A Six Degree-of-Freedom God-Object Method for Haptic Display of Rigid Bodies with Surface Properties

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Abstract—This paper describes a generalization of the god-object method for haptic interaction between rigid bodies. Our approach separates the computation of the motion of the six degree-of-freedom god-object from the computation of the force applied to the user. The motion of the god-object is computed using continuous collision detection and constraint-based quasi-statics, which enables high-quality haptic interaction between contacting rigid bodies. The force applied to the user is computed using a novel constraint-based quasi-static approach, which allows us to suppress force artifacts typically found in previous methods. The constraint-based force applied to the user, which handles any number of simultaneous contact points, is computed within a few microseconds, while the update of the configuration of the rigid god-object is performed within a few milliseconds for rigid bodies containing up to tens of thousands of triangles. Our approach has been successfully tested on complex benchmarks. Our results show that the separation into asynchronous processes allows us to satisfy the different update rates required by the haptic and visual displays. Force shading and textures can be added and enlarge the range of haptic perception of a virtual environment. This paper is an extension of [1].

Index Terms—Haptics, god-object, six degrees of freedom, rigid bodies, constraint-based quasi-statics.

1 INTRODUCTION

Haptic display of rigid bodies has the potential to improve the interaction between a human and a virtual environment by providing the user with the ability to touch and feel the geometric details of the virtual objects. Typical applications include CAD/CAM design, virtual prototyping, scientific visualization, and medical simulation.

Because of the high computational requirements of haptic rendering, however, finding effective methods is still a great challenge. A classical three degree-of-freedom method for haptic display of the interaction of a point and a virtual object was introduced by Zilles and Salisbury [2]. The two main benefits of their approach are 1) a nonpenetrating simulation of the motion of the point as it slides on the surface of the obstacles, and 2) a constraint-based computation of the force applied to the user, which results in a force orthogonal to the constraints. These features are highly desirable in that noninterpenetration of virtual objects is known to increase their perceived stiffness [3] and that an incorrect orientation of the force has been shown to perturb the perceived orientation of the virtual surfaces [4].

Although several six degree-of-freedom haptic rendering methods have been proposed (see Section 2), none seems to preserve all of the properties of the initial three degree-of-freedom approach introduced by Zilles and Salisbury [2]: These methods might allow the virtual objects to interpenetrate, or they use some form of virtual coupling [5] which can lead to disturbing force artifacts by modifying the orientation of the force applied to the user. In this paper, we propose what seems to be the first six degree-of-freedom constraint-based method that prevents both these visual and haptic artifacts. Especially, we make the following contributions:

- **Six degree-of-freedom god-object method**: We extend the three degree-of-freedom god-object method proposed by Zilles and Salisbury [2] to a six degree-of-freedom haptic interaction between rigid bodies.

- **High-quality god-object simulation**: Our god-object simulation method prevents any interpenetration between the virtual objects, while allowing the god-object to precisely contact and slide on the surface of the obstacles. This results in highly detailed haptic rendering of the objects geometries and increases the perceived stiffness of the virtual objects [3].

- **Constraint-based force computation**: We introduce a novel constraint-based quasi-static approach to compute the motion of the god-object and the force applied to the user. The constraint-based approach is physically-based, handles any number of simultaneous contact points, and yields constraint forces that are orthogonal to the constraints, thereby rendering correct surface orientations to the user. Furthermore, we show that our constraint-based quasi-static approach can only dissipate the energy transmitted to the god-object. This helps us improve the stability of the haptic display.
The paper is organized as follows: Section 2 provides a summary of related work. Section 3 gives an overview of our approach. Section 4 describes how we compute the motion of the god-object to ensure realistic haptic interaction with rigid bodies. Section 5 presents our novel constraint-based quasi-static approach to computing the force applied to the user. Section 6 discusses methods for producing haptic effects for surface perception such as force shading and textures. Section 7 demonstrates our approach on several benchmarks and shows how our approach is able to provide the user with a high-quality haptic display of contacting rigid bodies. We also discuss the benefits and limitations of our approach. Finally, Section 8 concludes and details several future research directions.

