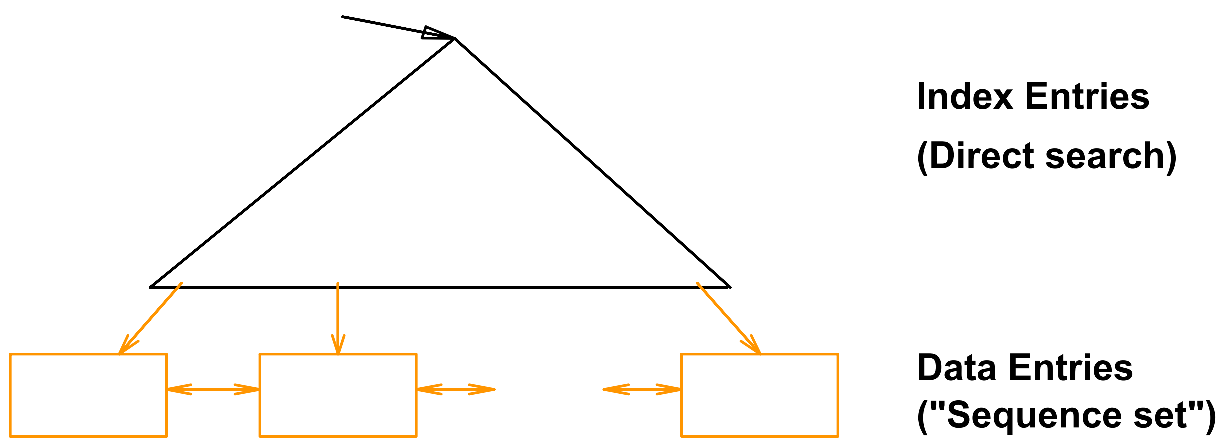
B+ Review

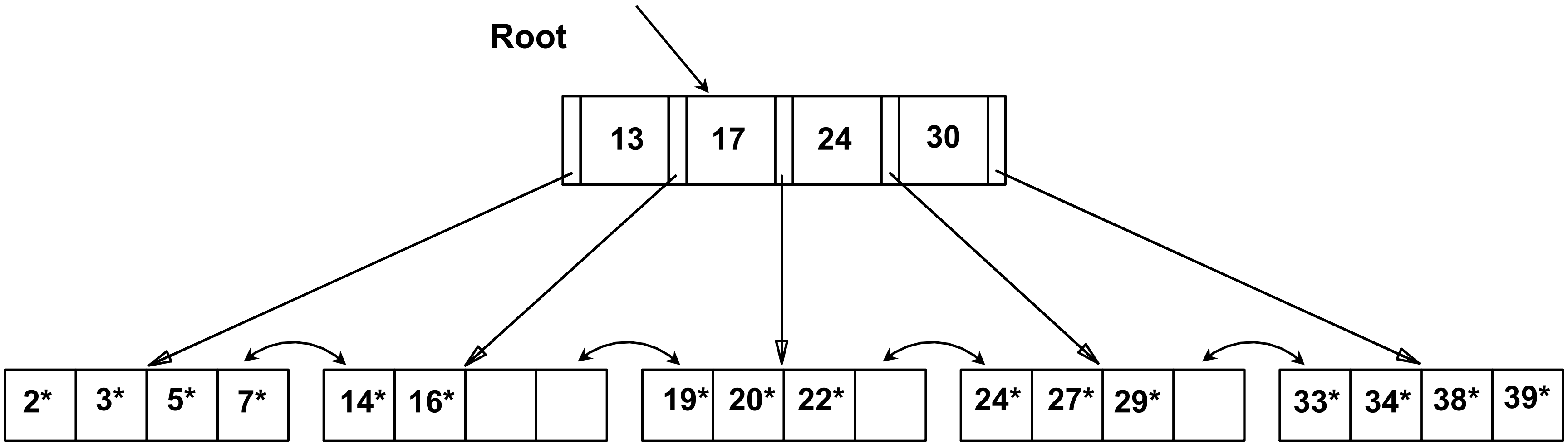
B+ Tree: Most Widely Used Index

- Insert/delete at $\log_F N$ cost; keep tree *height-balanced*. (F = fanout, N = # leaf pages)
- Minimum 50% occupancy (except for root). Each node contains **d** <= *m* <= 2**d** entries. The parameter **d** is called the *order* of the tree.
- Supports equality and range-searches efficiently.



Example B+ Tree

- Search begins at root, and key comparisons direct it to a leaf (as in ISAM).
- Search for 5*, 15*, all data entries >= 24* ...



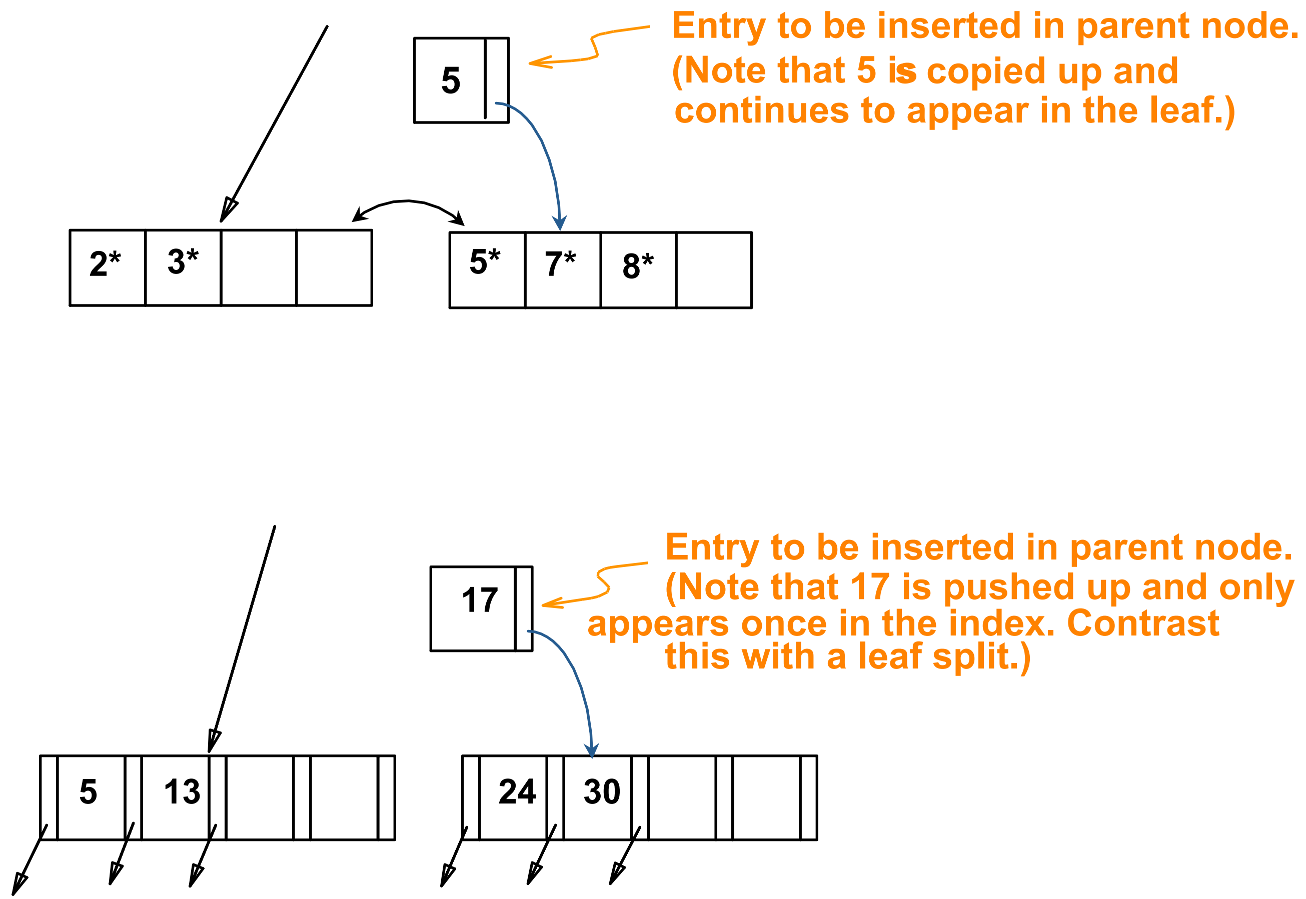
☛ *Based on the search for 15*, we <u>know</u> it is not in the tree!*

