

Stanford Continuing Studies, Course MATH 07, Fall 2005

Three views of mathematics

LECTURE 10

Keith Devlin

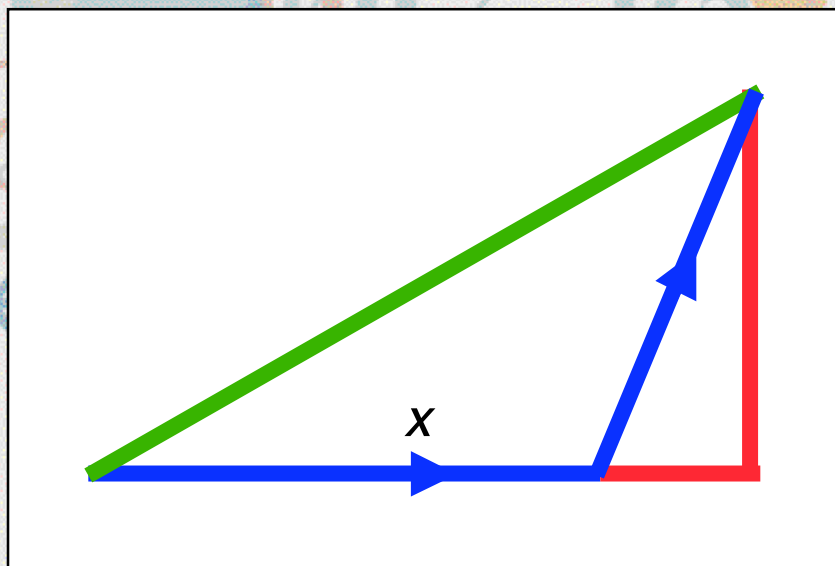
STANFORD UNIVERSITY

devlin@stanford.edu

Elvis: the dog that can do calculus

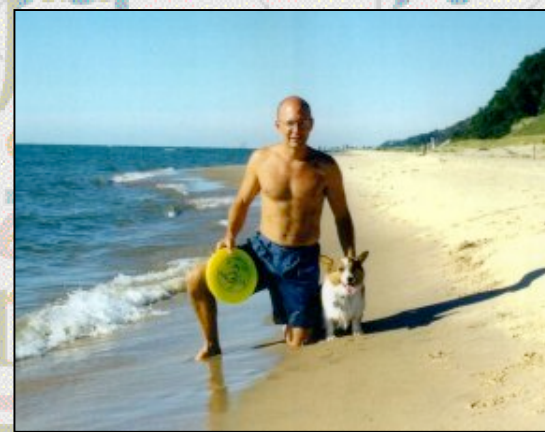


Elvis: the dog that can do calculus



