Lecture 2
Relationships between Categorical Variables

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DATASCI 112

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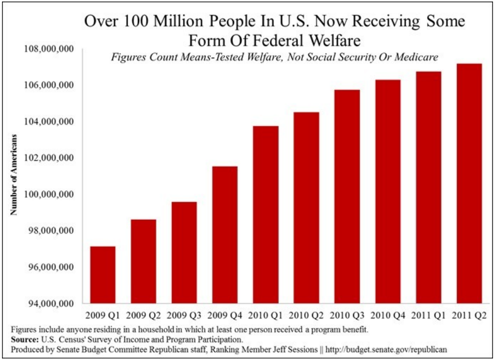


1. Review

2. Two (or More) Categorical Variables

3. Proportions and Probabilities

4. Joint and Conditional Distributions

Types of Data

Type	On Disk	In Python
tabular	CSV	`DataFrame`
hierarchical	JSON	`dict`
textual	plaintext	`str`
geospatial	???	???

Types of Variables

```python
import pandas as pd
df = pd.read_csv("https://datasci112.stanford.edu/data/titanic.csv")
df
```

variables

observational units

	name	pclass	survived	sex	age	sibsp	parch	ticket	fare	cabin	embarked	boat	body	home.dest
0	Allen, Miss. Elisabeth Walton	1	1	female	29.0000	0	0	24160	211.3375	B5	S	2	NaN	St Louis, MO
1	Allison, Master. Hudson Trevor	1	1	male	0.9167	1	2	113781	151.5500	C22 C26	S	11	NaN	Montreal, PQ / Chesterville, ON
2	Allison, Miss. Helen Loraine	1	0	female	2.0000	1	2	113781	151.5500	C22 C26	S	NaN	NaN	Montreal, PQ / Chesterville, ON
3	Allison, Mr. Hudson Joshua Creighton	1	0	male	30.0000	1	2	113781	151.5500	C22 C26	S	NaN	135.0	Montreal, PQ / Chesterville, ON
4	Allison, Mrs. Hudson J C (Bessie Waldo Daniels)	1	0	female	25.0000	1	2	113781	151.5500	C22 C26	S	NaN	NaN	Montreal, PQ / Chesterville, ON
...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1304	Zabour, Miss. Hileni	3	0	female	14.5000	1	0	2665	14.4542	NaN	C	NaN	328.0	NaN
1305	Zabour, Miss. Thamine	3	0	female	NaN	1	0	2665	14.4542	NaN	C	NaN	NaN	NaN
1306	Zakarian, Mr. Mapriededer	3	0	male	26.5000	0	0	2656	7.2250	NaN	C	NaN	304.0	NaN
1307	Zakarian, Mr. Ortin	3	0	male	27.0000	0	0	2670	7.2250	NaN	C	NaN	NaN	NaN
1308	Zimmerman, Mr. Leo	3	0	male	29.0000	0	0	315082	7.8750	NaN	S	NaN	NaN	NaN

1309 rows × 14 columns

quantitative variables

categorical variables

One Categorical Variable

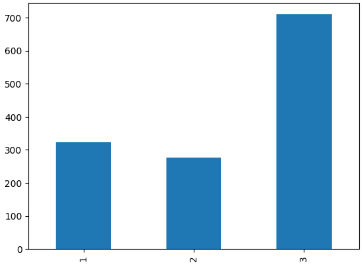
To *summarize* a categorical variable, we report the **counts** of each possible category.

```
df["pclass"].value_counts().sort_index()
```

```
1    323
2    277
3    709
Name: pclass, dtype: int64
```

To *visualize* a categorical variable, we make a **bar plot**.

```
df["pclass"].value_counts().sort_index().plot.bar()
```



Selecting Columns

Note that we selected a single column by passing the column name as a key to the `DataFrame`.

```
df["pclass"]
```

```
0       1
1       1
2       1
       ..
1306    3
1307    3
1308    3
Name: pclass, Length: 1309, dtype: int64
```

The result is a one-dimensional `pandas` object called a `Series`.
Both `DataFrame` and `Series` have a special *column* called `.index` which identifies the observational units.