Inserting a Data Entry into a B+ Tree

- Find correct leaf *L.*
- Put data entry onto *L*.
  - If *L* has enough space, *done*!
  - Else, must *split* *L (into L and a new node L2)*
    - Redistribute entries evenly, **copy up** middle key.
    - Insert index entry pointing to *L2* into parent of *L*.
- This can happen recursively
  - To split index node, redistribute entries evenly, but **push up** middle key. (Contrast with leaf splits.)
- Splits "grow" tree; root split increases height.
  - Tree growth: gets *wider* or *one level taller at top.*

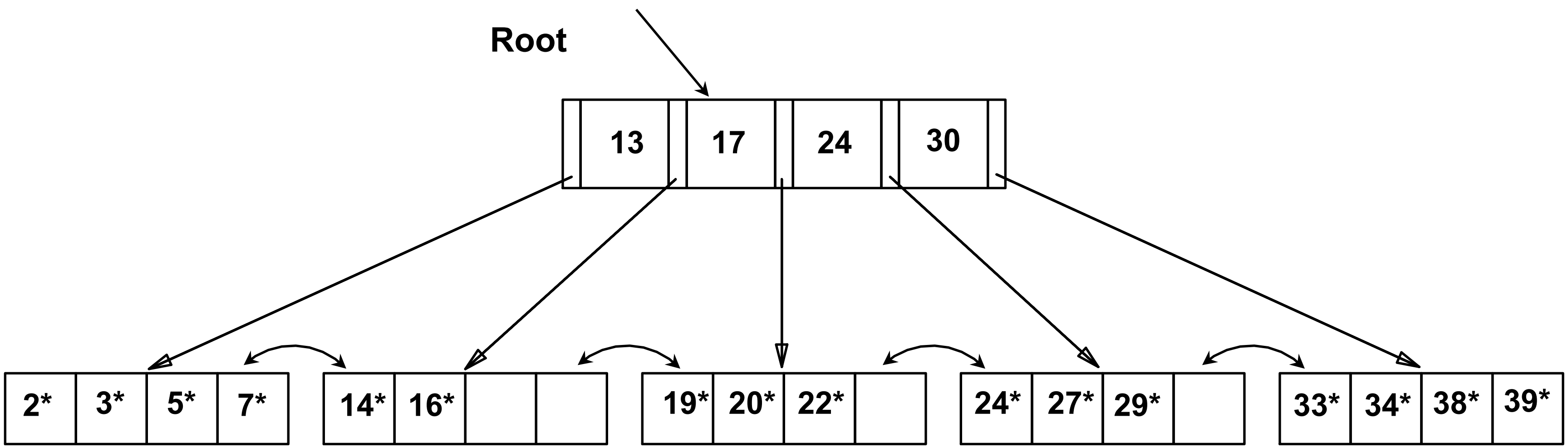
Inserting 8* into Example B+ Tree

- Observe how minimum occupancy is guaranteed in both leaf and index pg splits.
- Note difference between *copy-up* and *push-up*; be sure you understand the reasons for this.



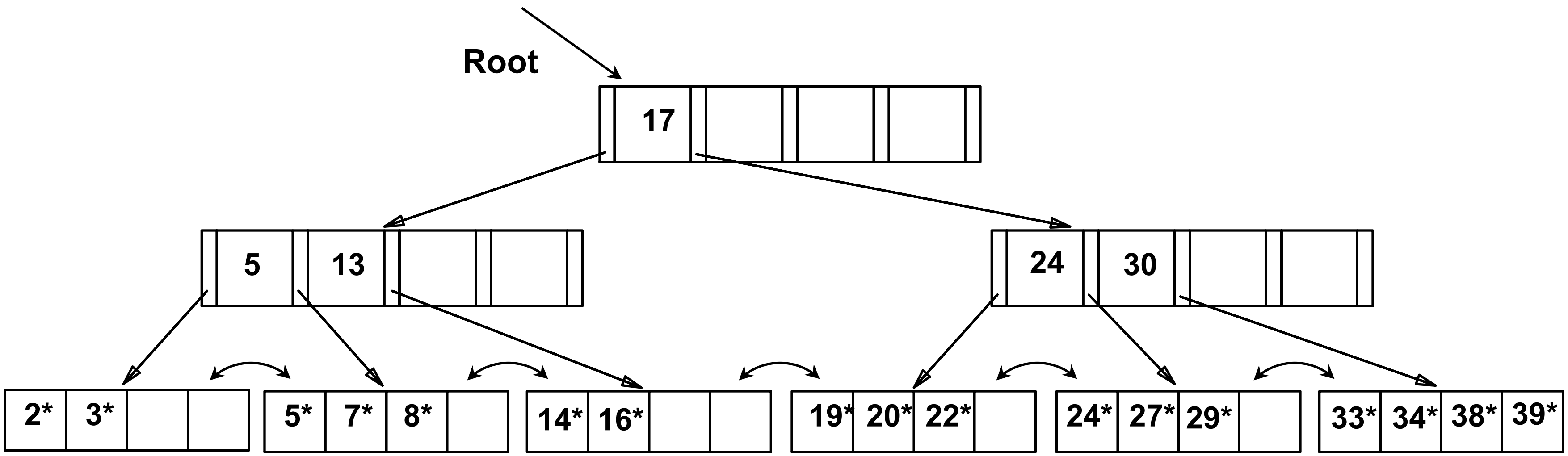
Example B+ Tree

- We're going to insert 8.



☛ *Based on the search for 15*, we <u>know</u> it is not in the tree!*

Example B+ Tree After Inserting 8*



❖ Notice that root was split, leading to increase in height.

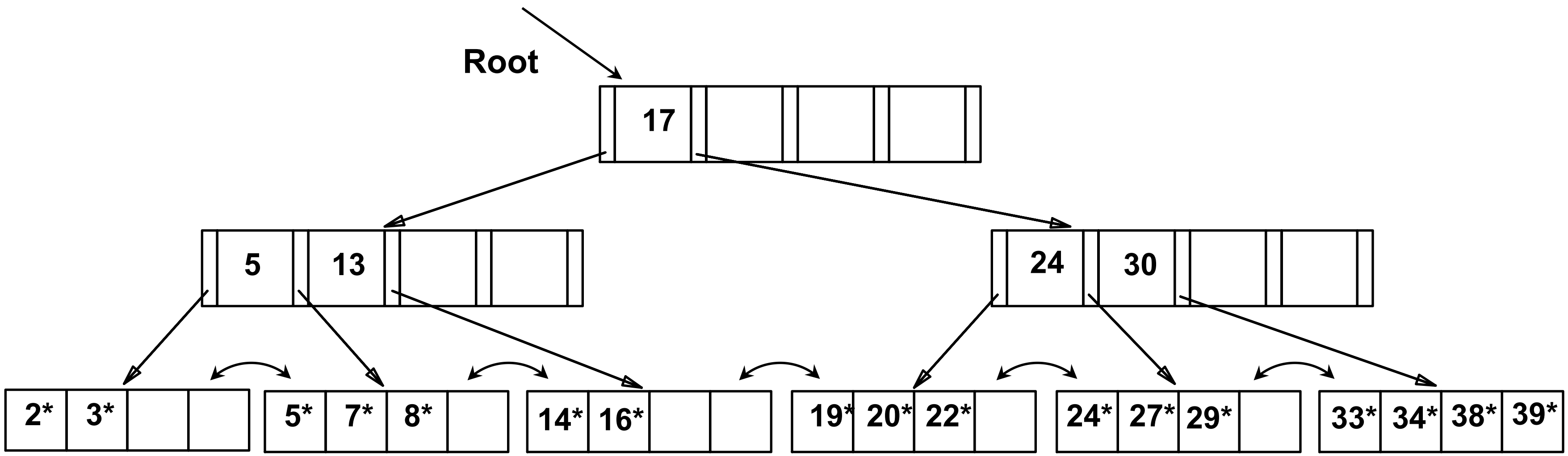
❖ In this example, we can avoid split by re-distributing entries; however, this is usually not done in practice.

Deleting a Data Entry from a B+ Tree

- Start at root, find leaf *L* where entry belongs.
- Remove the entry.
  - If L is at least half-full, *done!*
  - If L has only **d-1** entries,
    - Try to re-distribute, borrowing from *sibling* *(adjacent node with same parent as L)*.
    - If re-distribution fails, *merge* *L* and sibling.
- If merge occurred, must delete entry (pointing to *L* or sibling) from parent of *L*.
- Merge could propagate to root, decreasing height.