2 RELATED WORK

Haptic display of virtual objects has been an active area of research over the last decade. In 1995, Zilles and Salisbury [2] proposed what appears to be the first constraint-based method for three degree-of-freedom haptic rendering of generic polygonal objects. They introduced the god-object, an idealized representation of the position of the haptic device that is constrained to the surface of the obstacles. In their three degree-of-freedom approach, the location of the god-object minimizes at each time step the distance to the haptic device; the difference between the two positions provides the force direction. Ruspini et al. [6] extend this approach by replacing the god-object by a small sphere and propose methods to smooth the object surface and add friction. Niemayer and Mitra [7] propose dynamic proxies to better simulate dynamic effects. Several authors have proposed to extend the virtual proxy approach to a three degree-of-freedom interaction with objects defined by implicit representations [8], [9].

Some authors have proposed six degree-of-freedom haptic display algorithms. McNeely et al. [10] propose a voxel sampling method. Johnson et al. [11] use local minimum distances to compute the force applied to the user. Gregory et al. [12] extend the virtual proxy approach to six degrees of freedom and estimate the local penetration depth to compute the force and torque applied to the user. These methods, like most six degree-of-freedom haptic display methods [13], [14], [15], [16], [17], [18], do not attempt to prevent the interpenetration between the virtual objects, which might lead to missing some collisions between the virtual objects and can lead to the well-known pop-through effect, where the virtual proxy can traverse thin objects or objects parts [6], thereby degrading the perception of geometric details. Berkelman et al. [19] have proposed a general constraint-based method for a six degree-of-freedom interaction with rigid bodies. However, their approach includes a virtual coupling [5] which leads to perceptible force artifacts (see discussion in Section 7). Recent work on stable six degree-of-freedom interactions by Otaduy and Lin [20], however, has shown that the force artifacts created by a virtual coupling can be reduced through the use of an implicit integration method.

To the best of our knowledge, the approach described in this paper seems to be the first six degree-of-freedom constraint-based haptic rendering method that does not suffer from the visual or haptic artifacts of previous approaches (i.e., interpenetrations, forces felt at a distance, or artificial friction and sticking).

3 OVERVIEW

Our method extends the classical three degree-of-freedom constraint-based method by Zilles and Salisbury [2] by employing a six degree-of-freedom god-object, i.e., an idealized representation of the haptic device that is constrained to remain on the surface of the environment obstacles when the haptic device penetrates the environment obstacles (see Fig. 2). At each time step, we attempt to reduce the discrepancy between two rigid reference frames: one attached to the haptic device, and one attached to the virtual object. We typically place the origin at the center of gravity of the virtual object, although any point can be chosen. Only the god-object is displayed (and not the actual configuration of the haptic device), so that even when the haptic device penetrates the environment obstacles, the user...
4.2 Constraint-Based God-Object Quasi-Statics

Let \( \mathbf{a} = (\mathbf{a}_c, \mathbf{\alpha})^T \) denote the generalized (six-dimensional) acceleration of the god-object, where \( \mathbf{a}_c \) and \( \mathbf{\alpha} \) are, respectively, the linear acceleration and the angular acceleration of the god-object. The set of possible accelerations is easily determined from the contact positions and normals provided by the continuous collision detection algorithms. Let \( I_k \) and \( \mathbf{n}_k \), respectively, denote the position and normal of the \( k \)th contact point, \( 1 \leq k \leq m \). Assuming the normal \( \mathbf{n}_k \) is directed toward the exterior of the environment obstacle, the acceleration of the god-object must satisfy the following nonpenetration constraint [22]:

\[
\mathbf{a}_c^T \mathbf{n}_k + \mathbf{\alpha}^T (\mathbf{G} I_k \times \mathbf{n}_k) \geq 0,
\]

where \( \mathbf{G} \) is the vector from the center of inertia \( G \) of the god-object to the contact point \( I_k \). Note the absence of a velocity-dependent term in the nonpenetration constraint, as the quasi-static assumption implies that the velocity of the god-object is zero at all times. These \( m \) nonpenetration constraints can be concatenated to form a single constraint.
on the generalized acceleration of the god-object: $\mathbf{J}a \geq 0$, where $\mathbf{J}$ is a $m \times 6$ Jacobian.