May 2003 *College Mathematics Journal*

**Tim Pennings, Associate Professor
of Mathematics, Hope College, MI**



The Tunisian desert ant

Cataglyphis fortis



The Tunisian desert ant

Cataglyphis fortis



The Tunisian desert ant

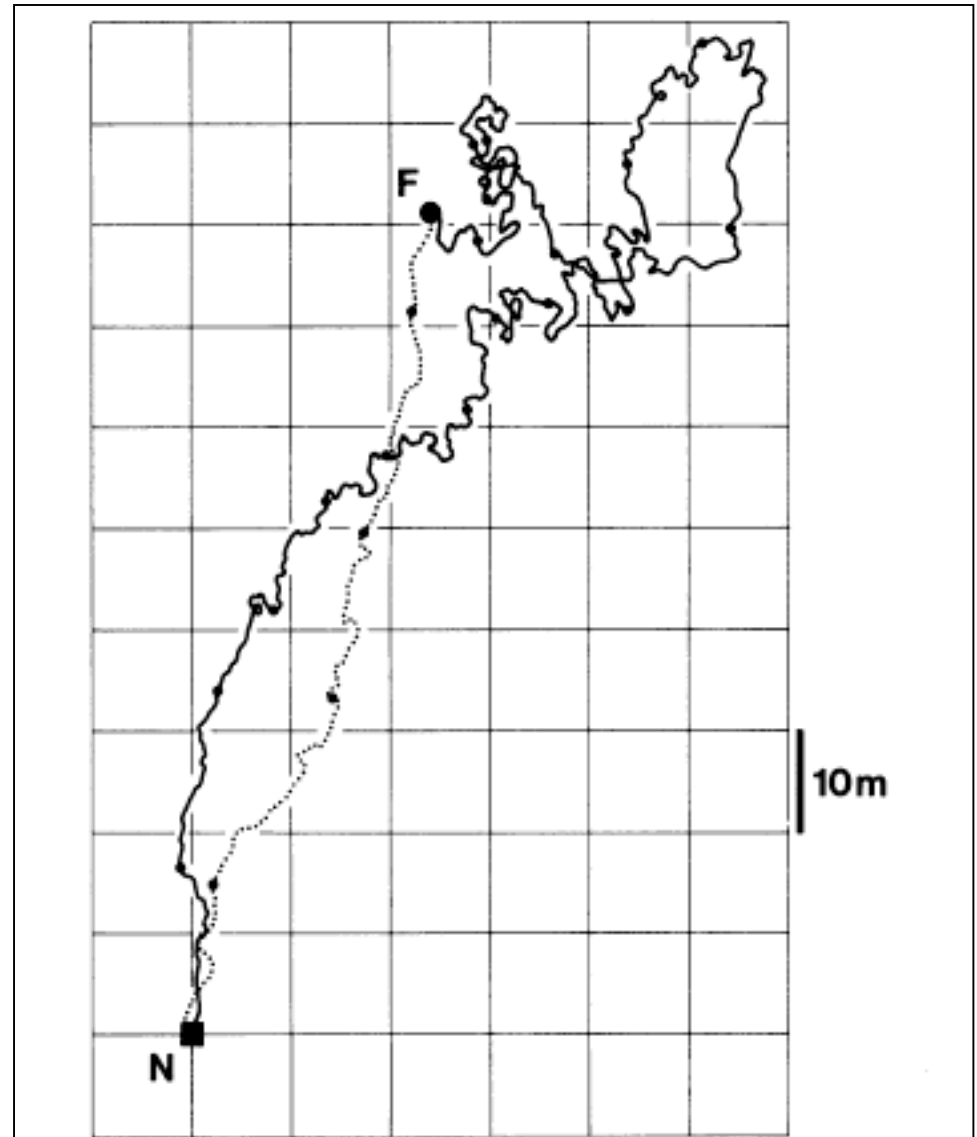
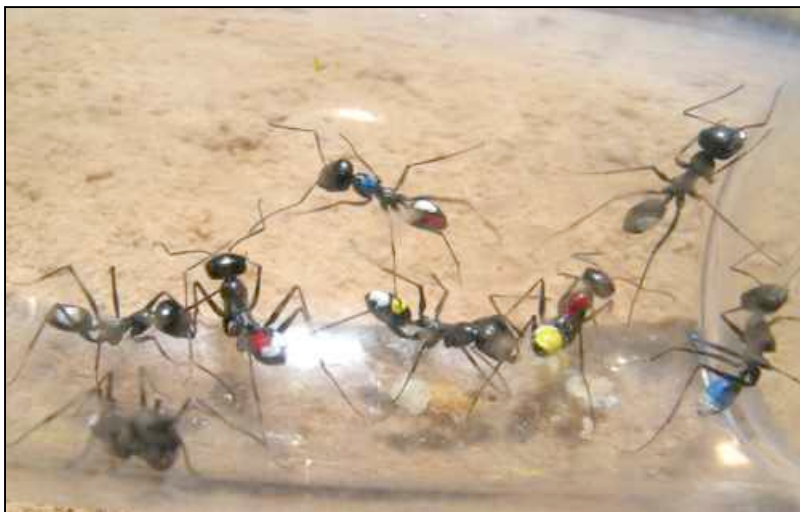


