Human Preference Models: Choice models (Continue)

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All Sections

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The ideal point model

- An embedding approach, assumes user item preference depends on distance
  - Let x_n denote a latent vector representing an individual n
  - Let v_i denote a latent vector representing choice (or item) i
  $U_{ni} = dist(x_n, v_i) + \epsilon_{ni}$
  - Model is equivalent to choosing the "closest" item

$$y_{ni} = \begin{cases} 1, & \text{if } U_{ni} > U_{nj} \, \forall j \neq i \\ 0, & \text{otherwise} \end{cases}$$

Ideal point model: the why

- Pros: Can sometimes learn preferences faster than attribute-based preference models by exploiting geometry (see refs)
- Cons:
  - Embedding assumption may be strong (can make more flexible via distance function choice)
  - However, have to select a distance function (usually use Euclidian distance in the embedding)

Jamieson, Kevin G., and Robert Nowak. "Active ranking using pairwise comparisons."
Tatli, Gokcan, Rob Nowak, and Ramya Korlakai Vinayak. "Learning Preference Distributions From Distance Measurements."

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Choice models in RL (and RLHF)

Collect

- Collect some choice/preference data

Reward

- Train a utility/reward model

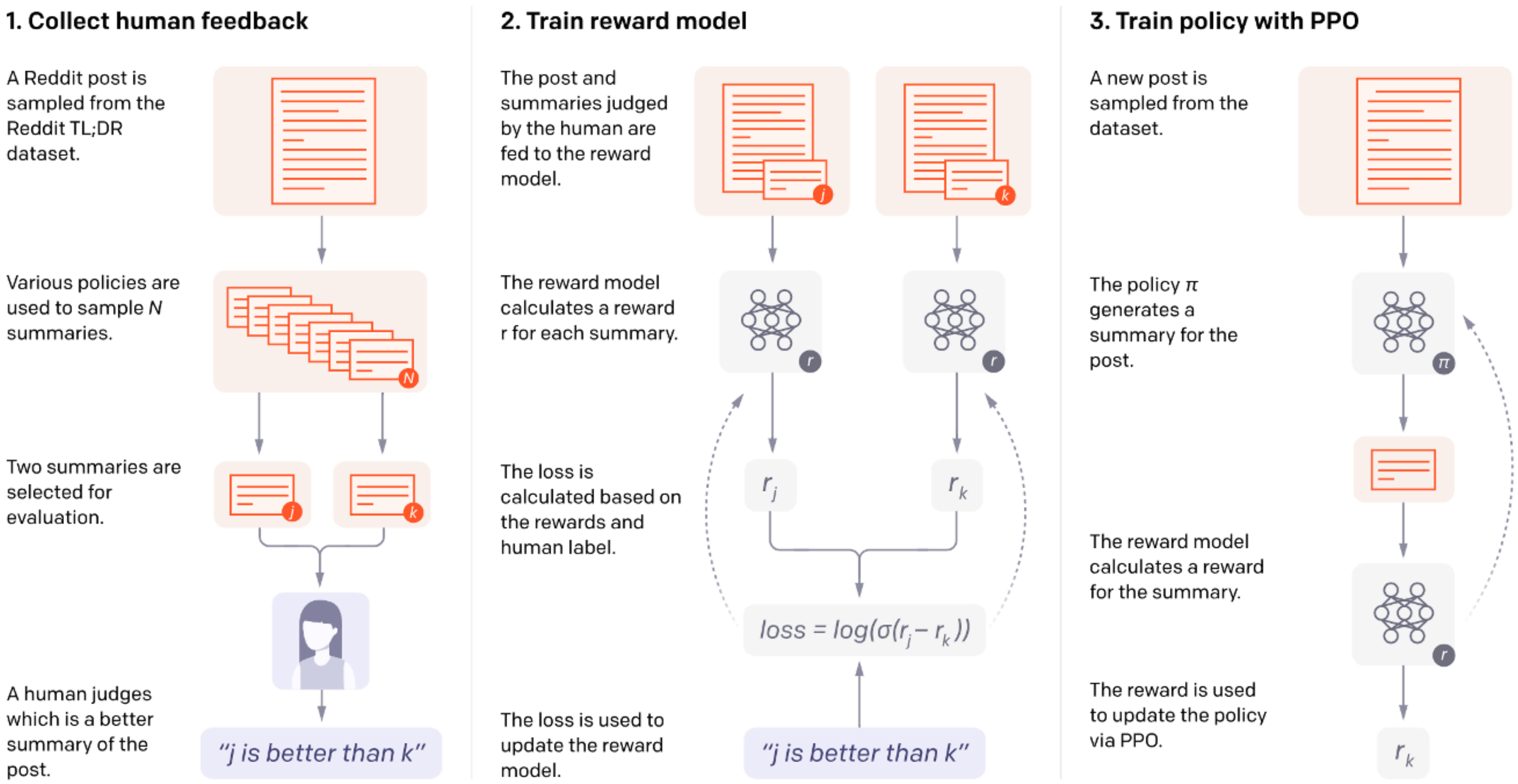
(optional) repeat

ML/RL

- Train a model to maximize this utility

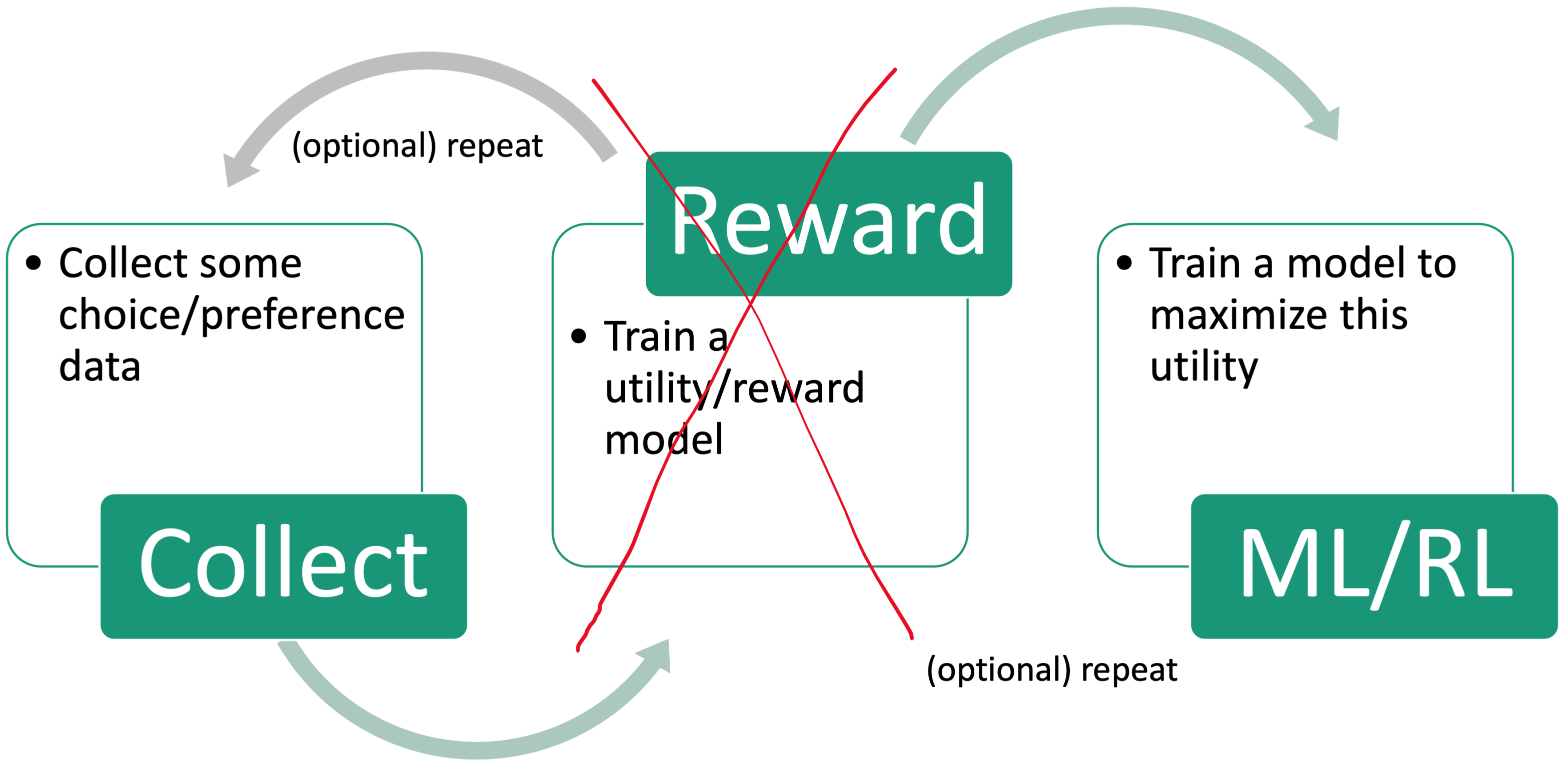
(optional) repeat

Application: RL and Language (Bradley-Terry model)