Gauss’ principle states that the constrained generalized acceleration $\dot{\mathbf{a}} = (\mathbf{a}_G^c, \dot{\mathbf{a}}^c)^T$ of the god-object minimizes the following function [23]:

$$G(\dot{\mathbf{a}}) = \frac{1}{2}(\dot{\mathbf{a}} - \dot{\mathbf{a}}^c)^T \mathbf{M}(\dot{\mathbf{a}} - \dot{\mathbf{a}}^c) = \frac{1}{2}||\dot{\mathbf{a}} - \dot{\mathbf{a}}^c||^2_{\mathbf{M}}, \quad (1)$$

that is, the kinetic distance $||\dot{\mathbf{a}} - \dot{\mathbf{a}}^c||_{\mathbf{M}}$ between the constrained acceleration $\dot{\mathbf{a}}^c$ and the unconstrained acceleration $\dot{\mathbf{a}}^u$ over the set of possible accelerations $\{\dot{\mathbf{a}}: \mathbf{J}a \geq 0\}$. In other words, the constrained acceleration $\dot{\mathbf{a}}^c$ is the (non-euclidean) projection of the unconstrained acceleration $\dot{\mathbf{a}}^u$ onto the set of possible accelerations. This projection problem is solved using Wilhelmsen’s projection algorithm [24]. Note that the matrices $\mathbf{M}$ and $\mathbf{J}$ contain all the necessary and sufficient information to compute the constrained motion of the god-object.

5 Constraint-Based Force Computation

The constraint-based coupling loop determines the forces applied to the user based on the configuration of the haptic device and the contact information sent by the god-object simulation loop. Essentially, the constraint-based coupling loop performs the same constraint-based quasi-static computations as in the god-object simulation loop, but assuming the configuration of the god-object is fixed. This suppresses the need for collision detection in the constraint-based coupling loop, and allows us to compute the constraint-based force applied to the user within a few microseconds (see Section 7). Precisely, the constraint-based force applied to the user is computed according to the following constraint-based force computation algorithm:

1. Data retrieval: The configuration $\mathbf{x}_h$ of the haptic device and the configuration $\mathbf{x}_g$ of the god-object are read from the shared data, as well as the matrices $\mathbf{M}$ and $\mathbf{J}$, computed in the god-object simulation loop, which describe the local quasi-statics of the god-object.

2. Unconstrained acceleration computation: As in the god-object simulation loop, the unconstrained six-dimensional acceleration $\dot{\mathbf{a}}^u$ of the god-object is computed from $\mathbf{x}_h$ and the six-dimensional configuration $\mathbf{x}_g$ of the god-object ($\dot{\mathbf{a}}^u = \mathbf{k}_h(\mathbf{x}_h - \mathbf{x}_g)$).

3. Constraint-based force computation: The constrained acceleration $\dot{\mathbf{a}}^c$ of the god-object is computed from the unconstrained acceleration $\dot{\mathbf{a}}^u$ and the matrices $\mathbf{M}$ and $\mathbf{J}$ retrieved from the shared data by solving Gauss’ projection problem. The constraint-based force to be applied to the user is then $\mathbf{F}^c = \mathbf{k}_c \mathbf{M}(\dot{\mathbf{a}}^c - \dot{\mathbf{a}}^u)$, where $\mathbf{k}_c$ is a coupling constant.\(^1\)

4. Force transmission: The constraint-based force $\mathbf{F}^c$ is written to the shared data. It will be read by the haptic loop for application to the user.