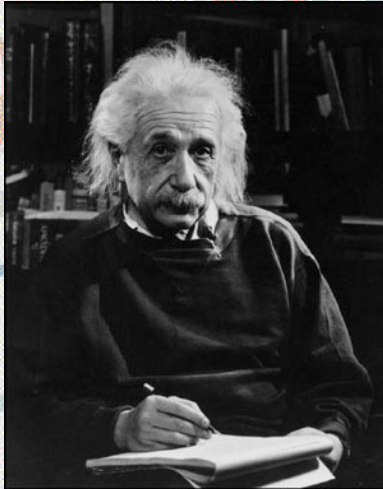
FIG. 1. Foraging trip of an individual ant, *Cataglyphis fortis*. Outbound and inbound trajectories are depicted by solid and stippled lines, respectively. *N*, nest; *F*, location of food item found by the searching ant. The length of the outbound path is 354.5 m; the maximal distance from the nest is 113.2 m. Time marks (small filled circles) are given every 60 s.

Migration

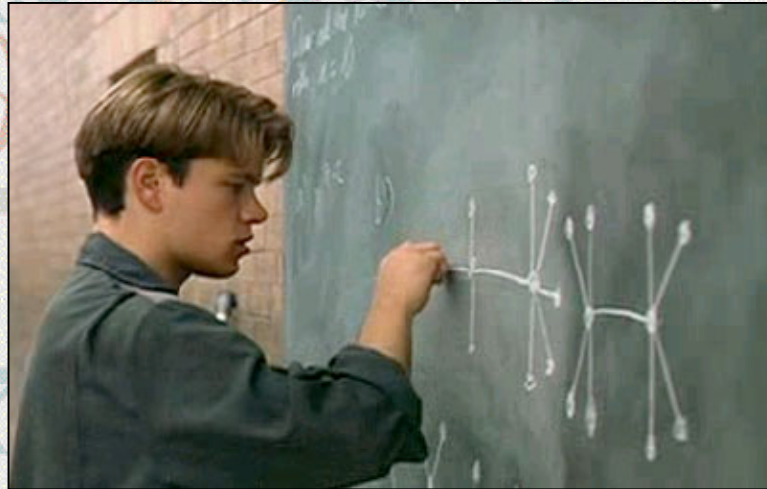
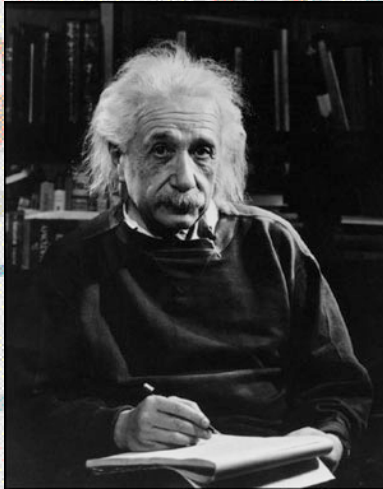


