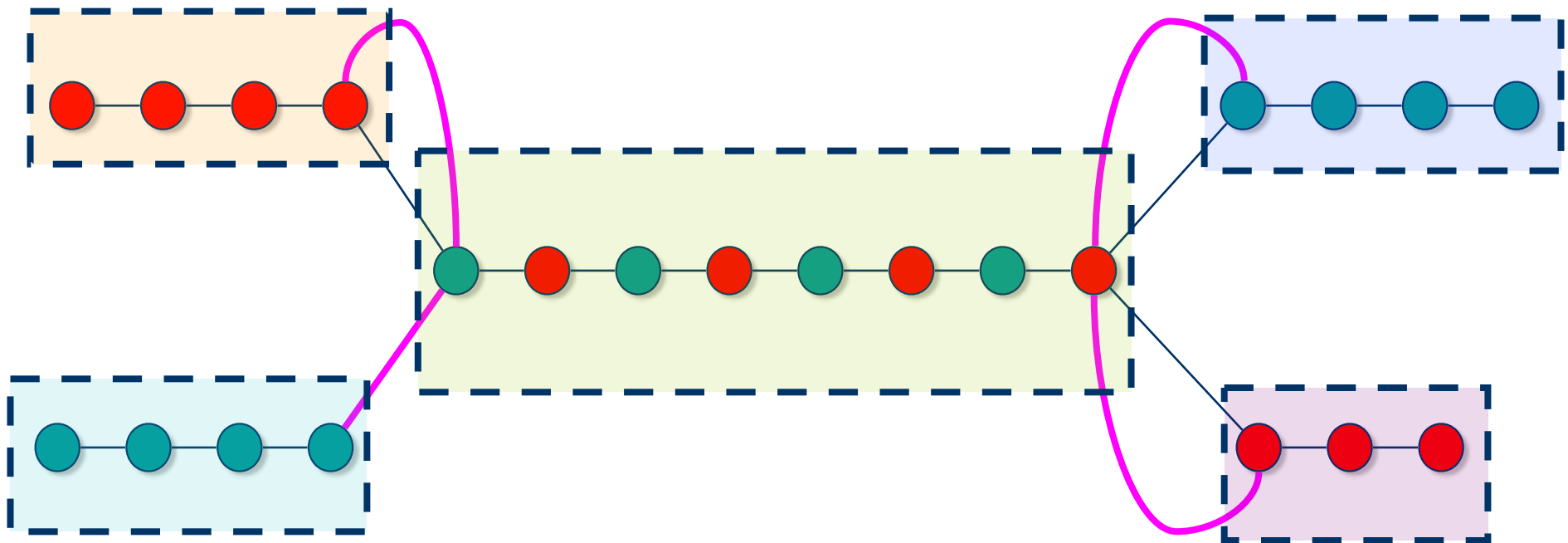


- Remove transitively inferable overlaps
 - If read r overlaps to the right reads r_1 , r_2 , and r_1 overlaps r_2 , then (r, r_2) can be inferred by (r, r_1) and (r_1, r_2)





2. Merge Reads into Contigs





Repeats, errors, and contig lengths

- Repeats shorter than read length are easily resolved
 - Read that spans across a repeat disambiguates order of flanking regions
- Repeats with more base pair diffs than sequencing error rate are OK
 - We throw overlaps between two reads in different copies of the repeat
- To make the genome **appear** less repetitive, try to:
 - Increase read length
 - Decrease sequencing error rate

Role of error correction:

Discards up to 98% of single-letter sequencing errors
decreases error rate
⇒ decreases effective repeat content
⇒ increases contig length