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1. Different constants can be used for the translational and rotational parts, but this might lead to constraint forces that are not orthogonal to the nonpenetration constraints (see Section 7).

Fig. 4 demonstrates this algorithm in the case of a god-object in contact with an obstacle. For clarity, only two degrees of freedom are allowed: a vertical translation and a rotation whose axis is orthogonal to the plane of the figure. Fig. 4a shows the god-object contacting the obstacle (in blue) and four successive configurations of the haptic device (in green), as well as the resulting unconstrained accelerations $\dot{\mathbf{a}}_1^u, \ldots, \dot{\mathbf{a}}_4^u$. Fig. 4b shows the corresponding two-dimensional motion-space, i.e., the space of accelerations, and the linearized nonpenetration constraint resulting from the contact point (the diagonal line). The possible accelerations are above this diagonal line. Projecting the unconstrained accelerations $\dot{\mathbf{a}}_1^u, \ldots, \dot{\mathbf{a}}_4^u$ on the set of possible accelerations yields the constrained accelerations $\dot{\mathbf{a}}_1^c, \ldots, \dot{\mathbf{a}}_4^c$ as well as the corresponding constraint forces $\mathbf{F}_1^c, \ldots, \mathbf{F}_4^c$ applied to the user. Haptic configurations 1 and 2 result in a force and a torque which attempt to bring the haptic device back to a configuration reachable by the god-object, while haptic configurations 3 and 4, which correspond to accelerations satisfying the nonpenetration constraint, do not generate any force.

Note that, because the configuration $\mathbf{x}_g$ of the god-object is not updated in the constraint-based coupling loop, the matrices $\mathbf{M}$ and $\mathbf{J}$ do not have to be updated either.\(^2\) Hence, only the configuration of the haptic device changes, and the main computation involved is the determination of the constrained acceleration $\dot{\mathbf{a}}^c$, which can be performed very efficiently (see Section 7).

When a new set of constraints is available, some of the new nonpenetration constraints might not be satisfied by the current configuration of the haptic device (see Fig. 6a). This might create a large constraint force if the user has already penetrated those new constraints. In order to smooth the constraint-based force applied to the user and reduce potentially large forces created by delays in the
update of the set of constraints, we generalize the method introduced by Mark et al. [25]. Assume a new constraint \( J_k a \geq 0 \) on the acceleration \( a \) of the god-object occurs, where \( J_k \) is a six-dimensional row vector (a row of the Jacobian). Assume that this constraint is not satisfied at time 0, when the new set of constraints becomes available, i.e., that the configuration of the haptic device is such that \( J_k a^u = d_k < 0 \). We initially offset this constraint: the constraint becomes \( J_k a^u = f_k(t) \), where \( f_k \) is a monotonously increasing time-dependent function such that \( f_k(0) = d_k \) and \( f_k(\Delta t) = 0 \). This constraint is thus satisfied when the set of constraints progressively turns into the constraint that should be enforced (i.e., after a time \( \Delta t \), see Figs. 6b, 6c, and 6d). In order to provide the user with a slight force discontinuity and improve the perception of new constraints, however, we perform this interpolation only if \( d_k \leq \varepsilon \), where \( \varepsilon \) acts as a user-defined discontinuity threshold (\( \varepsilon < 0 \)). We leave the formal evaluation of the influence of \( \varepsilon \) on the haptic perception of new contacts and on the overall stability of the algorithm for future work.

The combination of the god-object simulation loop and the constraint-based coupling loop results in the perception of six degree-of-freedom constraint forces as the user manipulates the virtual object and slides on the virtual obstacles.

### 6 Haptic Surface Properties

The six degree-of-freedom constraint-based method proposed here provides a force orthogonal to the nonpenetration constraints. No force artifacts are felt by the user, such as artificial friction or sticking effect. The force vector direction can now be controlled and perturbed for providing haptic surface properties like force shading or texture. The two following sections demonstrate how such effects can be added by modifying either the constraints or the force applied to the user.