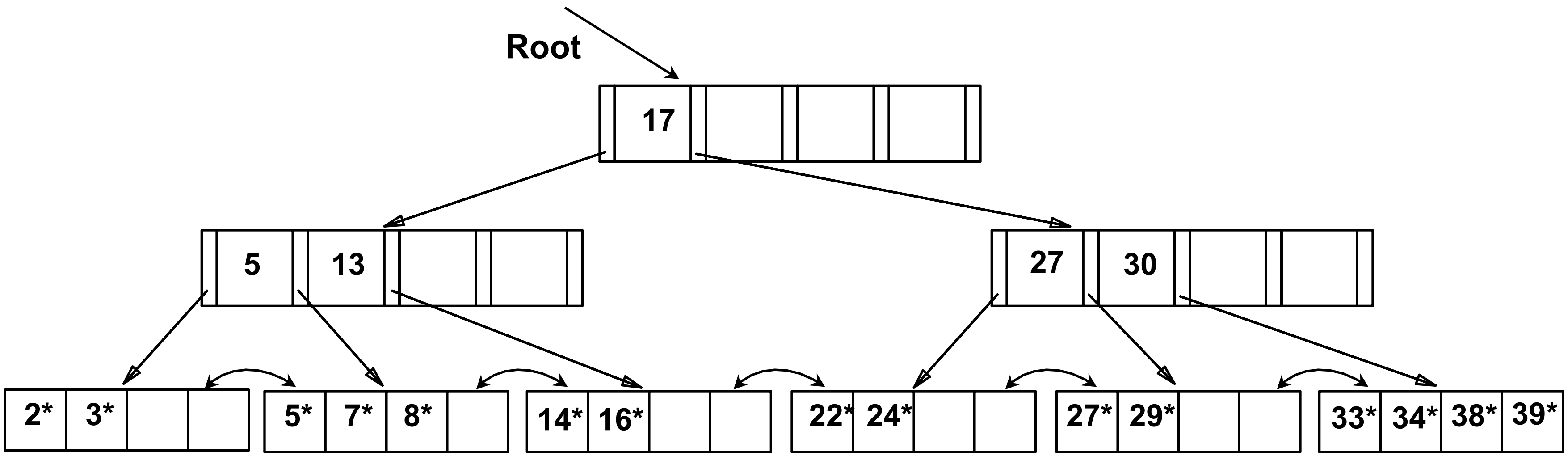
Delete

Example B+ Tree After Inserting 8*



❖ We're going to delete 19 and 20

Example Tree After (Inserting 8*, Then) Deleting 19* and 20* ...

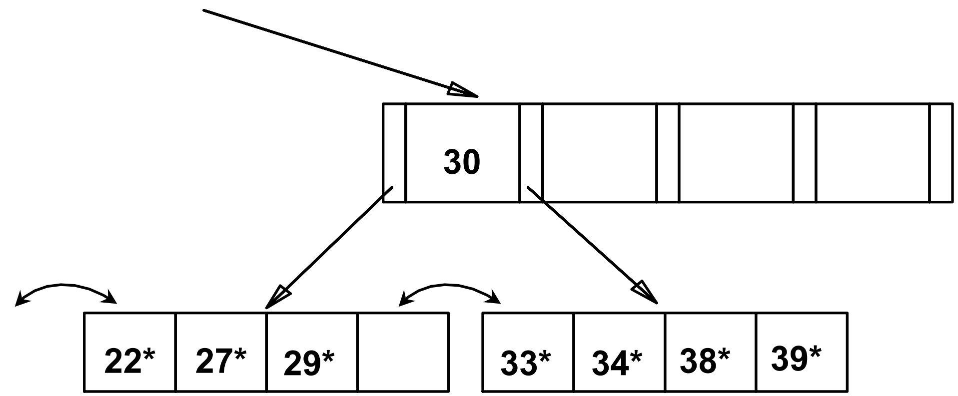


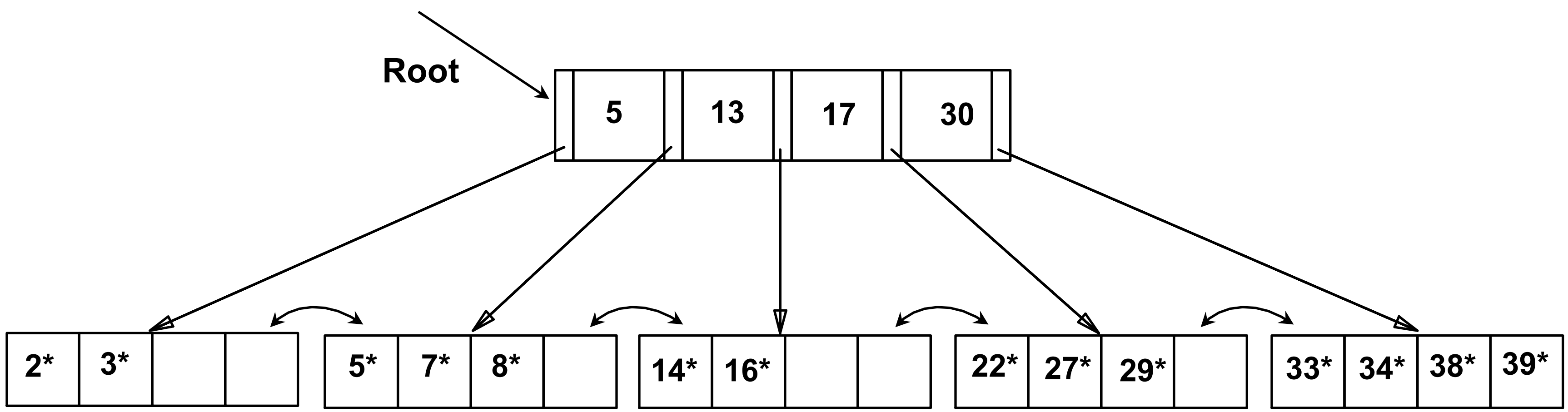
- Deleting 19* is easy.
- Deleting 20* is done with re-distribution. Notice how middle key is *copied up*.

Next, we delete 24

… And Then Deleting 24*

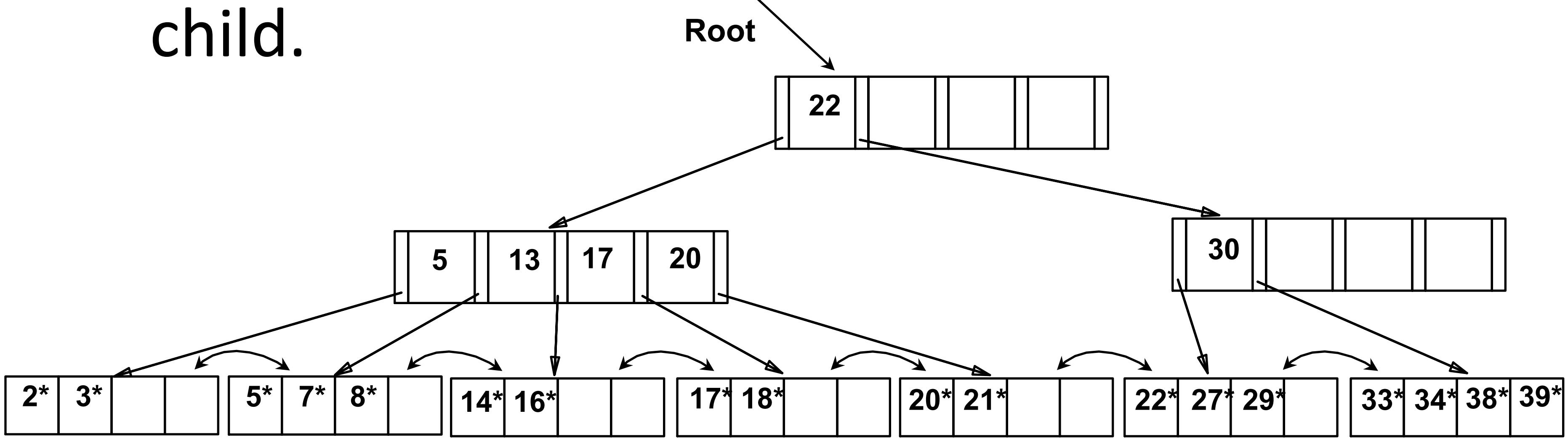
- Must merge.
- Observe *`toss'* of index entry (on right), and *`pull down'* of index entry (below).