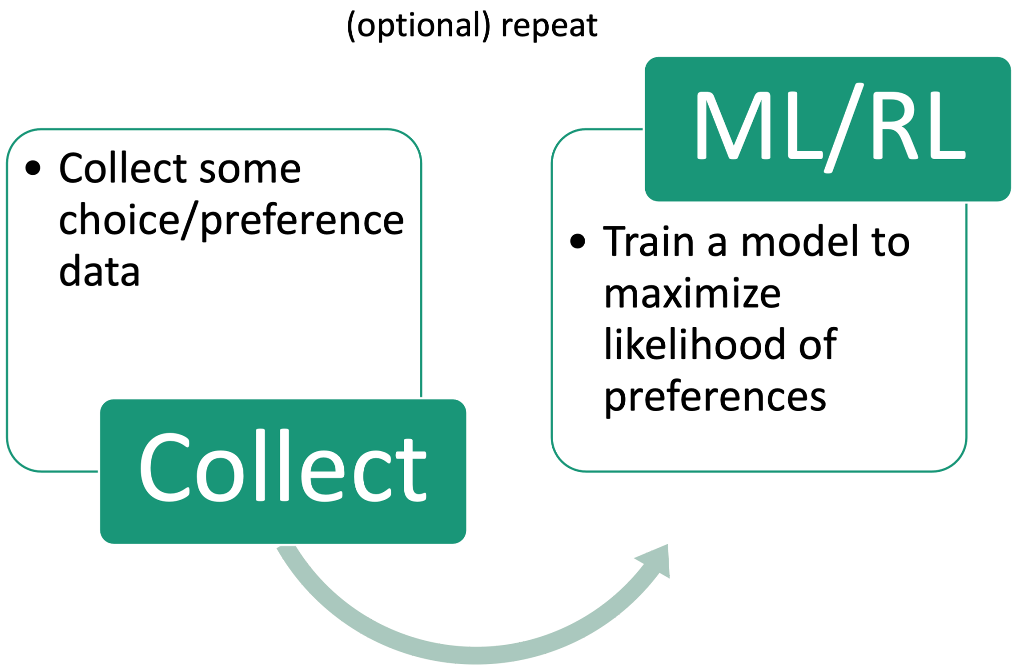


https://openai.com/research/learning-to-summarize-with-human-feedback

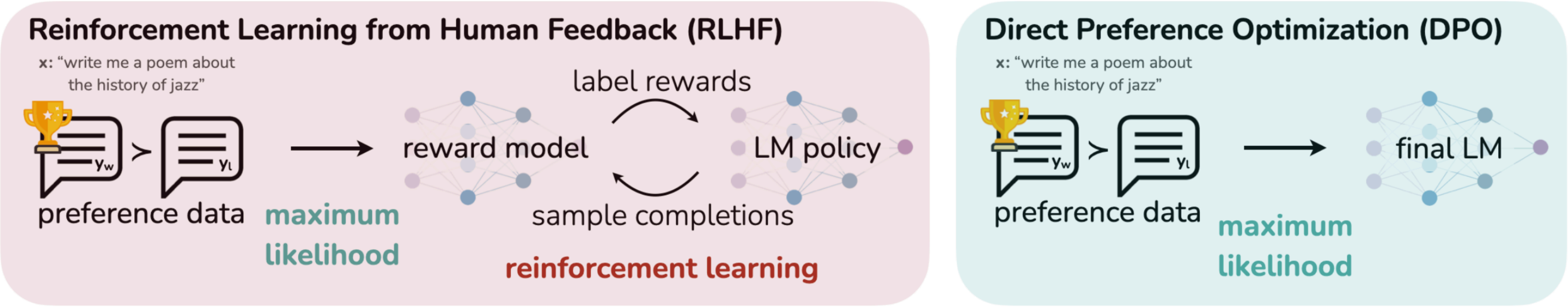
Choice models in ML (recommender systems, bandits, Direct Preference Optimization)



Choice models in ML (recommender systems, bandits, DPO)



Why DPO?



- RLHF pipeline is complex and unstable due to the reward model optimization.
- DPO is more stable and can be used to optimize the reward model directly.

Rafael Rafaelov, Archit Sharma, Eric Mitchell, Stefano Ermon, Christopher D. Manning, and Chelsea Finn, "Direct preference optimization: Your language model is secretly a reward model."