**But are they
REALLY
doing math?**

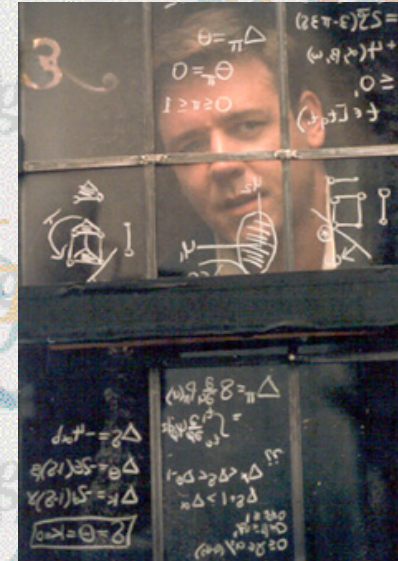
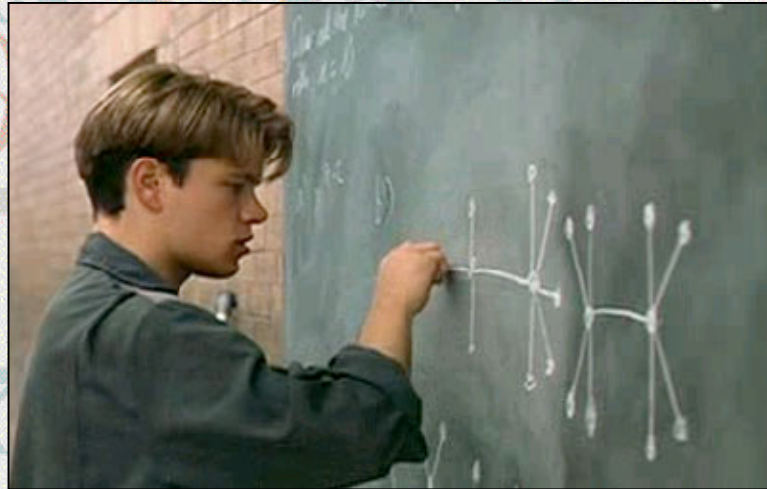
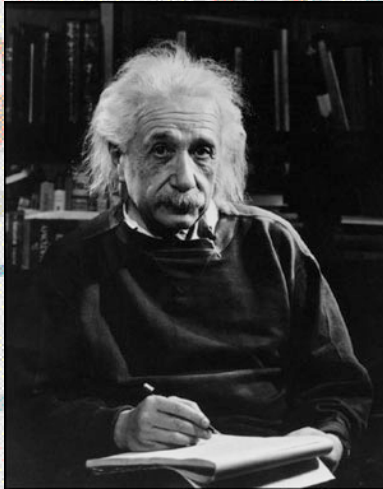
What does it mean to “do math”?



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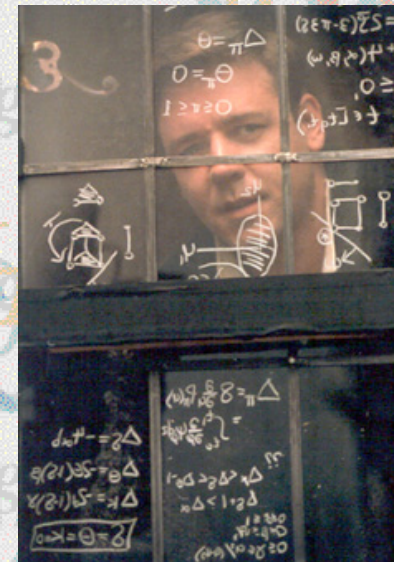
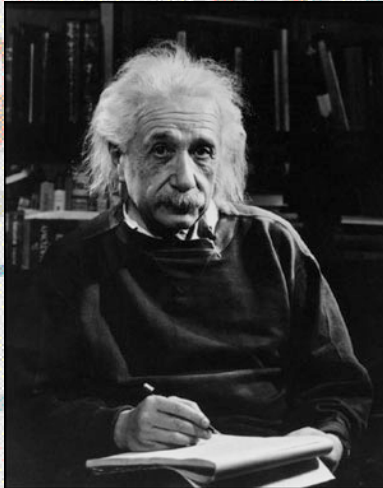
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IS THIS THE ONLY WAY TO DO MATH?



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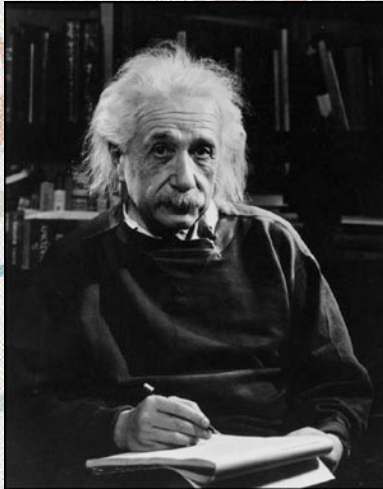