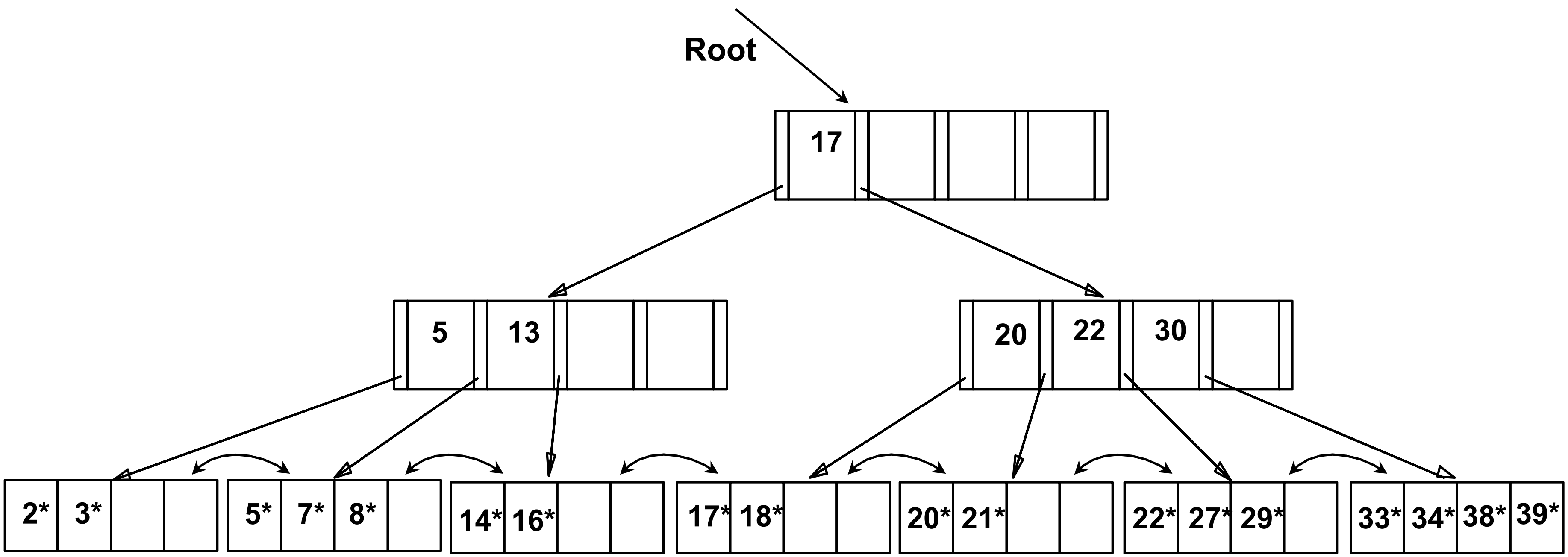
Example of Non-leaf Re-distribution

- Tree is shown below *during deletion* of 24*. (What could be a possible initial tree?)
- In contrast to previous example, can re-distribute entry from left child of root to right child.



After Re-distribution

- Entries are re-distributed by *`pushing through'* the splitting entry in the parent node.
- It suffices to re-distribute index entry with key 20; we've re-distributed 17 as well

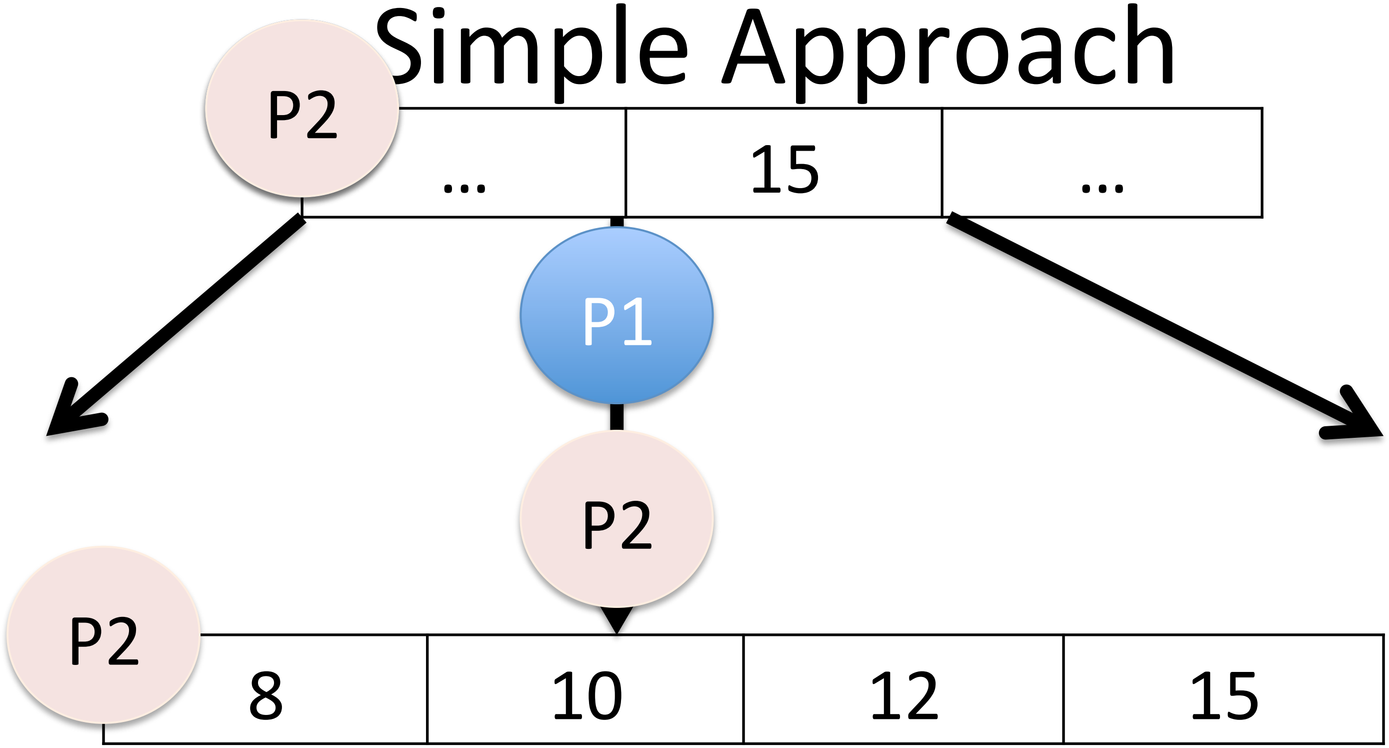


B+ Concurrency

Model

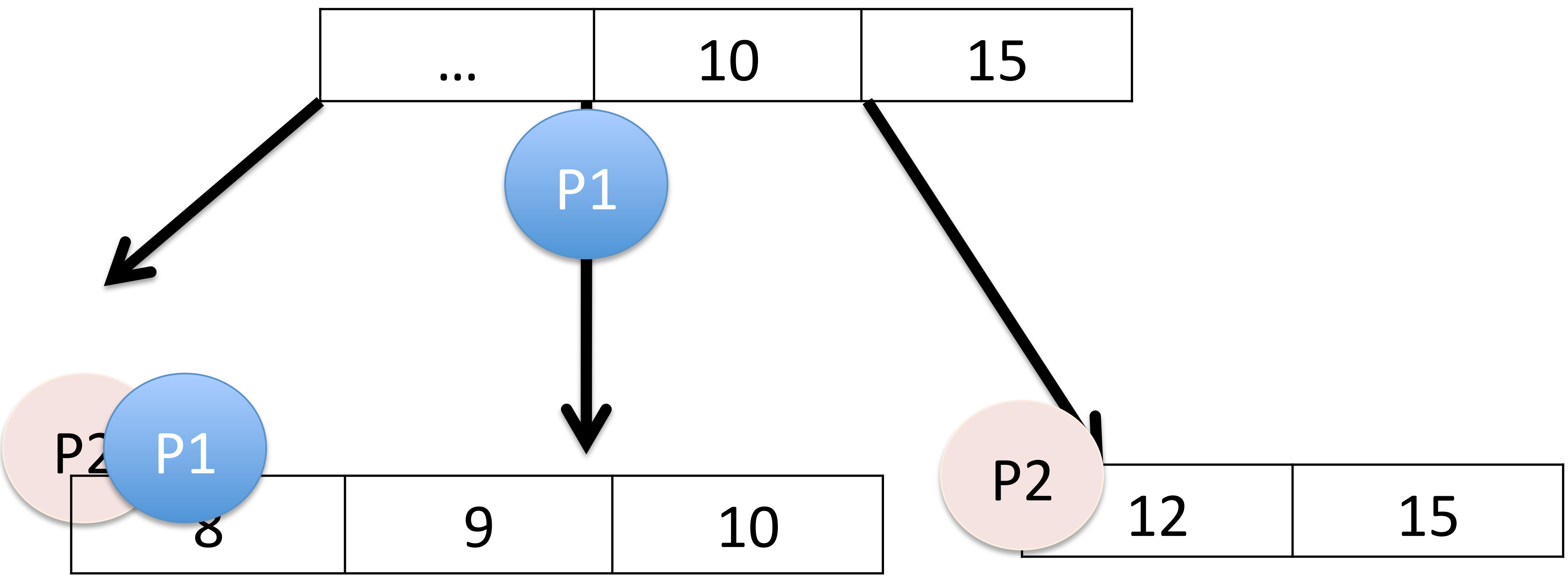
- We consider page lock(x)/unlock(x) of pages (only for writes!)
- We copy into our memory and then atomically update pages.

