Mining Data Streams (Part 2)

CS345a: Data Mining
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Filtering Data Streams

- Each element of data stream is a tuple
- Given a list of keys S
- Determine which elements of stream have keys in S
- Obvious solution: hash table
  - But suppose we don't have enough memory to store all of S in a hash table
  - e.g., we might be processing millions of filters on the same stream

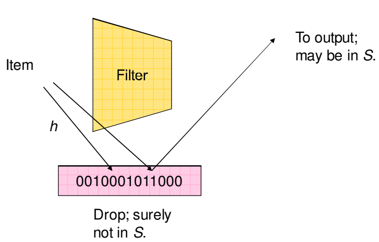
Applications

- Example: email spam filtering
  - We know 1 billion "good" email addresses
  - If an email comes from one of these, it is NOT spam
- Publish-subscribe
  - People express interest in certain sets of keywords
  - Determine whether each message matches a user's interest

First Cut Solution – (1)

- Create a bit array *B* of *m* bits, initially all 0's.
- Choose a hash function *h* with range [0,m)
- Hash each member of *S* to one of the bits, which is then set to 1
- Hash each element of stream and output only those that hash to a 1

First Cut Solution – (2)



First Cut Solution – (3)

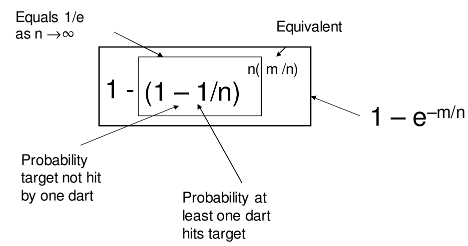
- |S| = 1 billion, |B|= 1GB = 8 billion bits
- If a string is in *S*, it surely hashes to a 1, so it always gets through
- Approximately most 1/8 of the bit array is 1, so about $1/8^{th}$ of the strings not in *S* get through to the output (*false positives*)
  - Actually, less than $1/8^{th}$, because more than one key might hash to the same bit

Throwing Darts

- If we throw *m* darts into *n* equally likely targets, what is the probability that a target gets at least one dart?
- Targets = bits, darts = hash values

Throwing Darts – (2)

m darts, n targets



Throwing Darts – (3)

- Fraction of 1's in array = probability of false positive = $1 - e^{-m/n}$
- Example: 10^9 darts, $8*10^9$ targets.
  - Fraction of 1's in B = $1 - e^{-1/8} = 0.1175$.
  - Compare with our earlier estimate: 1/8 = 0.125.

Bloom Filter

- Say |S| = *m*, |B| = *n*
- Use *k* independent hash functions $h_1, \ldots, h_k$
- Initialize B to all 0's
- Hash each element *s* in S using each function, and set $B[h_i(s)] = 1$ for $i = 1,..,k$
- When a stream element with key x arrives
  - If $B[h_i(x)] = 1$ for $i = 1,..,k$, then declare that x is in S
  - Otherwise discard the element

Bloom Filter -- Analysis

- What fraction of bit vector B is 1's?
  - Throwing km darts at n targets
  - So fraction of 1's is $(1 - e^{-km/n})$
- k independent hash functions
- False positive probability = $(1 - e^{-km/n})^k$

Bloom Filter – Analysis (2)

- m = 1 billion, n = 8 billion
  - k = 1: $(1 - e^{-1/8}) = 0.1175$
  - k = 2: $(1 - e^{-1/4})^2 = 0.0493$
- What happens as we keep increasing *k*?
- "Optimal" value of *k*: *n/m* ln 2

Bloom Filter: Wrap-up

- Bloom filters guarantee no false negatives, and use limited memory
  - Great for pre-processing before more expensive checks
  - E.g., Google's BigTable, Squid web proxy
- Suitable for hardware implementation
  - Hash function computations can be parallelized

Counting Distinct Elements

- Problem: a data stream consists of elements chosen from a set of size n. Maintain a count of the number of distinct elements seen so far.
- Obvious approach: maintain the set of elements seen.

Applications

- How many different words are found among the Web pages being crawled at a site?
  - Unusually low or high numbers could indicate artificial pages (spam?)
- How many different Web pages does each customer request in a week?

Using Small Storage

- Real Problem: what if we do not have space to store the complete set?
- Estimate the count in an unbiased way.
- Accept that the count may be in error, but limit the probability that the error is large.