#### 6.1 Smooth Surfaces

Our current implementation uses a continuous collision detection method suitable for polygonal objects. As a result, smooth shaped objects approximated by polygonal meshes feel like polyhedral surfaces due to the discontinuity at the polygon edges. To avoid that, Morgenbesser and Srinivasan [26] have been the first to adapt the well-known Phong method [27] for smoothing polygonal meshes. They demonstrated that a similar haptic effect, called force shading, can give the illusion of a haptically smooth shape.

More recently, Ruspini et al. [6] also proposed to adapt the graphical methods using the virtual proxy approach. Compared to the Morgenbesser approach, their force shading method allows them to handle situations involving multiple intersections between the proxy and shaded surfaces at the same time.

Like the Ruspini et al. approach, the constraint-based method proposed in this paper allows us to adapt the Phong method [27]. At each point on a mesh polygon, a new vector is computed by interpolating the normals from the vertices of the polygon. This new normal is used to compute the illumination of the model at this point. Consequently, the edges of the polygonal mesh do not appear, and the shape appears to be smooth. The same idea is used for force shading.

The following sections explain the link between the force vector direction and the surface normal, followed by the description of the force shading algorithm. Finally, they show how force shading can be efficiently computed in our asynchronous algorithm.

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**Fig. 5. Haptic interaction with Stanford bunnies.** The user manipulates the green bunny. (a) The ear of the green bunny slides in a ridge of the blue bunny. (b) Continuous collision detection and constraint-based quasi-statics allows the manipulated object to precisely contact and slide on the obstacles. (c) and (d) Our method provides the user with the ability to precisely feel the contact between pairs of triangles, resulting in a highly detailed haptic display of contacting rigid bodies.

**Fig. 6. Constraints adaptation.** When a new constraint (here, a vertical plane) appears that would create too large a constraint force, it is first translated so that the constraint is satisfied by the current haptic device configuration, then progressively returned to its initial position. This helps us smooth the force felt by the user, while ensuring that small discontinuities signaling new contact points are felt.
As described in Sections 4 and 5, the computation of the force directly results from the computation of the constrained acceleration, which itself uses both the unconstrained acceleration and the contact information (or constraint space). The latter is mainly defined by the surface normal for each contact point between the god-object and the shape. Consequently, changing the surface normals in the contact information will change the direction of the force vector.

6.1.2 Basic Algorithm

Using contact positions, similarly to the Phong approach, the algorithm proceeds by first computing the interpolated contact normals at each position of the contact points. These vectors are used to create a new constraint space, called force shading constraint space. The rest of the algorithm consists of two computation passes (cf. Fig. 7), i.e., the computation of the new direction of the force vector and the computation of the new god-object configuration.

- **Force vector direction.** First, a force shading constrained acceleration is computed from the unconstrained acceleration and the force shading constraint space. Next, the computation of the force is done with this new acceleration and the original unconstrained acceleration. At this point, and without the next stage, the force rendered by the haptic device will give the illusion of a nonflat mesh polygon, but the edges are still felt. The next stage explains how to avoid that.

- **Constrained acceleration.** As seen in Fig. 8a, with the six degree-of-freedom god-object method, a discontinuity occurs when the user reaches an edge of the shape. Indeed, such an effect is provided by the computation of the constraint acceleration, which is always as close as possible to the unconstrained acceleration. Even with the computation of the perturbed force direction described in the stage before, this sudden change in the configuration of the god-object makes the user feel the edges of the polygonal mesh. To avoid that, the force shading constraint acceleration is used as an unconstrained acceleration and combined with the original constraint space to compute a final acceleration for the god-object (cf. Fig. 7). Fig. 8b shows the successive god-object configurations when such an approach is used.

6.1.3 Optimization with the Asynchronous Process

The computation described above is a time-consuming computation, because of the double constraint-based quasi-static computation. This can be optimized by exploiting the asynchronous aspect of the proposed algorithm and by implementing one step in each process (i.e., the simulation and the coupling loops).