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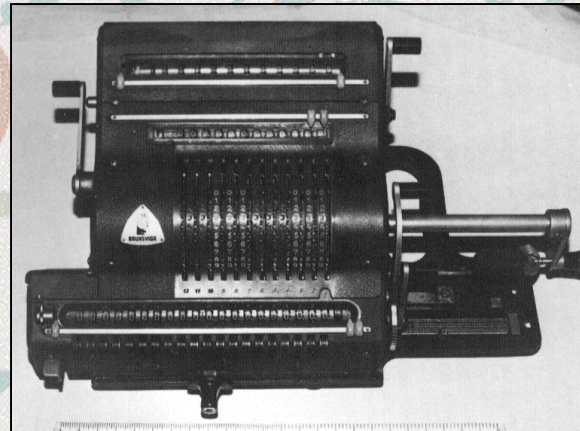
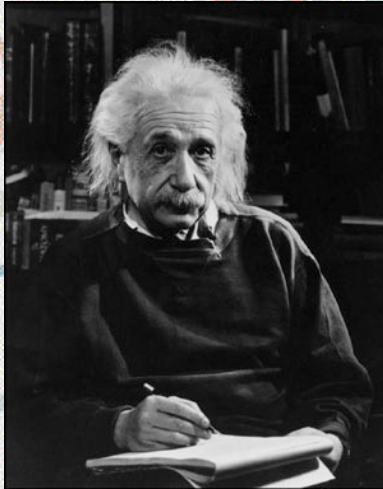
**CAN ONLY HUMANS
DO IT?**