We can select multiple columns by passing a `list` of column names.

```
df[["pclass", "survived"]]
```

	pclass	survived
0	1	1
1	1	1
2	1	0
3	1	0
4	1	0
...	...	...
1304	3	0
1305	3	0
1306	3	0
1307	3	0
1308	3	0

1309 rows × 2 columns

The result is two-dimensional, another smaller `DataFrame`.
You can see its size by the `.shape` attribute.

How do we make sense of multiple variables at once?

Summarizing Multiple Categorical Variables

To summarize multiple categorical variables, we report the **counts** of every possible combination of categories.

We can use the `.value_counts()` method of `DataFrame`.

```
df[["pclass", "survived"]].value_counts()
```

```
pclass  survived
3       0           528
1       1           200
3       1           181
2       0           158
1       0           123
2       1           119
dtype: int64
```

Note that the result is a `Series`, with a multi-level index, one for each variable!

Summarizing Multiple Categorical Variables

```
pclass  survived
3       0           528
1       1           200
3       1           181
2       0           158
1       0           123
2       1           119
dtype: int64
```

Let's make this information easier to read by arranging one variable along the rows and the other along the columns.

```
(df[["pclass", "survived"]].value_counts().
 unstack())
```

survived	0	1
pclass		
1	123	200
2	158	119
3	528	181

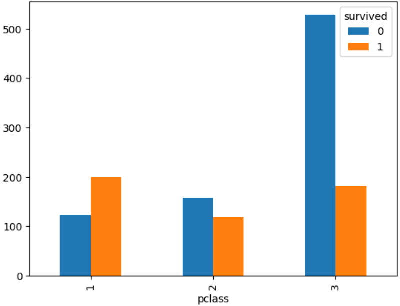
This representation is called a **two-way table** or a **crosstab** (short for "cross-tabulation").

Visualizing Multiple Categorical Variables

survived	0	1
pclass		
1	123	200
2	158	119
3	528	181

From a crosstab, we can make a bar plot to visualize the data.

```
(df[["pclass", "survived"]].value_counts().
 unstack().
 plot.bar())
```



This is called a **grouped bar plot**.

Marginal Counts

How do we recover the counts for each individual variable from a crosstab?

```
crosstab = df[["pclass", "survived"]].value_counts().unstack()
crosstab
```

survived	0	1
pclass		
1	123	200
2	158	119
3	528	181

We could sum over the columns (across each row) to obtain the counts for **pclass**...

```
crosstab.sum(axis="columns")
```

```
pclass
1    323
2    277
3    709
dtype: int64
```

Marginal Counts

How do we recover the counts for each individual variable from a crosstab?

```
crosstab = df[["pclass", "survived"]].value_counts().unstack()
crosstab
```

survived	0	1
pclass		
1	123	200
2	158	119
3	528	181

...or sum over the rows (down each column) to obtain the counts for **survived**.

```
crosstab.sum(axis="rows")
```

```
survived
0    809
1    500
dtype: int64
```

Proportions

Instead of counts, it can be useful to report **proportions**, where we normalize by the total.

$$\text{proportion} = \frac{\text{count}}{\text{total}}.$$

For example, the proportions of the three passenger classes are:

```
df["pclass"].value_counts() / df.shape[0]
```

```
3    0.541635
1    0.246753
2    0.211612
Name: pclass, dtype: float64
```

Notice that the values in a distribution add up to 1.0!

Together, the proportions of a categorical variable are called the **distribution** of the variable **pclass**.

Probabilities

What does it mean to say, "The proportion of passengers in 3rd class is 0.541635?"

One interpretation is as a **probability**.
"If we were to pick a passenger on the Titanic at random, the probability that they are in 3rd class is 0.541635."

We notate this as

$$P(\text{3rd class}) = 0.541635$$

or, if we want to be explicit about the variable and the category,

$$P(\textbf{pclass} = 3) = 0.541635$$

Vectorization

Let's take a closer look at the code for calculating the proportions.

```
df["pclass"].value_counts() / df.shape[0]
```

Notice that we divided a `Series` by a number! Is that even legal?

In `pandas`, operations are **vectorized** (sometimes referred to as **broadcasting**). A `Series` behaves like a vector.

