

In applications it is often the case that descriptive analyses of the individual trajectories, rates of improvement, etc. are of the greatest substantive value, and Timepath97 provides extensive descriptive output. The topic here is to motivate the (subsequent) estimation of variance components and model parameters. Some key quantities, which are linked with generic *Research Questions*, are described below:

longitudinal research questions and parameters of interest

1. Individual and Group Growth. Description of the form and amount of change, estimation of the individual (or group) growth curve, heterogeneity (individual differences) in the individual growth curves, and the statistical and psychometric properties of these estimates.

Parameters: $f(\theta; t)$, μ_θ σ_θ^2 $\rho(\hat{\theta})$ $\rho_{\eta(t_I)\theta}$

a. The first two moments of the rate of change over the population of individuals, μ_θ and σ_θ^2 , which for the straight-line growth model address questions about typical rates of change and heterogeneity (individual differences) in rates of change.

b. The reliability of the growth curve estimates of θ_p , which for psychometric purposes addresses questions about accuracy of the estimates.

For the unbiased OLS estimate of θ_p , the reliability is denoted as $\rho(\hat{\theta})$.

c. The correlation between change θ_p and true initial status $\eta_p(t_I)$, where t_I indicates a designated time of initial status.

The correlation $\rho_{\eta(t_I)\theta}$ is used to investigate whether those with lowest initial status make the most progress (negative value) or those with the highest initial status make the most progress (positive value). As discussed in Rogosa and Willett (1985), the choice of t_I is of critical importance because $\rho_{\eta(t)\theta}$ is functionally dependent on time.

One omission in this listing is the estimation of the individual θ_p beyond simple OLS. Methodology for improvements upon the unbiased estimate ($\hat{\theta}_p$) has a prominent history and central focus in empirical Bayes methodology (e.g., Morris, 1983) and can be incorporated into the computational program.

2. Correlates and Predictors of Change. systematic individual differences in growth: "What kind of persons learn (grow) fastest?".

Parameters: $\rho_{\theta W}$ $\beta_{\theta W}$

The exogenous variable regression parameter $\beta_{\theta W}$ in (2) and the standard error of its estimate. $\beta_{\theta W}$ is taken to represent the "influence" of W (on θ) and is often of primary interest in applications.

3. Stability over Time.

Questions about temporal stability fall into two broad headings-- Is an individual consistent over time? and Are individual differences consistent over time? (Rogosa, Willett, and Floden 1984) .

consistency of individual differences over time. The question, "Are individual differences consistent over time?" addressed by index of tracking γ proposed by Foulkes and Davis (1981): γ is used to assess consistency of individual differences over a specified time interval. The tracking index estimates the probability that two randomly chosen individual's trajectories do not cross in the time interval specified. "*tracking*" if index $> .50$ (significantly).

Parameter: Foulkes-Davis $\gamma = \text{Pr}(\text{two growth curves do not intersect})$
Estimation. Fit individual trajectories (straight-line or polynomial etc). For each individual compute the proportion of other trajectories not crossed. Point estimate is the average over individuals of these proportions. Standard error of parameter estimate from a jackknife approximation (Foulkes-Davis, 1981 p.441 use standard deviation of individual estimates divided by $\text{Sqrt}[n]$ as the standard error and construct normal theory CI) or from bootstrap resampling.

Common Claims:

- *everyone changes at the same rate (people interchangeable)
- *change can't be measured reliably/accurately
- *correlation of change and initial status is negative; regression toward mean pertains etc

Other research questions

4. Comparing Experimental Groups.
5. Comparing Nonexperimental Groups.
(note: Dental, Wisc compare intact groups via W code)
6. Analysis of Reciprocal Effects.
7. Growth in Multiple Measures.