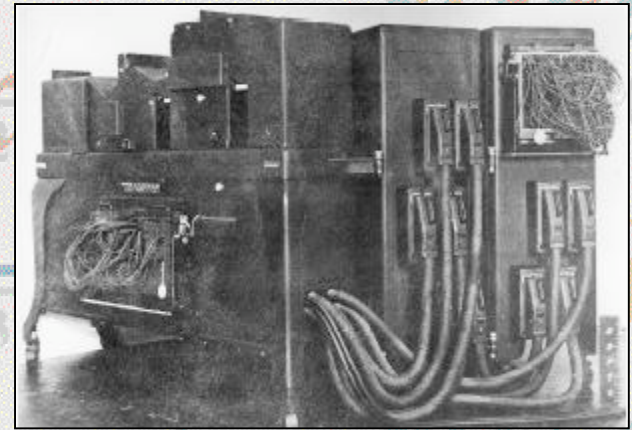
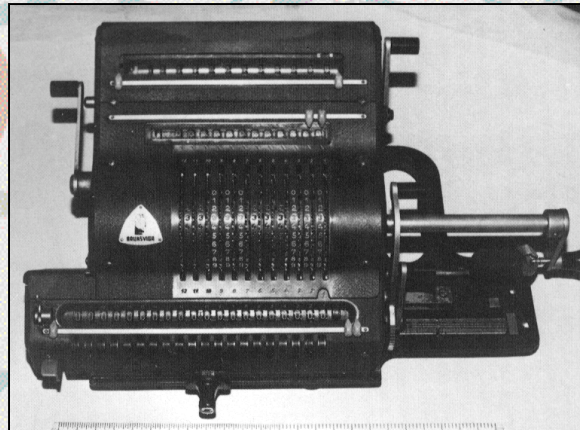
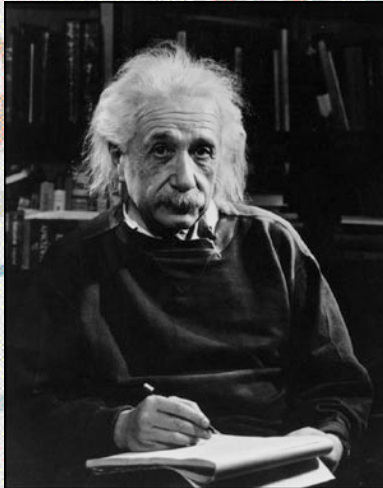
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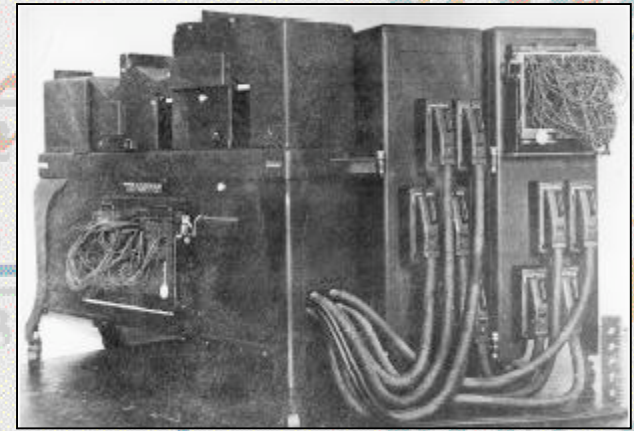
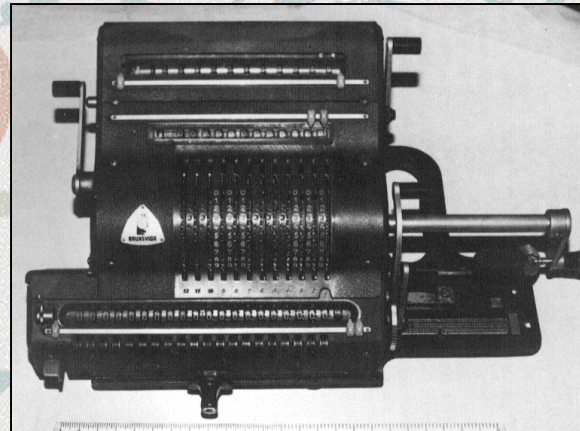
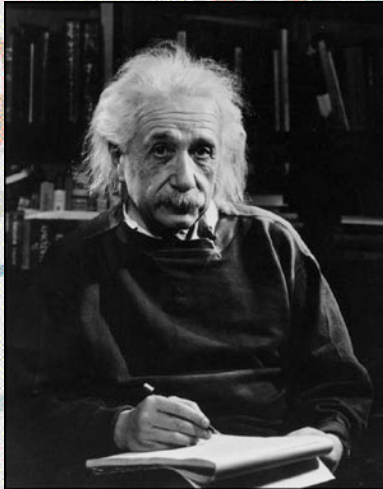
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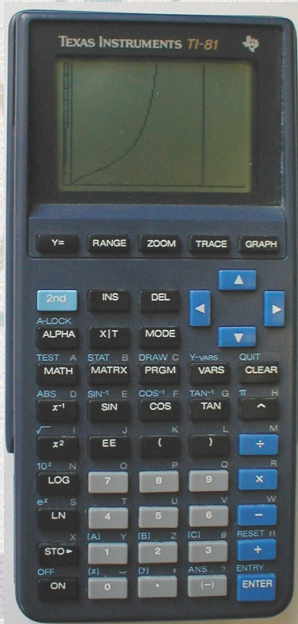
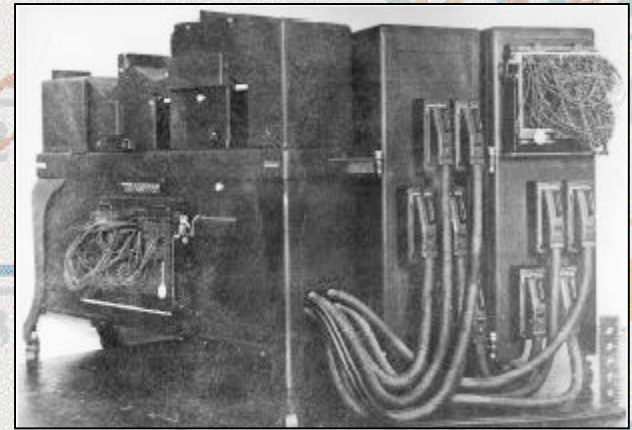