Simple Approach



- P1 searches for 15
- P2 inserts 9

After the Insertion



- P1 searches for 15
- P2 inserts 9

P1 Finds no 15!

How could we fix this?

B-Link Trees

Two important Conventions

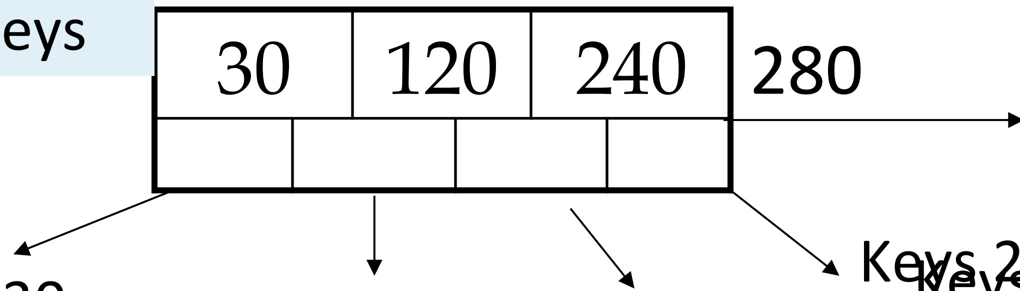
- Search for B-link trees root to leaf, left-to-right in nodes

- Insertions for B-link trees proceed bottom-up.

Internal Nodes

- Parameter d = the *degree*

Internal Node has
s >= d and <= 2d keys



Add right pointers.

We add a High key

Idea: If we get to this page, looking for 300. What can we conclude happened?

Valid Trees & Safe Nodes

- A node may not have a parent node, but it must have a left twin.
- We introduce the right links before the parent.
- A node is safe if it has [k,2k-1] pointers.

Scannode

scannode(u, A) : examine the tree node in A for value u and return the appropriate pointer from A.

Appropriate pointer may be the right pointer.

Searching for v

```
current = root;
A = get(current);
while (current is not a leaf) {
      current = scannode(v, A);
      A = get(current);}
while ((t = scannode(v,A)) == link pointer of A) {
      current = t;
      A = get(current);}
Return (v is in A) ? success :  failure;
```

Find the leaf w/ v

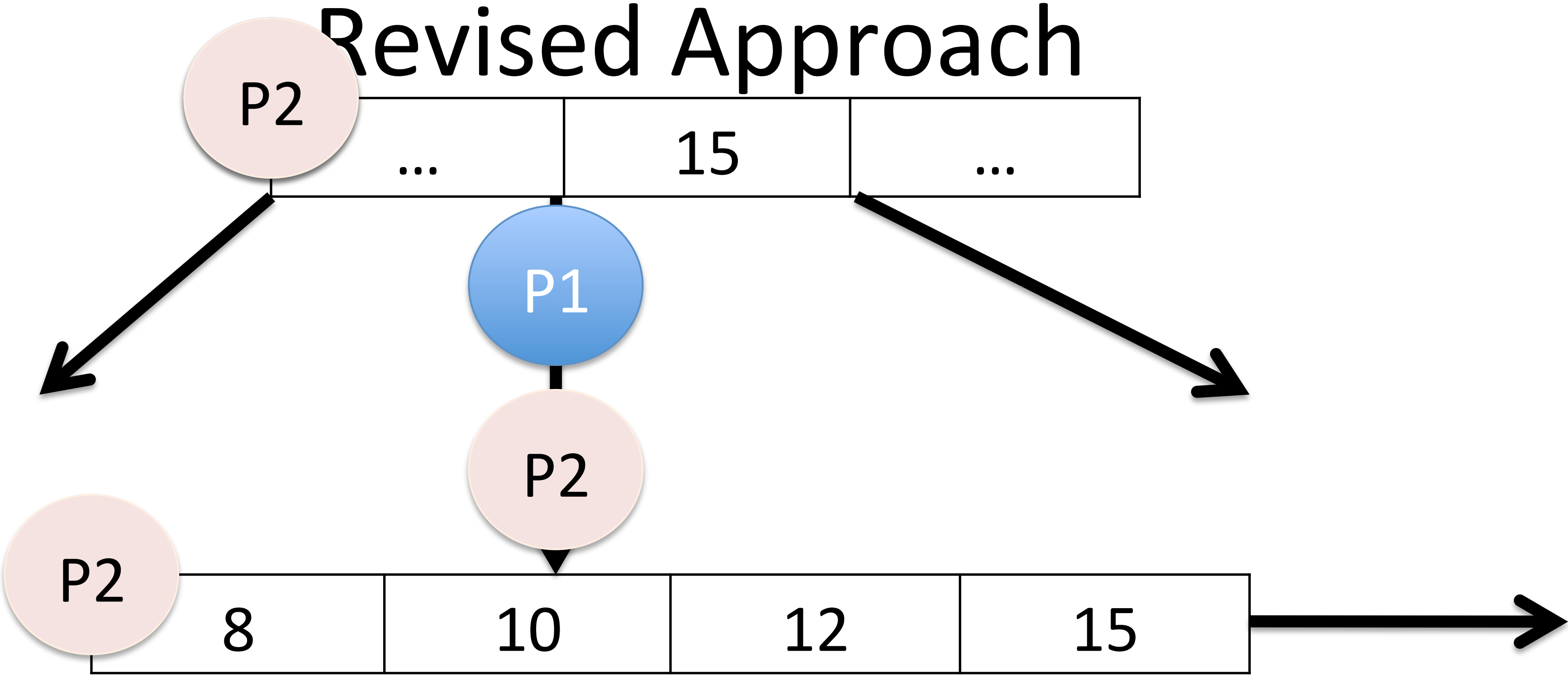
Find the leaf w/ v

Only modify scannode – No locking?!?