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DPO: Bradley-Terry model

- Given prompt x and completions y_w and y_l the choice model gives the preference

$$p^*(y_w > y_l|x) = \frac{\exp(r^*(x, y_w))}{\exp(r^*(x, y_w)) + \exp(r^*(x, y_l))}$$

where $r^*(x, y)$ is some latent reward model that we do not have access to (i.e., the human preference)

DPO: Bradley-Terry model

Luckily, we can use parameterize the reward model with some neural networks with parameters ϕ:

Let us start with the Reward Maximization Objective in RL:

$$\max_{\pi_\theta} \mathbb{E}_{x\sim\mathcal{D}, y\sim\pi_\theta(y|x)}[r_\phi(x,y) - \beta D_{KL}(\pi_\theta(y|x)||\pi_{\text{ref}}(y|x))]$$

- Where $\pi_\theta(y|x)$ is the language model, and $\pi_{\text{ref}}(y|x)$ is the reference model (e.g., the language model before fine-tuning)

$$\max_{\pi_\theta} \mathbb{E}_{x\sim\mathcal{D}, y\sim\pi_\theta(y|x)}[r_\phi(x,y) - \beta D_{KL}(\pi_\theta(y|x)||\pi_{\text{ref}}(y|x))]$$

Recall the definition of KL divergence:

$$D_{KL}(p||q) = \sum_{x\in\mathcal{X}} p(x)\log\frac{p(x)}{q(x)} = \mathbb{E}_{x\sim\mathcal{X}}\left[\log\frac{p(x)}{q(x)}\right]$$

Then we can rewrite the objective as:

$$\max_{\pi_\theta} \mathbb{E}_{x\sim\mathcal{D}, y\sim\pi_\theta(y|x)}\left[r_\phi(x,y) - \beta\mathbb{E}_{y\sim\pi_\theta(y|x)}\left[\log\frac{\pi_\theta(y|x)}{\pi_{\text{ref}}(y|x)}\right]\right]$$

$$= \max_{\pi_\theta} \mathbb{E}_{x\sim\mathcal{D}}\mathbb{E}_{y\sim\pi_\theta(y|x)}\left[r_\phi(x,y) - \beta\log\frac{\pi_\theta(y|x)}{\pi_{\text{ref}}(y|x)}\right]$$

Then, we can continue to derive the objective as:

$$\max_{\pi_\theta} \mathbb{E}_{x\sim\mathcal{D}}\mathbb{E}_{y\sim\pi_\theta(y|x)}\left[r_\phi(x,y) - \beta \log \frac{\pi_\theta(y|x)}{\pi_{\text{ref}}(y|x)}\right]$$

$$\propto \min_{\pi_\theta} \mathbb{E}_{x\sim\mathcal{D}}\mathbb{E}_{y\sim\pi_\theta(y|x)}\left[\log \frac{\pi_\theta(y|x)}{\pi_{\text{ref}}(y|x)} - \frac{1}{\beta} r_\phi(x,y)\right] // \text{ reverse and divide } \beta$$

$$= \min_{\pi_\theta} \mathbb{E}_{x\sim\mathcal{D}}\mathbb{E}_{y\sim\pi_\theta(y|x)}\left[\log \frac{\pi_\theta(y|x)}{\frac{1}{Z(x)}\pi_{\text{ref}}(y|x)\exp\left(\frac{1}{\beta} r_\phi(x,y)\right)} - \log Z(x)\right]$$

$$\text{with} \quad Z(x) = \sum_y \pi_{\text{ref}}(y|x)\exp\left(\frac{1}{\beta} r_\phi(x,y)\right)$$

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Because $Z(x)$ is a constant with respect to π_θ, we can define:

$$\pi^*(y|x) = \frac{1}{Z(x)}\pi_{\text{ref}}(y|x)\exp\left(\frac{1}{\beta}r_\phi(x,y)\right)$$

Then, we can rewrite the optimization problem as:

$$\min_{\pi_\theta} \mathbb{E}_{x\sim\mathcal{D}}\mathbb{E}_{y\sim\pi_\theta(y|x)}\left[\log\frac{\pi_\theta(y|x)}{\pi^*(y|x)} - \log Z(x)\right]$$

$$= \min_{\pi_\theta} \mathbb{E}_{x\sim\mathcal{D}}\mathbb{E}_{y\sim\pi_\theta(y|x)}\left[\mathbb{D}_{KL}(\pi_\theta(y|x)\|\pi^*(y|x)) - \log Z(x)\right]$$