Vectors in `pandas` work like vectors in math!

Math Review

To multiply a vector

$$\vec{v} = (v_1, v_2, ..., v_n)$$

by a scalar (a.k.a. number) a,

$$a\vec{v} = (av_1, av_2, ..., av_n),$$

we multiply each component of the vector by a.

Joint Distributions

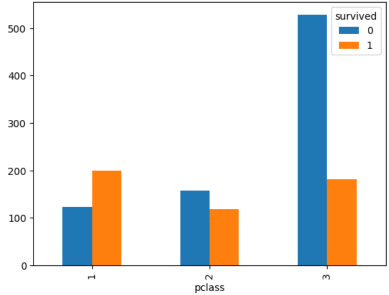
We can also calculate the distribution of multiple variables, called a **joint distribution**.

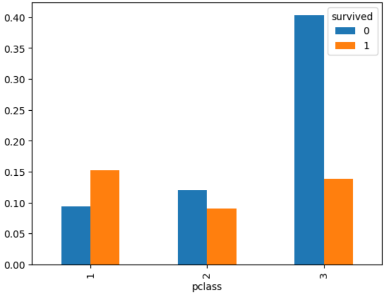
```
df[["pclass", "survived"]].value_counts().unstack() / df.shape[0]
```

survived	0	1
pclass		
1	0.093965	0.152788
2	0.120703	0.090909
3	0.403361	0.138273

Notice that the values in the joint distribution also sum to 1.0!

Visualizing Joint Distributions





How does the bar plot change if we plot the joint distribution instead of the counts?

The y-axis scale changes, but the shape is the same.

To appreciate the power of proportions, we need to look at conditional distributions.

Conditional Distributions

To compare survival across the classes, we should normalize by the total in each class.

survived	0	1
pclass		
1	123	200
2	158	119
3	528	181

```
pclass
1      323
2      277
3      709
dtype: int64
```

```
crosstab
```

```
pclass_marginal = crosstab.sum(axis="columns")
```

Next, we divide the crosstab by the total in each class.

```
crosstab.divide(pclass_marginal, axis="rows")
```

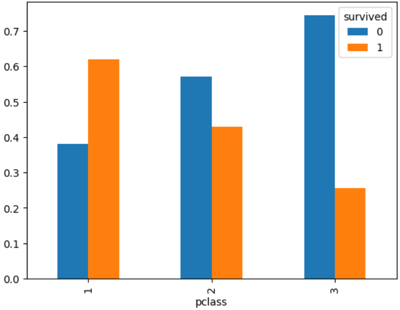
survived	0	1
pclass		
1	0.380805	0.619195
2	0.570397	0.429603
3	0.744711	0.255289

These are the **conditional distributions** of **survived** given **pclass**.

Visualizing Conditional Distributions

To visualize the conditional distributions, we could make a grouped bar plot...

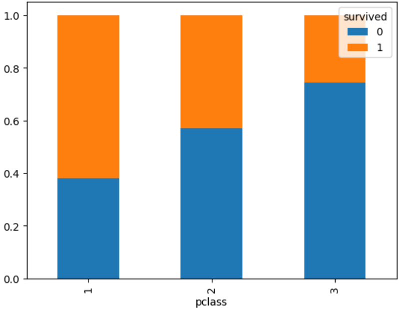
```
(crosstab.divide(pclass_marginal, axis="rows").
 plot.bar())
```



Visualizing Conditional Distributions

To visualize a conditional distribution, we could make a grouped bar plot...

```
(crosstab.divide(pclass_marginal, axis="rows").
 plot.bar(stacked=True))
```



...but it is better to make a **stacked bar plot**.

Conditional Probabilities

What does it mean to say, "The conditional proportion of survival given 3rd class is 0.255289"?

One interpretation is as a **conditional probability**.
"If we were to pick a 3rd class passenger on the Titanic at random, the probability that they survived is 0.255289."

We notate this as

$$P(\text{survived}|\text{3rd class}) = 0.255289$$

or, if we want to be explicit about the variable and the category,

$$P(\textbf{survived} = 1|\textbf{pclass} = 3) = 0.255289$$