Flajolet-Martin* Approach

- Pick a hash function h that maps each of the n elements to at least $\log_2 n$ bits
- For each stream element a, let $r(a)$ be the number of trailing 0's in $h(a)$
- Record R = the maximum $r(a)$ seen
- Estimate = 2^R.

* Really based on a variant due to AMS (Alon, Matias, and Szegedy)

Why It Works

- The probability that a given $h(a)$ ends in at least r 0's is 2^{-r}
- Probability of NOT seeing a tail of length r among m elements: $(1 - 2^{-r})^m$

Prob. All end in fewer than r 0's.

Prob. a given h(a) ends in fewer than r 0's.

Why It Works – (2)

- Since 2^{-r} is small, prob. of NOT finding a tail of length r is:
- If m<< 2^r, tends to 1. So probability of finding a tail of length r tends to 0.
- Ifm>> 2^r, tends to 0. So probability of finding a tail of length r tends to 1.
- Thus, 2^R will almost always be around *m*.

Why It Doesn't Work

- E(2^R) is actually infinite.
  - Probability halves when *R* ->*R* +1, but value doubles.
- Workaround involves using many hash functions and getting many samples.
- How are samples combined?
  - Average? What if one very large value?
  - Median? All values are a power of 2.

Solution

- Partition your samples into small groups
- Take the average of groups
- Then take the median of the averages

Generalization: Moments

- Suppose a stream has elements chosen from a set of *n* values.
- Let m_i be the number of times value *i* occurs.
- The k^{th} *moment* is

Special Cases

- 0^{th} moment = number of distinct elements
  - The problem just considered.
- 1^{st} moment = count of the numbers of elements = length of the stream.
  - Easy to compute.
- 2^{nd} moment = *surprise number* = a measure of how uneven the distribution is.

Example: Surprise Number

- Stream of length 100; 11 distinct values
- Item counts: 10, 9, 9, 9, 9, 9, 9, 9, 9, 9, 9
  Surprise # = 910
- Item counts: 90, 1, 1, 1, 1, 1, 1, 1 ,1, 1, 1
  Surprise # = 8,110.

AMS Method

- Works for all moments; gives an unbiased estimate.
- We'll just concentrate on 2nd moment.
- Based on calculation of many random variables *X*.
  - Each requires a count in main memory, so number is limited.

One Random Variable (X)

- Assume stream has length *n*.
- Pick a random time to start, so that any time is equally likely.
- Let the chosen time have element *a* in the stream
- Maintain a count c of the number *a'*s in the stream starting at the chosen time
- $X = n*(2c-1)$
  - Store *n* once, count of *a*'s for each *X*.

Expectation Analysis

- $X = n(2c - 1)$
- $E[X] = (1/n)\Sigma_{\text{all times } t}\, n\,(2c - 1)$
  $= \Sigma_{\text{all times } t}\,(2c - 1)$
  $= \Sigma_a\,(1 + 3 + 5 + \ldots + 2m_a - 1)$
  $= \Sigma_a (m_a)^2$

Combining Samples

- Compute as many variables *X* as can fit in available memory.
- Average them in groups.
- Take median of averages.

Problem: Streams Never End

- We assumed there was a number *n*, the number of positions in the stream.
- But real streams go on forever, so *n* is a variable – the number of inputs seen so far.

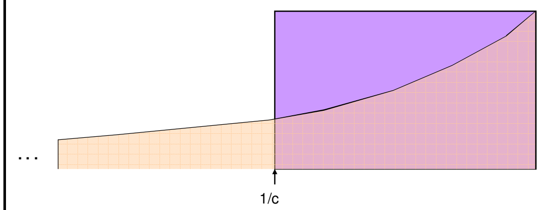
Fixups

1. The variables *X* have *n* as a factor – keep *n* separately; just hold the count in *X*
2. Suppose we can only store *k* counts. We must throw some *X*'s out as time goes on.
   - Objective: each starting time *t* is selected with probability *k* /*n*
   - How can we do this?

Exponentially Decaying Windows

- Stream $a_1, a_2, \dots$
- Define exponentially decaying window at time *t* to be: $\Sigma_{i=1,2,\dots,t} a_i (1-c)^{t-i}$
- *c* is a constant, presumably tiny, like 10^{-6} or 10^{-9}.

Sliding Versus Decaying Windows



Applications

- Key use case is when the stream's statistics can vary over time
- Finding the most popular elements "currently"
  - Stream of Amazon items sold
  - Stream of topics mentioned in tweets
  - Stream of music tracks streamed