Thus, the optimal solution (i.e., the optimal language model) is:

$$\pi_\theta(y|x) = \pi^*(y|x) = \frac{1}{Z(x)}\pi_{\text{ref}}(y|x)\exp\left(\frac{1}{\beta}r_\phi(x,y)\right)$$

With some algebra, we can show that the optimal reward model is:

$$\pi_\theta(y|x) = \frac{1}{Z(x)}\pi_{\text{ref}}(y|x)\exp\left(\frac{1}{\beta}r_\phi(x,y)\right)$$

$$\log \pi_\theta(y|x) = \log \pi_{\text{ref}}(y|x) + \frac{1}{\beta}r_\phi(x,y) - \log Z(x) // \text{ perform } \log(.)$$

$$r_\phi(x,y) = \beta \log \frac{\pi_\theta(y|x)}{\pi_{\text{ref}}(y|x)} + \beta \log Z(x)$$

Recall the Bradley-Terry model with parameterized reward model:

$$p_\phi(y_w > y_l | x) = \frac{\exp(r_\phi(x, y_w))}{\exp(r_\phi(x, y_w)) + \exp(r_\phi(x, y_l))}$$

We also have the optimal reward model:

$$r_\phi(x, y) = \beta \log \frac{\pi_\theta(y|x)}{\pi_{\text{ref}}(y|x)} + \beta \log Z(x)$$

Thus, we can rewrite the choice model as:

$$\begin{aligned} p_\phi(y_w \succ y_l | x) &= \frac{1}{1 + \exp\left(\beta \log \frac{\pi_\theta(y_l|x)}{\pi_{\text{ref}}(y_l|x)} - \beta \log \frac{\pi_\theta(y_w|x)}{\pi_{\text{ref}}(y_w|x)}\right)} \\ &= \sigma\left(\beta \log \frac{\pi_\theta(y_w|x)}{\pi_{\text{ref}}(y_w|x)} - \beta \log \frac{\pi_\theta(y_l|x)}{\pi_{\text{ref}}(y_l|x)}\right) \end{aligned}$$

DPO: Bradley-Terry model

Recall our objective to maximize the reward model, we can rewrite the objective as maximizing the likelihood of the choice model:

$$\mathcal{L}(r_\theta, \mathcal{D}) = -\mathbb{E}_{(x,y_w,u_l)\sim\mathcal{D}}\left[\log p_\phi(y_w \succ y_l|x)\right]$$

Finally, we can rewrite the objective as:

$$\mathcal{L}_{DPO}(\pi_\theta; \pi_{\text{ref}}) = -\mathbb{E}_{(x,y_w,u_l)\sim\mathcal{D}}\left[\log p_\phi(y_w \succ y_l|x)\right]$$
$$= -\mathbb{E}_{(x,y_w,y_l)\sim D}\left[\log \sigma\left(\beta \log \frac{\pi_\theta(y_w|x)}{\pi_{\text{ref}}(y_w|x)} - \beta \log \frac{\pi_\theta(y_l|x)}{\pi_{\text{ref}}(y_l|x)}\right)\right]$$

Rafael Rafaelov, Archit Sharma, Eric Mitchell, Stefano Ermon, Christopher D. Manning, and Chelsea Finn, "Direct preference optimization: Your language model is secretly a reward model."