Insert

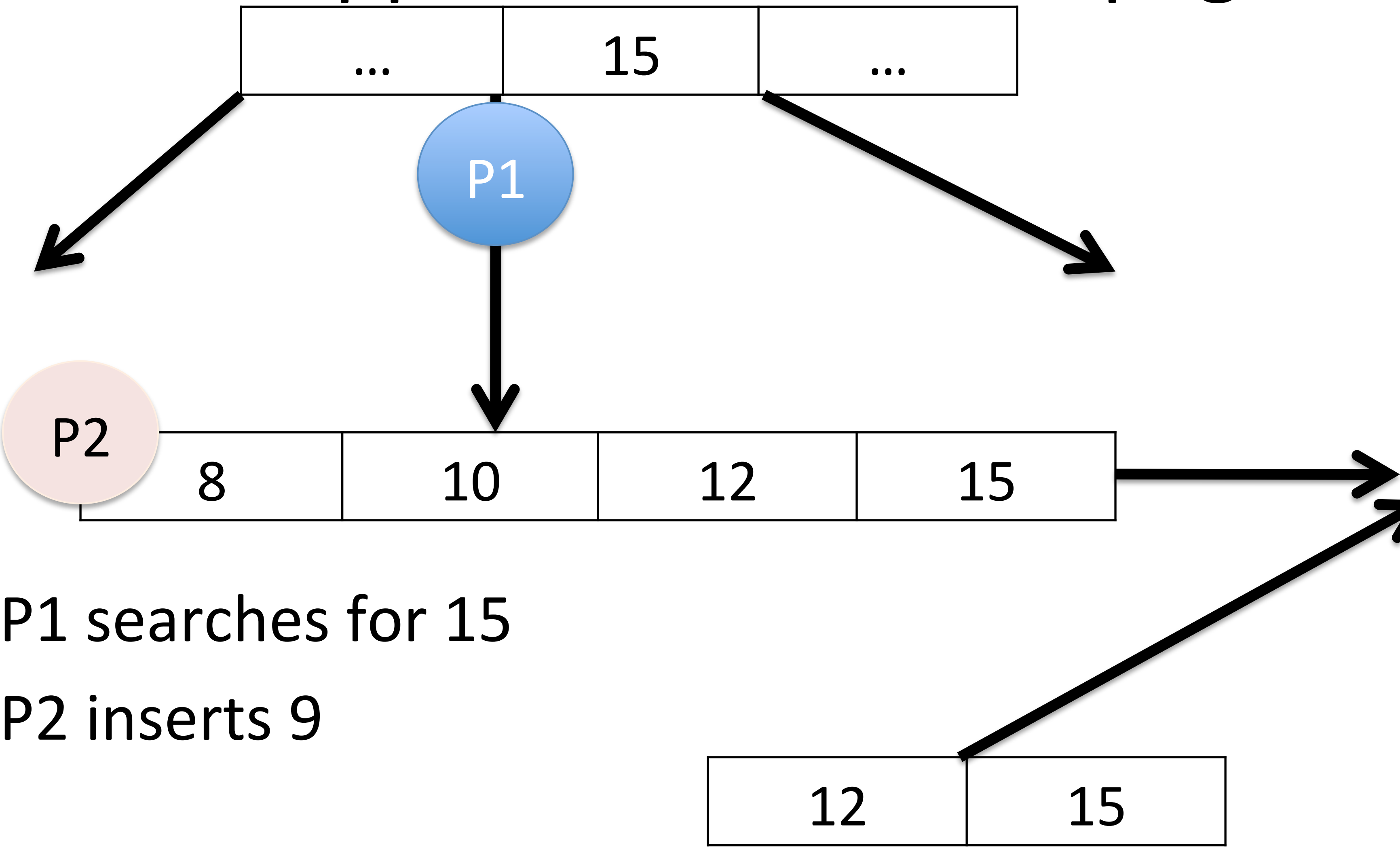
High Key Omitted

Revised Approach



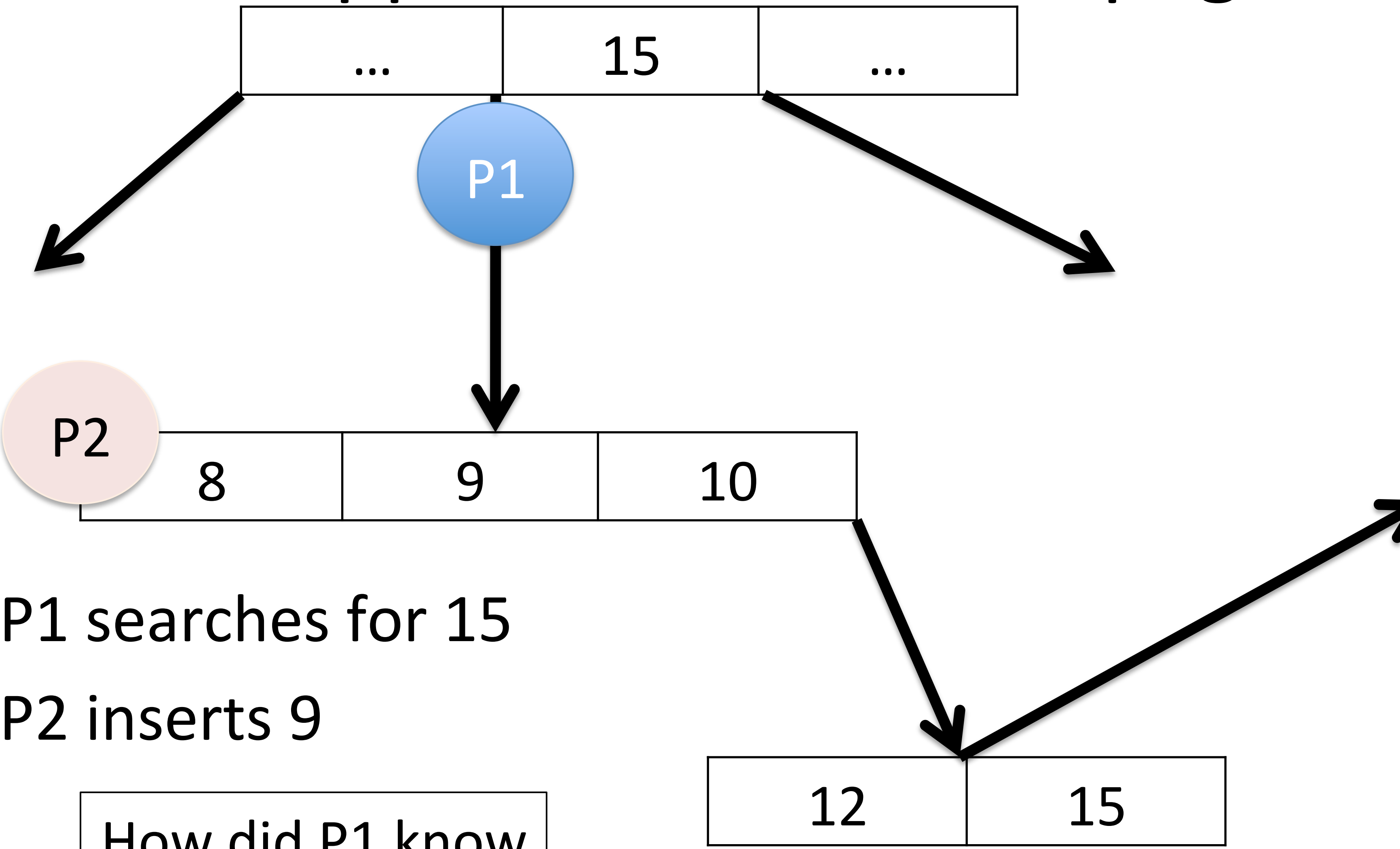
- P1 searches for 15
- P2 inserts 9

Revised Approach: Build new page



- P1 searches for 15
- P2 inserts 9

Revised Approach: Build new page



- P1 searches for 15
- P2 inserts 9

How did P1 know to continue?

Start Insert

```
initialize stack; current = root;
A = get(current);
while (current is not a leaf) {
    t = current;
    current = scannode(v,A);
    if (current not link pointer in A)
        push t;
    A = get(current);}
```

Keep a stack of the rightmost node we visited at each level:

A subroutine: move_right

```
While t = scannode(v,A) is a link pointer of A do
   Lock(t)
   Unlock(current)
   Current = t
   A = get(current);
end
```

How many locks held here?

The move_right procedure scans right across the leaves with lock coupling.

Easy case:

DoInsert:

```
if A is safe {
    insert new key/ptr pair on A;
    put(A, current);
    unlock(current);
}
```

Fun Case: Must split

u = allocate(1 new page for B);

redistribute A over A and B ;

y = max value on A now;

make high key of B equal old high key of A;

make right-link of B equal old right-link of A;

make high key of A equal y;

make right-link of A point to B;

Insert

put (B, u);

put (A, current);

oldnode = current;

new key/ptr pair = (y, u); // high key of new page, new page

current = pop(stack);

lock(current); A = get(current);

move_right();

unlock(oldnode)

goto Doinsertion;

may have 3 locks: oldnode, and two at the parent level while moving right

Deadlock Free

Total Order < on Nodes

Consider pages a,b define a total order <

1. a < b if b is closer to the root than a (different height)
2. If a and b are at the same height, then a < b if b is reachable.

"Order is bottom-up"

Observation: Insert process only puts down locks satisfying this order. Why is this true?

Deadlock Free

Since the locks are placed by every process in a total order, there can be no deadlock. Why?

Is it possible to get the cycle:
T1(A) T2(B) T1(B) T2(A)?