The force shading constraint space is created by the simulation loop and written to the shared data. In parallel, the coupling loop uses the last force-shaded constraint space retrieved and computes the force shading constrained acceleration which is also written in the shared data. Consequently, instead of the unconstrained acceleration, the force shading constrained acceleration is computed by the simulation loop using the original constraint space to create the new constraint acceleration of the god-object.

6.2 Textures

Except for some recent methods for six degree-of-freedom haptic texture rendering [28], [29], which are not usable...
allowing the user to feel the bumps and holes defined by this function. The update rate of the simulation loop.

This can be easily explained by the fact that 1) new contact points rarely occur exactly simultaneously and 2) compared to other approaches using the interpenetration between multiple and potentially redundant contact points occurring during the task [12].

A similar effect can be produced by perturbing the force computed by the six degree-of-freedom constraint-based god-object approach, using a discrete or continuous function at the contact point position. For example, a sine function along one axis could be sufficient for providing bumps and holes along this axis (cf. Fig. 9). In the case of multiple contact points, the perturbation vector used to modify the force vector direction is defined by averaging the perturbation vector at each contact point.

This method provides high-frequency textures and can be mixed with the force shading effect described above. However, similar to the Minsky approach, if the speed of the god-object is too high, or the update rate of the simulation loop is too low, the contact point positions can pass from a hole directly to another one without feeling the bump in between. This implies a limitation in the texture frequency according to the exploration speed and the update rate of the simulation loop.

7 Results and Discussion

The validation of our approach is performed on a Stringed Haptic Workbench in which the SPIDAR-G, a tension-based six degree-of-freedom force-feedback device [35], allows a user to interact intuitively on a large two-screen display [36]. The entire algorithm is executed on a 3.2 GHz dual-processor Xeon PC, to which the haptic device is connected. This PC communicates with a cluster of PCs only dedicated to the stereo display on both screens of the Stringed Haptic Workbench. The communication between the Xeon PC and the cluster of PCs is ensured by UDP protocols.

Each of the three main loops is launched in its separate thread. The haptic device thread frequency is fixed by the device: The constraint-based force computed by the constraint-based coupling loop is read from the shared data and applied to the user at 1,000 Hz. The frequencies of the constraint-based coupling thread and the god-object simulation thread vary over time, depending on the complexity of the models and the task being performed (see below).

7.1 Peg-in-a-Hole Benchmark

We first evaluate the quality and the stability of the haptic interaction in a simple but classical case: the peg-in-a-hole benchmark (see Fig. 10). This benchmark is well-known because, although it involves only very simple geometry (here, 288 triangles for the peg and 280 triangles for the box), it has typically been a challenge to provide a stable and realistic haptic display of the insertion of the peg due to the multiple and potentially redundant contact points occurring during the task [12].

Fig. 11 reports several timings and statistics measured during a typical interaction. The first row reports several key configurations tested during the interaction, including

a. sliding the tip of the peg on the top side of the box,
b. laying the peg on the top side of the box and sliding it on the box,
c. pushing on the left side of the box,
d. exploring the right extremity of the hole, and
e. inserting the peg in the hole.

The second row reports the time required to compute the constraint-based force (see Section 5) during the interaction. It can be seen that the constraint-based force is computed in less than 25 microseconds throughout the manipulation. The third row shows that the time required to update the configuration of the god-object is always smaller than 10 milliseconds, which is sufficient to prevent any visual lag throughout the manipulation. The fourth row reports the number of simultaneous contact points during the interaction, which can be seen to be fairly limited throughout the manipulation. This can be easily explained by the fact that 1) new contact points rarely occur exactly simultaneously and 2) compared to other approaches using the interpenetration between virtual objects, constraint-based quasi-static computations tend to limit the apparition of new contact points, since at
most 12 of them can be independent (each constraint removes half a degree of freedom). This greatly contributes to the efficiency of the constraint-based coupling loop. Finally, the fifth and sixth rows report the Y and Z components of the constraint-based force applied to the user during the interaction. As expected, the Y component is nonzero only when the user pushes the peg on the left side of the box (step c) or explores the right extremity of the hole (step d) and remains equal to zero whenever the peg is sliding on the top side of the box or inside the hole. The Z component has high values when the user pushes the peg on the top side of the box, and has little variations when the peg is inside the hole, due to user movement precision. In other words, the user does not feel any artificial friction force or any artificial sticking during the manipulation (e.g., the Y component of the force is never positive during step c).