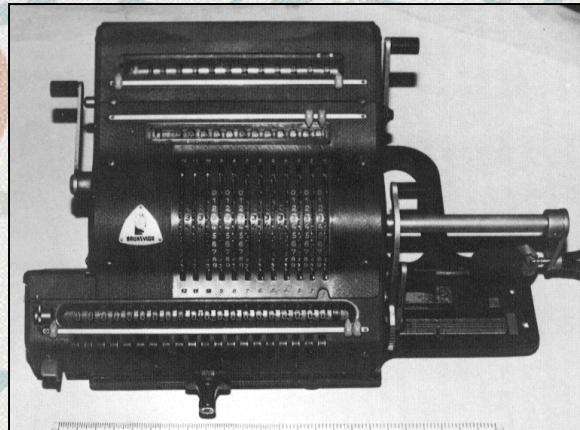
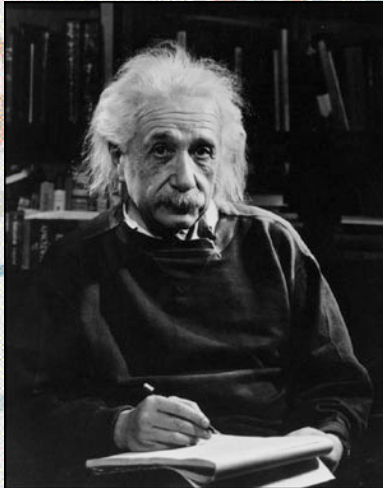
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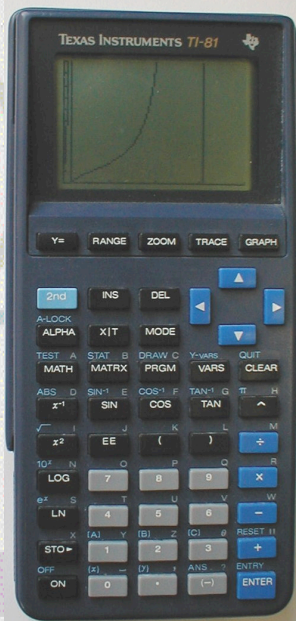
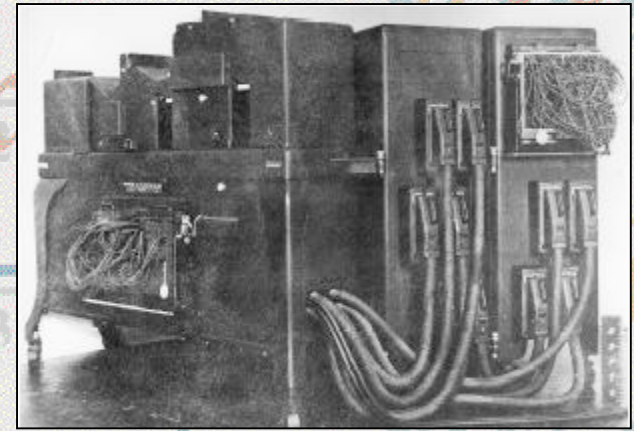
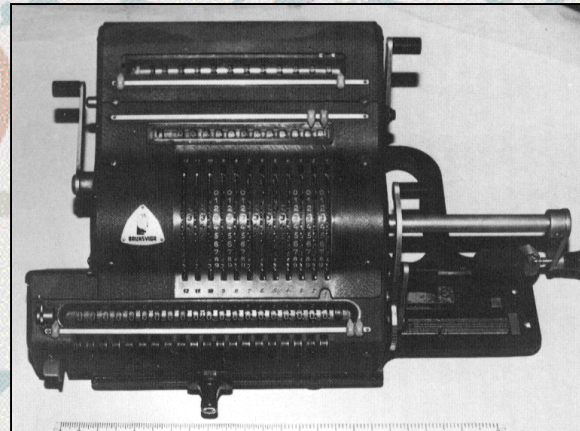
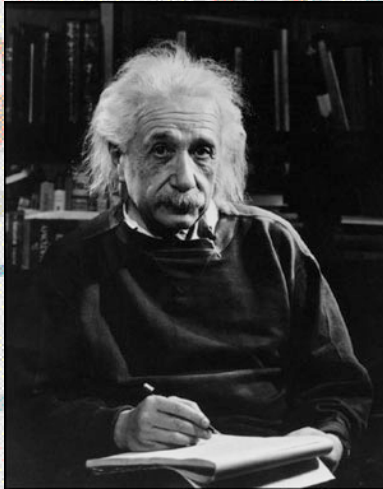
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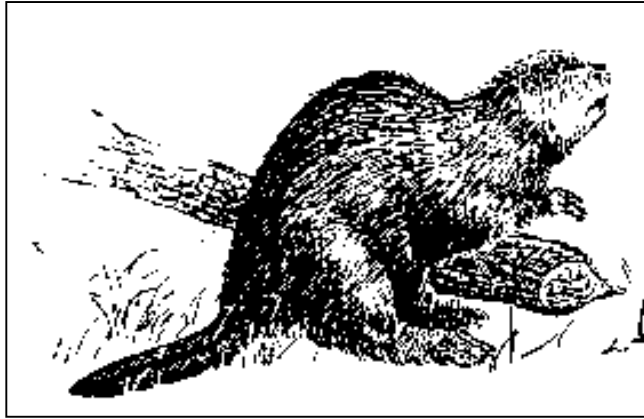