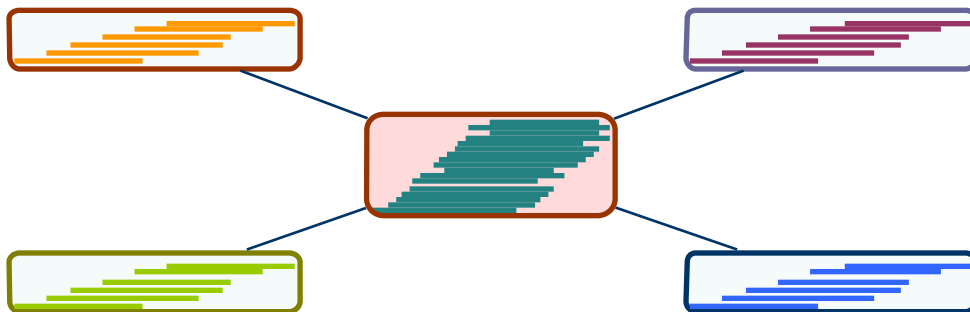
3. Link Contigs into Supercontigs



Normal density



Too dense
⇒ Overcollapsed



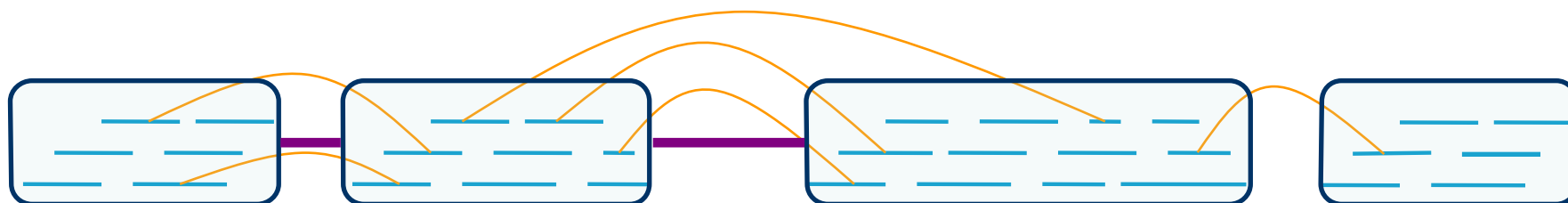
Inconsistent links
⇒ Overcollapsed?



3. Link Contigs into Supercontigs

Find all links between unique contigs

Connect contigs incrementally, if ≥ 2 forward-reverse links



supercontig
(aka scaffold)