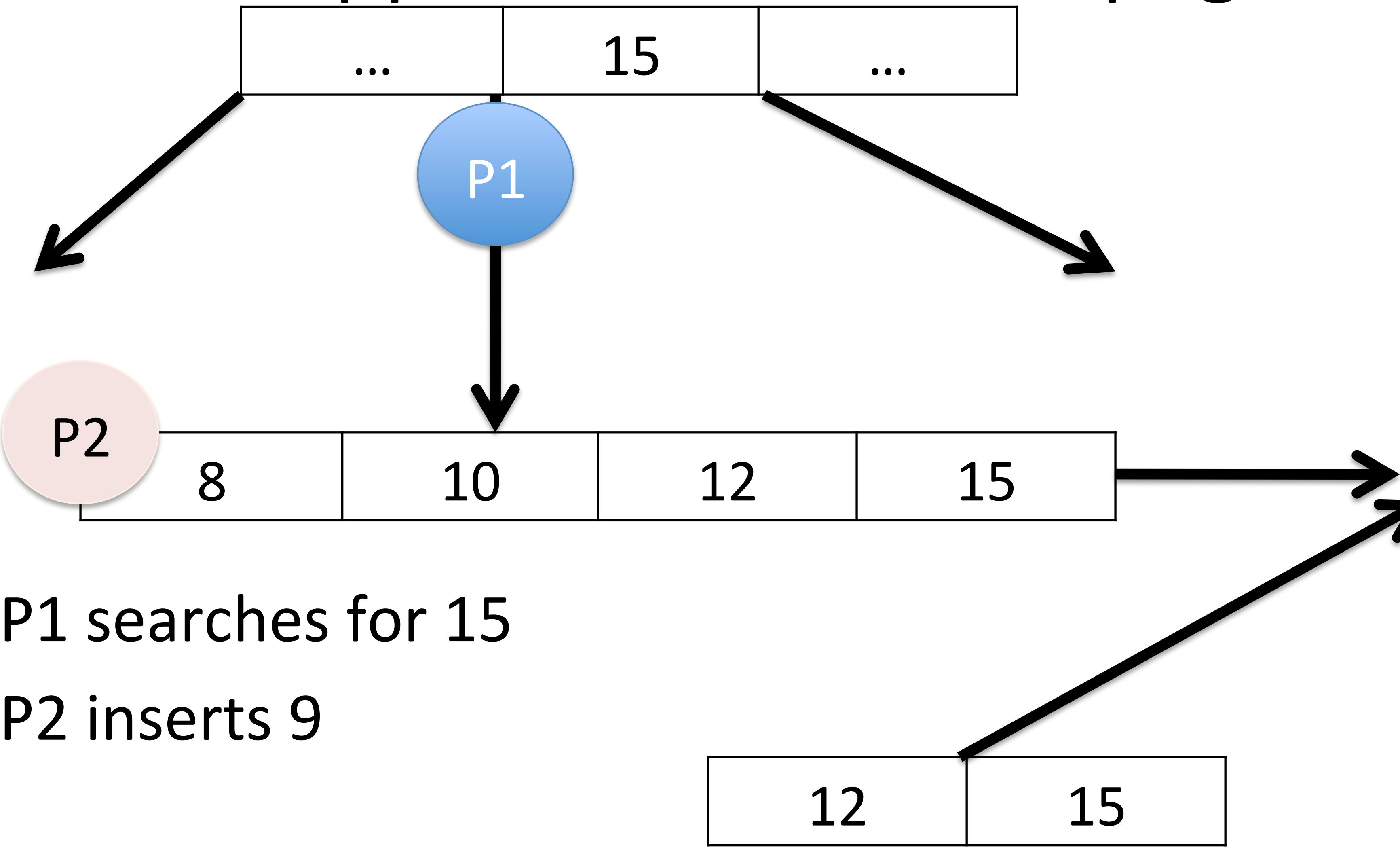
Tree Modification

Tree Modifications

Thm: All operations correctly modify the tree structure.

Observation 1: put(B,u) and put(A, current) are one operation (since put(B,u) doesn't change tree. Proof by pictures (again).

Revised Approach: Build new page



- P1 searches for 15
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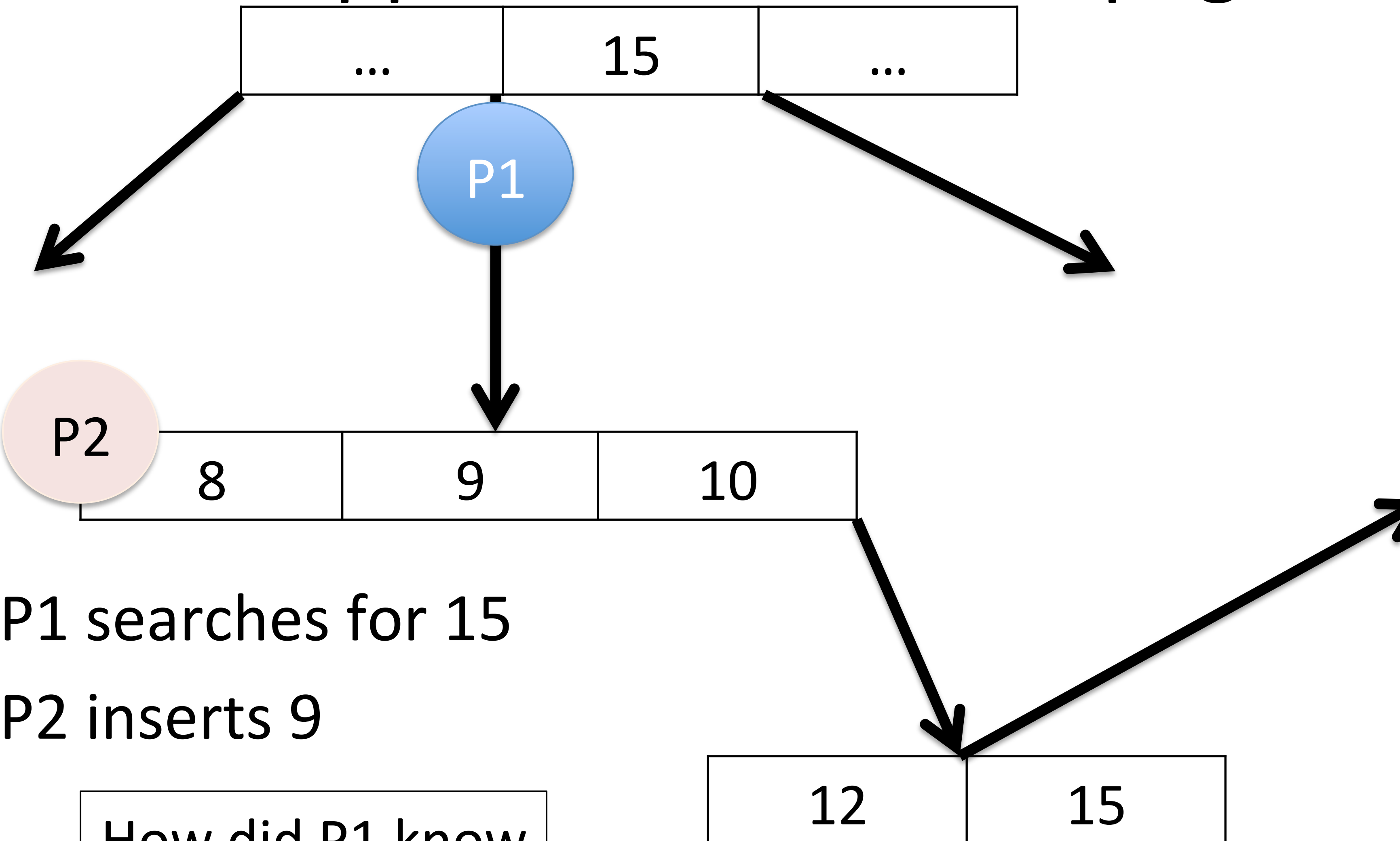
Revised Approach: Build new page



- P1 searches for 15
- P2 inserts 9

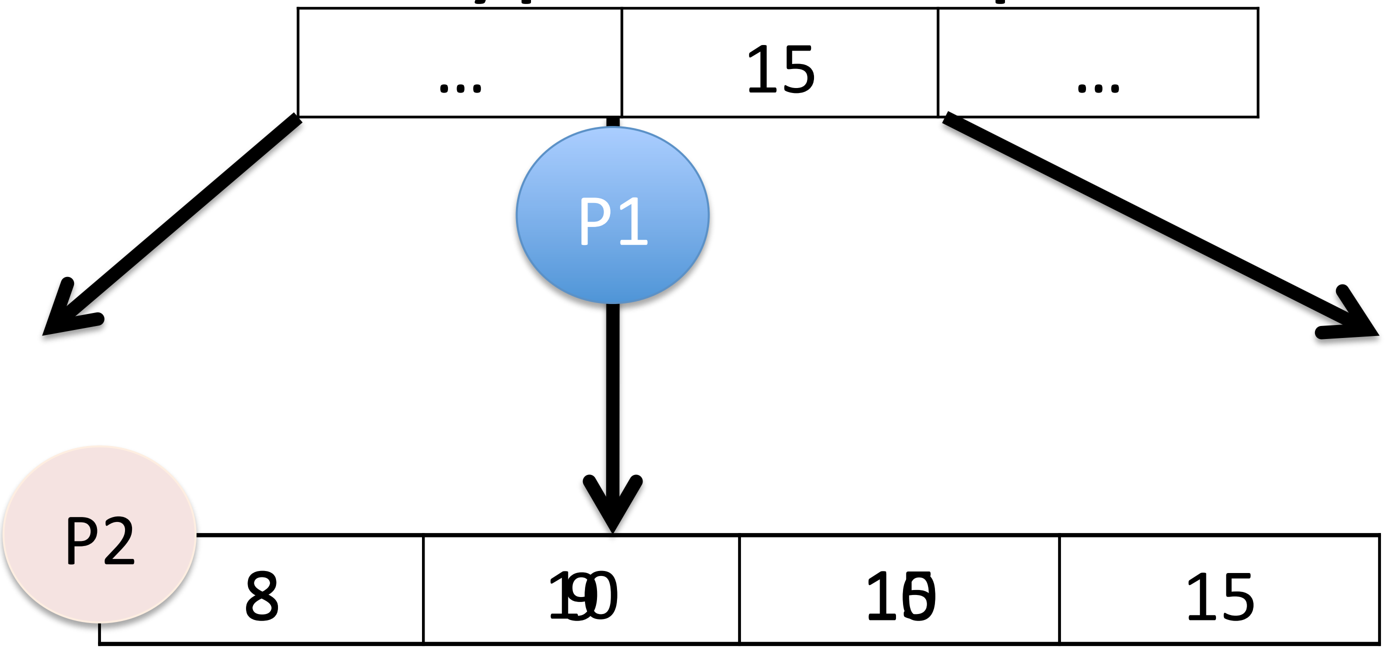
How did P1 know to continue?