Overall, the combination of continuous collision detection, constraint-based quasi-statics, and constraint-based force computation makes it very easy for the user to accomplish the task by allowing the peg to slide on the surface of the box and the hole, while providing the user with a high-quality haptic display.
7.2 Stanford Bunnies Benchmark

The second benchmark involves two Stanford bunnies (27,000 triangles per bunny, see Fig. 1). One bunny is static, and the second bunny is manipulated by the user. Fig. 5 shows several key steps of the interaction: Fig. 5a shows the ear of the mobile bunny sliding in a ridge of the static bunny; Fig. 5b demonstrates how the constraint-based god-object simulation provides realistic contacting configurations during the interaction; similarly, Figs. 5c and 5d show how our approach is able to provide the user with high-quality haptic display of contacting rigid bodies, where the details of the geometry can be felt by the user.

Fig. 12 reports on the performance of our approach during a typical interaction session with the bunnies, which includes the configurations represented in Fig. 5. Again, the force applied to the user is computed within a few microseconds, while an update of the configuration of the mobile bunny, which includes continuous collision detection and constraint-based quasi-statics, is performed within a few milliseconds, resulting in the absence of any visual lag during the interaction.

7.3 Discussion

7.3.1 Benefits

The main benefits of our approach stem from the combination of three key elements:

- **Continuous collision detection** allows the user to feel the details of the geometry of the rigid bodies and potentially feel the contact between vertices, edges, and faces of the contacting objects. Furthermore, the ability to produce visually convincing nonpenetrating but tangent contacting configurations (e.g., Fig. 5b) helps us improve the perceived stiffness of the objects [3].

- **Asynchronous updates** of the configuration of the god-object and the force applied to the user help us satisfy the different update rates required by the haptic and the visual displays.

- **Constraint-based quasi-statics** allow the user to slide on the environment obstacles and haptically feel the reduced motion subspace resulting from the simultaneous nonpenetration constraints, thus providing the user with a realistic haptic display of surfaces, corners, ridges, and object/object contact in general.

Especially, the physically-based computation of the force applied to the user guarantees that no artificial friction or sticking is felt, and that no force is applied when the god-object is in free space. This is to be contrasted to what would occur if some kind of virtual coupling was involved in the computation of the force applied to the user. Fig. 13 shows such a comparison in which the god-object (in blue) is constrained to remain above the surface of the obstacle. In the case depicted in Fig. 13a, where the haptic device (in green) has penetrated the environment, a virtual coupling would attempt to bring the haptic device back to the configuration of the god-object, which would result in an artificial tangential friction applied to the user. As mentioned before, this would degrade the perceived orientation of the surface of the obstacle [4]. In contrast, the constraint-based approach guarantees that the perceived orientation is correct, since the contact forces are always orthogonal to the constraints. Furthermore, in the case depicted in Fig. 13b, where the user moves away from the obstacle, a virtual

\[ F^c = M(a^c - a^u) \]

\[ a^c = a^u \]

\[ F^c = 0 \]

\[ X_s, F^c = k(x_s - x_h) \]
coupling would attempt to bring the god-object back to the surface of the obstacle, which would result in a sticky feeling. In this case, however, the constraint-based approach yields the correct force \( F^v = 0 \), since moving away from the obstacle surface satisfies the nonpenetration constraint (hence, \( a^v = a^u \)).