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A CAUTION

Beavers



**Busy, but maybe
not so bright**



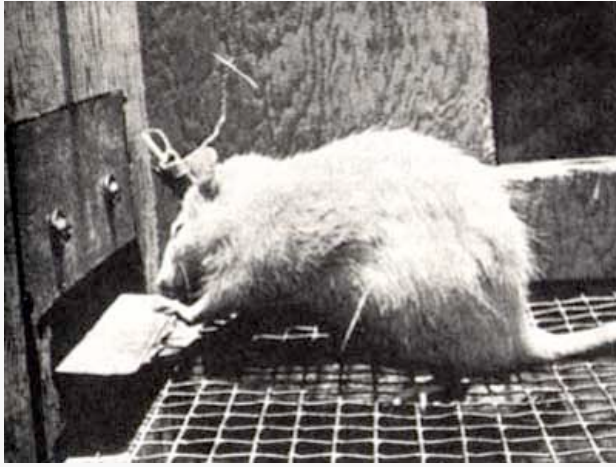
**What about explicit
number skills?**

Lion study: Karen McComb

Serengeti National Park in Tanzania.



Training animals to count



Rats: Francis Mechner

Raven: Otto Koehler (Jakob)



Parrot: Irene Pepperberg and Alex (African Grey)

Chimpanzees



Guy Woodruff and David Premack. Taught chimps fractions, using partially full glasses and pieces of an apple. Subjects could distinguish $1/2$, $1/4$, and $3/4$. This required lots of training. Without training, when offered trays with two piles of chocolates, they chose the tray with a pile of 3 and a pile of 4 over one with piles of 5 and 1. [$3 + 4 > 5 + 1$]

Sarah Boyson. Sheba chooses the card with the correct numeral 0, ..., 9, when shown objects on a tray. Sheba can also point to, say, 5 when shown the cards with 2 and 3.

Tetsuro Matsuzawa. Taught Ali to point to the correct numerals 0, ..., 9 when shown a number of objects on a tray. Ali also learned to order the numerals.



Children



Karen Wynn, 1992. Five-month old children know:

- $1 + 1 = 2$
- $1 + 2 = 3$
- $2 - 1 = 1$
- $3 - 1 = 2$
- $3 - 2 = 1$

Street Mathematics



Street Mathematics and School Mathematics, by Terezinha Nunes, Analucia Dias Schliemann, and David William Carraher, C.U.P. Recife, Brazil.

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Child asked to compute $243 - 75$.

“You just give me two hundred. I’ll give you twenty-five back. Plus the forty-three that you have, the hundred and forty-three, that’s one hundred and sixty-eight.”



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In other words, he changes a very difficult subtraction $243 - 75$ into a less difficult (though still challenging) addition $143 + 25$.