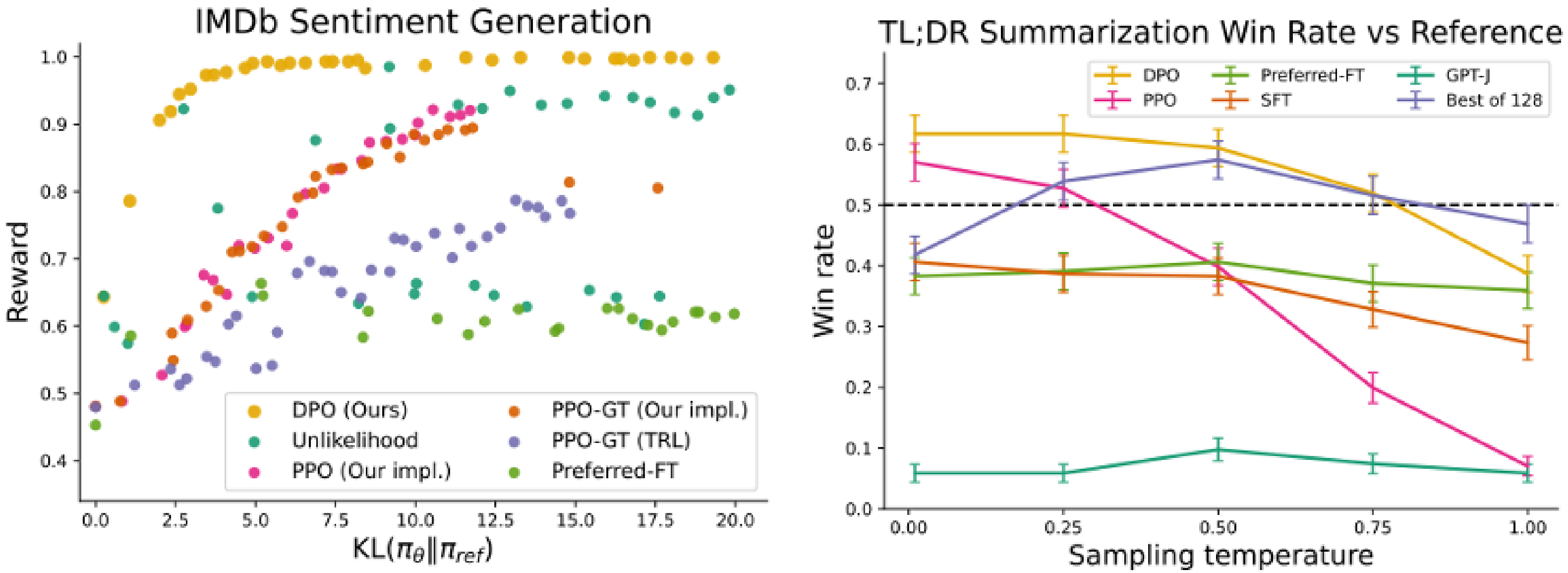

Figure 2: **Left.** The frontier of expected reward vs KL to the reference policy. DPO provides the highest expected reward for all KL values, demonstrating the quality of the optimization. **Right.** TL;DR summarization win rates vs. human-written summaries, using GPT-4 as evaluator. DPO exceeds PPO's best-case performance on summarization, while being more robust to changes in the sampling temperature.

Should your ML application use an explicit utility / reward model?

- Pro:
  - Reward models can be re-used (in principle)
  - Reward model can be examined to infer properties of human(s), and measure the quality of the preference model(s)
  - Reward model(s) add useful inductive biases to the training pipeline
- Cons:
  - The extra step of reward modeling can introduce (unnecessary?) errors
  - Reward model optimization can be unstable (e.g., in RLHF, as argued by DPO)

Some criticisms of choice modeling more broadly

- Real-world choices often appear to be highly situational or context-dependent e.g., way choice is posed, emotional states, other factors not well modeled.
  - Arguably what is exploited by marketing. Related to framing effects (more later).
  - A partial rebuttal: In principle, can always add more context to the model.
- Many choices are intuitive rather than rational, so utility optimization models do not apply
  - Please have limited attention and cognitive capability, especially for less salient choices
  - Default choices are powerful, e.g., in 401K, or opt-in organ donors

Q & A

- What are some key assumptions in (discrete) choice models?
  - Rationality (existence of a utility function that determines choices)
  - Parametric model for utility and choice noise
  - Finite set of choices, and explicit alternatives
- How does one apply discrete choice models to ML/RL applications with changing context (input)
  - Model utility via generic models (e.g., deep neural networks)
- What are some criticisms of discrete choice models?
  - Humans display context-dependent choices
  - Humans often make intuitive (or irrational) choices

What is not covered

- **Details of estimation, analysis**
  - Maximum likelihood is generally equivalent to standard classification/ranking
  - Existing analysis (though often interesting) is mostly for linear (or simpler) utilities
  - Many of the interesting theoretical questions are for active querying settings
- **Beyond discrete choice models**
  - With equivalent alternatives ($U_1 > U_2, U_1 \approx U_3$)
  - Continuous "choices" e.g., pricing, demand/supply
  - Dynamic discrete choice (for time varying choices) ≈ RL
- **Experimental design for "stated preferences"**
  - How to design a survey to measure alternatives, conjoint analysis
- **Active querying** (future discussion)

Summary

- **Today:** Overview of discrete choice models
  - Basics of discrete choice and rationality assumptions
  - Benefits and criticisms of discrete choice
  - Some special cases and applications of discrete choice models to ML
- **Next Lecture:** Student discussion on Human Decision Making and Choice Models

References

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- Additional:
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