Correct Interaction of Readers and Writers

Correct Interaction

Thm: Actions of an insertion process do not impair the correctness of the actions of other processes.

Type 1: No split



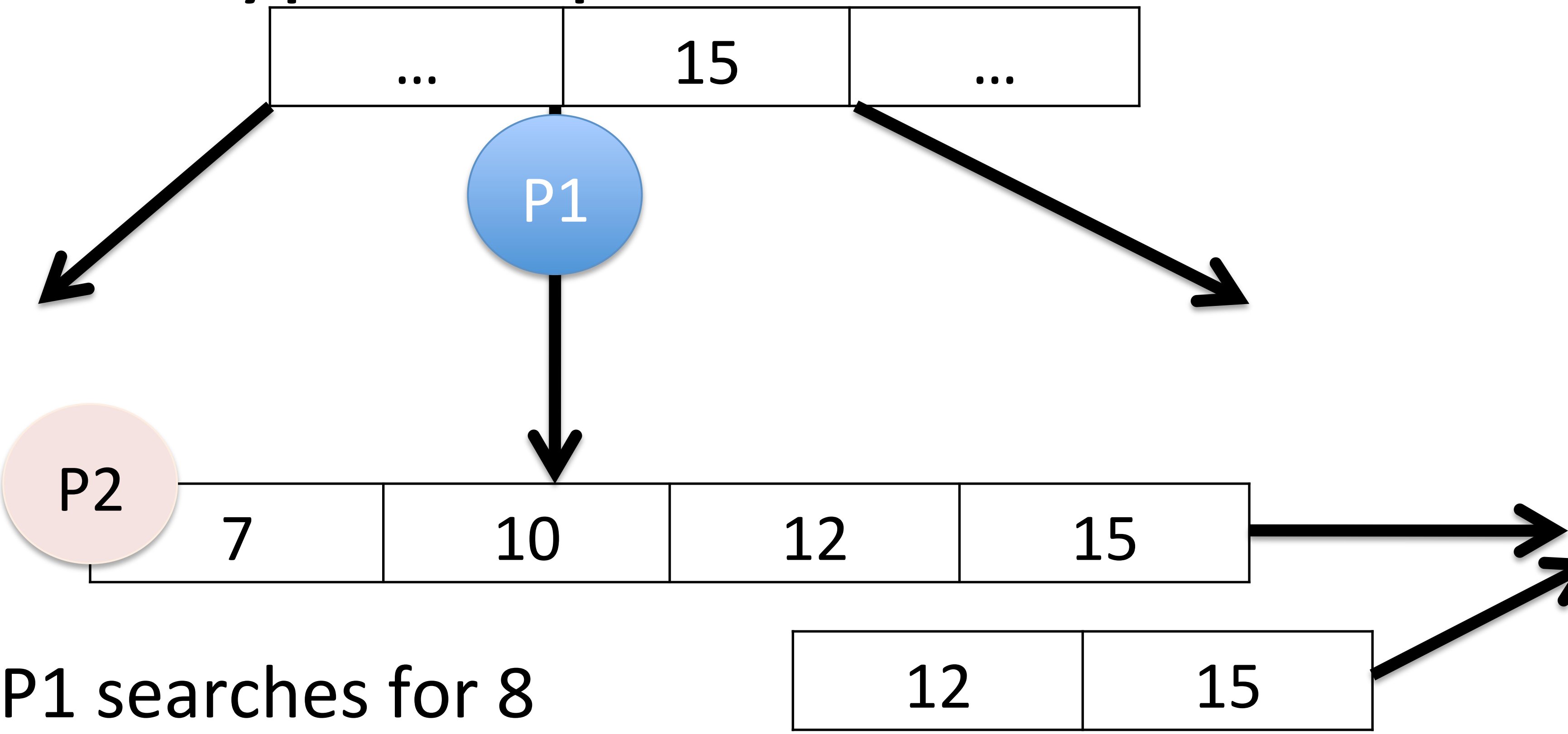
- P1 searches for 15
- P2 inserts 9

P2 reads the page.
What schedule is this?
Why can't P1,P2 conflict again?

What if P2 reads after P1?

Type 2: Split. insert into left Node

Type 2: Split. Insert LHS.



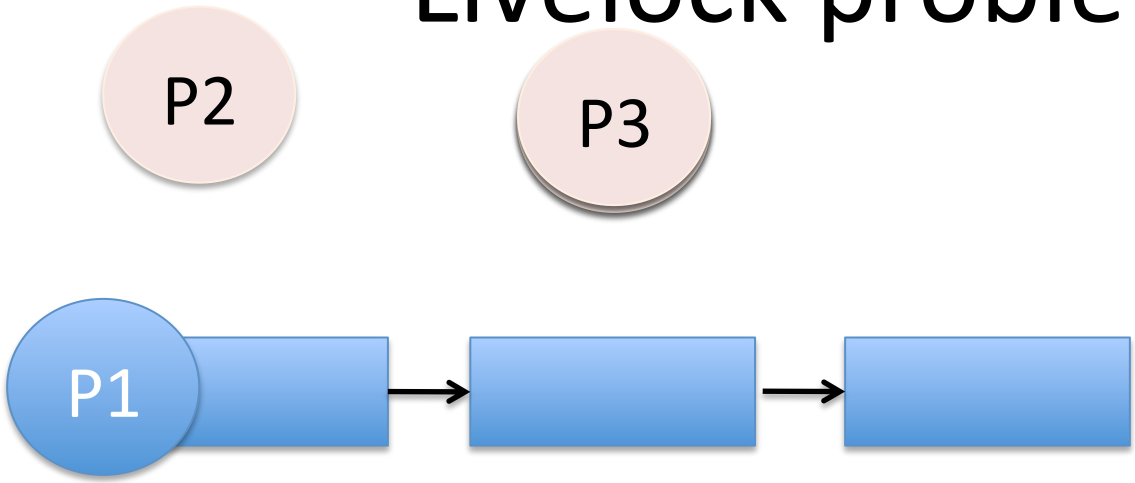
- P1 searches for 8
- P2 inserts 9

Notice that P1 would have followed 9s pointer!

How will P1 find 8?

Livelock

Livelock problem



Poor P1 never gets its value!
P1 is livelocked!

Chaining Example

Can we get down below 3 locks?

Consider the Alternative Protocol
(without lock coupling)

read A;

find out that there is room;

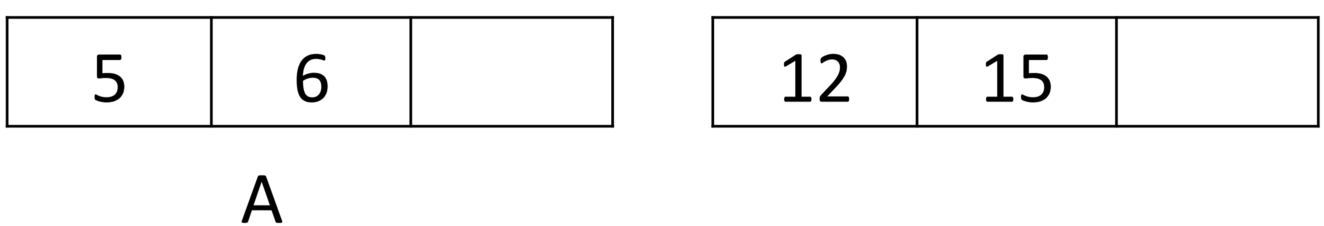
lock and re-read A;

find there is still room, and insert 9

unlock A;

Large # of inserts. A splits and after there is room!

What prevents this in Blink?



Further Reading

- Recent HP Tech Report is great source (Graefe)

http://www.hpl.hp.com/techreports/2010/HPL-2010-9.pdf

- Extensions: R-trees and GiST

Marcel Kornacker, Douglas Banks: High-Concurrency Locking in R-Trees. VLDB 1995: 134-145

Marcel Kornacker, C. Mohan, Joseph M. Hellerstein: Concurrency and Recovery in Generalized Search Trees. SIGMOD Conference 1997: 62-72