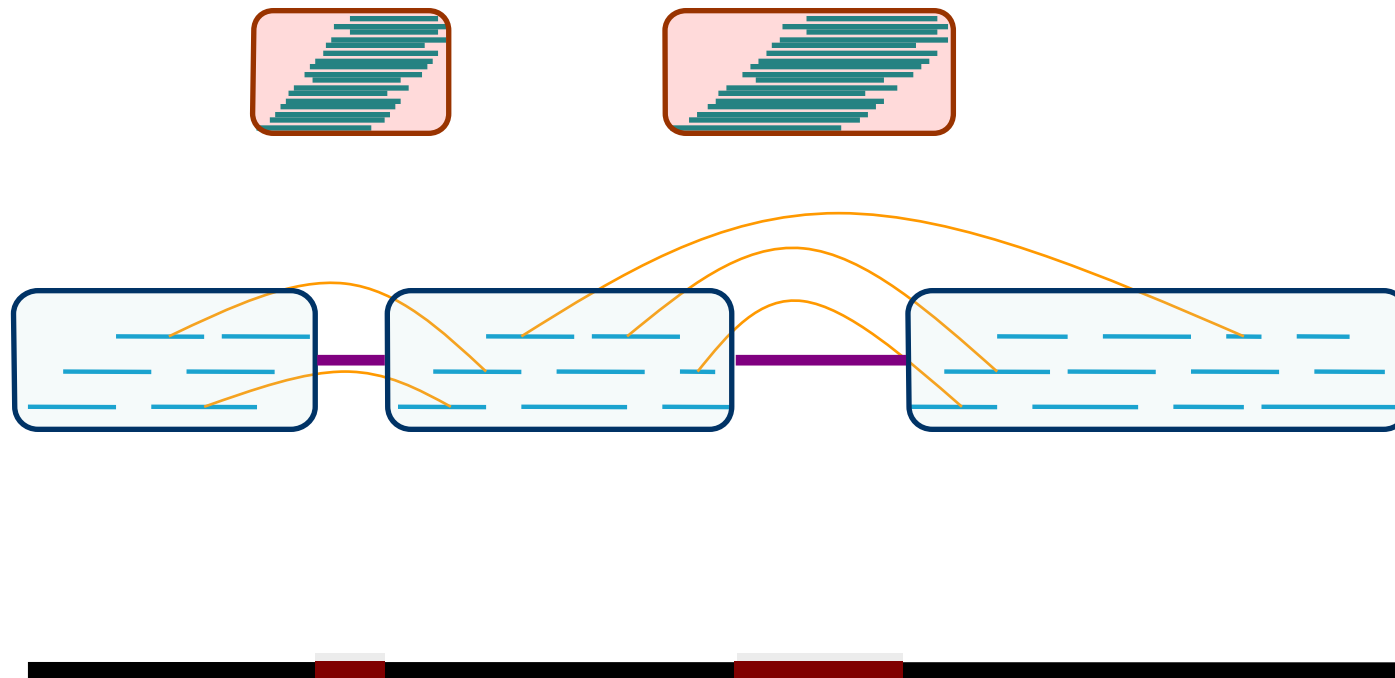


3. Link Contigs into Supercontigs

Fill gaps in supercontigs with paths of repeat contigs

Complex algorithmic step

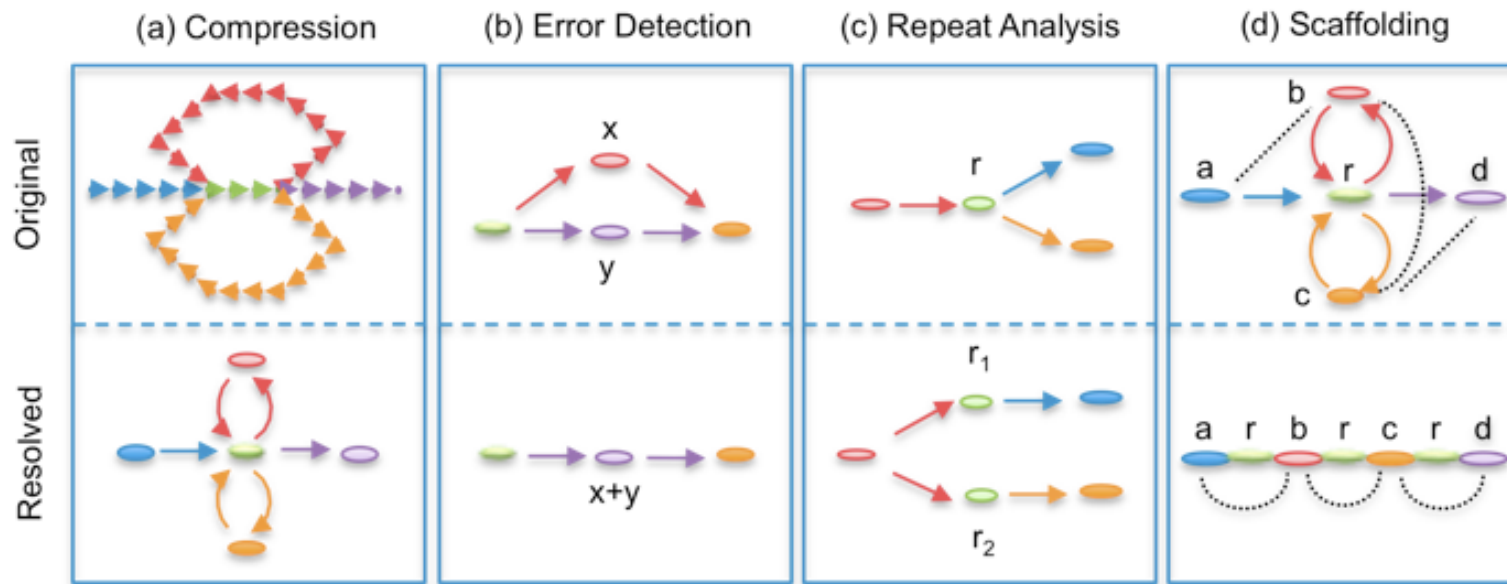
- Exponential number of paths
- Forward-reverse links





De Bruijn Graph formulation

- Given sequence $x_1 \dots x_N$, k-mer length k ,
Graph of 4^k vertices,
Edges between words with $(k-1)$ -long overlap





4. Derive Consensus Sequence



Derive **multiple alignment** from pairwise read alignments

Derive each consensus base by weighted voting

(Alternative: take maximum-quality letter)