Finally, although a complete stability analysis is outside the scope of this paper and is left as future work, we believe that the asynchronous constraint-based approach helps us improve the stability of the interaction. Indeed, it can be shown that the simulation of the god-object is purely dissipative, i.e., that the force \( F^v = Ma^v \) applied to the god-object is such that

\[
(F^v)^T a^u \leq (F^v)^T a^u.
\]

Thus, the nonpenetration constraints can only dissipate the energy transmitted to the god-object. Our tests have shown that the user is able to, e.g., release the handle of the haptic device while the peg is inside the hole (cf. Fig. 11, step e).

7.3.2 Limitations

Our approach has two main limitations:

- **Linearized constraints**: In order to efficiently compute the quasi-statics of the god-object and the constraint-based force applied to the user, the nonpenetration constraints are linearized. This might reduce the quality of the force applied to the user when a large discrepancy between the configurations of the god-object and the haptic device occurs. It would be interesting to investigate some more sophisticated force computation methods to address this problem, involving, for example, an explicit formulation of the nonpenetration constraints.

- **Potentially low update rate of the set of constraints**: We do not guarantee that our approach is able to update the set of nonpenetration constraints at 1,000 Hz. This might lead to missing some high-frequency details when the user slides rapidly on the surface of the environment obstacles.

The potentially low update rate of the set of constraints is the main reason for the separation of the god-object simulation and the constraint-based force computation into asynchronous processes, in our approach and several previous ones (e.g., [16], [25]). Because the complexity of any collision detection method which reports all the contacting features is output-dependent, however, it seems arguable that, whichever collision detection method is used, it will always be possible to find a scenario such that the time required to determine all the contact points will take more than one millisecond. We have thus preferred to rely on a god-object simulation method which offers precise interaction with rigid bodies and, especially, precisely contacting configurations. Although this might limit the rate at which the set of nonpenetration constraints is updated (sometimes as low as 70 Hz in the Stanford bunnies benchmark, and about 300 Hz on average), this approach allows us to compute a constraint-based force consistent with the current set of simultaneous constraints at extremely high rates (always higher than 80,000 Hz in the Stanford bunnies benchmark). Furthermore, it should be emphasized that the constraint-based computations performed in the constraint-based coupling loop implicitly include some collision detection. Returning to the example depicted in Fig. 4, it can be seen that, if between two updates of the set of nonpenetration constraints, the haptic device switches from a state where all currently known nonpenetrating constraints are satisfied (in which case \( F^v = 0 \)) to one where at least one of the currently nonpenetrating constraint is not satisfied (in which case \( F^v \neq 0 \)), the user will feel this collision. In summary, collision detection is implicitly performed in the constraint-based coupling loop, for the current set of nonpenetration constraints, at extremely high rates.

8 Conclusion and Future Work

This paper has introduced a new method for the six degree-of-freedom haptic display of rigid bodies, which generalizes the classical three degree-of-freedom god-object method introduced by Zilles and Salisbury [2]. As in their initial approach, the god-object is able to contact and slide on the environment obstacles without penetrating them, and the forces applied to the user are orthogonal to the nonpenetration constraints. Our approach has been successfully tested on the classically difficult peg-in-a-hole benchmark and on some more complex models—two Stanford bunnies with 27,000 triangles each. We have shown that our method is able to provide a high-quality haptic display of contacting rigid bodies in both cases. We also demonstrate that the approach is compatible with the generation of textures and force shading. In this case, our constraint-based approach ensures that no force artifacts are felt by the user.

There are several directions for future work. Besides addressing the limitations described above, we would like to extend our approach to multiple dynamic objects (although it can be argued that quasi-static interaction is preferable for the simulation of many tasks, as few manipulation tasks seem to require using the inertia of the manipulated object to accomplish the task). One possible direction to do this could be to generalize the approach suggested by Niemayer and Mitra [7] to six degree-of-freedom haptic interaction. Finally, we plan to investigate actual industrial scenarios such as virtual prototyping and assembly tasks.

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References


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