EE102B Spring 2018-19 Pauly
Signal Processing and Linear Systems II

Review Problems

Discussed in Class June 3
Do not hand in

1. IIR Filter Design
   Assume we have a continuous time filter,

   $$h_c(t) = \frac{1}{\tau} e^{-t/\tau}$$

   which has a Laplace transform

   $$H_c(s) = \frac{1}{1 + s\tau}$$

   First use the impulse invariant approach to design a discrete time filter.

   (a) Solve for the impulse response $h[n]$. Let $a = e^{-T/\tau}$ for convenience.
   (b) Find the transfer function $H(z)$
   (c) Draw the pole-zero plot. Assume $\tau = 2$ and $T = 1$, so that $e^{-T/\tau} \approx 0.61$.
   (d) Sketch the frequency response. Label values at $\omega T = 0$ and π.

   Repeat the design using the bilinear transform.

   $$s = \frac{2}{T} \frac{1 - z^{-1}}{1 + z^{-1}}$$

   For convenience let

   $$b = \frac{T - 2\tau}{T + 2\tau}$$

   (e) Find the transfer function $H(z)$.
   (f) Find the impulse response $h[n]$
   (g) Draw the pole-zero plot, again assuming $\tau = 2$ and $T = 1$, so that $b = -0.6$.
   (h) Sketch the frequency response. Label values at $\omega T = 0$ and π.

2. *Solving Difference Equations*
   A discrete time system has the following difference equation

   $$y[n] - 2y[n-1] - 3y[n-2] = x[n]$$

   where $y[-1] = 1$, $y[-2] = 1$, and

   $$x[n] = u[n]$$

   Find the solution $y[n]$

3. *z-Transforms*
   Find the z-transform of
$$x[n] = \left(\frac{4}{3}\right)^n u[n] + \left(\frac{1}{2}\right)^n u[-n]$$
   and find the region of convergence. If it does not exist, explain why. A sketch of the $x[n]$ may be helpful.

4. *Step Response Equalization*

   A system has an impulse response
$$h[n] = 12\left(\left(-\frac{1}{3}\right)^n - \left(-\frac{1}{2}\right)^n\right) u[n]$$

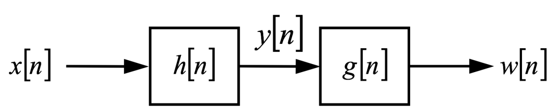
   (a) Find $H(z)$.

   (b) Sketch the pole-zero diagram.

   (c) Is this system stable? If so sketch the magnitude of the frequency response $H(e^{j\Omega})$.

   (d) Find the step response of this system
$$y_s[n] = h[n] * u[n]$$

   (e) This step response shows significant overshoot. We can fix this by cascading another filter, and allowing the output to be a delayed unit step $u[n-1]$. The system now looks like this



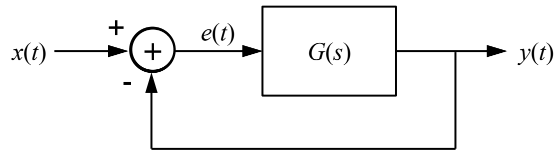
   The output is now
$$w[n] = x[n] * h[n] * g[n]$$
   Assume $g[n]$ is described by the difference equation
$$w[n] = a\, y[n] + b\, y[n-1] + c\, y[n-2]$$
   Find a, b, and c so that when $x[n] = u[n]$, then $w[n] = u[n-1]$
$$w_s[n] = u[n] * h[n] * g[n] = u[n-1]$$

   (f) What is the impulse response and transfer function for the complete system?

The basic idea is that by allowing for a delay in the output, we can design a compensation filter that corrects for the deficits of a system. This is widely used in practice.

5. *Continuous Time Tracking Filters*

The following system is designed to track the input.



Ideally we would like the error $e(t)$ to go to zero. The tracking filter we use depends on what sort of signals we would like to track with zero error. In this case we are going to assume an input of

$$x(t) = t\,u(t)$$

which is a ramp.

(a) Find the Laplace transform of the *error*, $E(s)$. You can think of this as a feedback system with input $X(s)$, a unity forward system, and a feedback system $G(s)$. You can also find the difference equation from the block diagram, take the Laplace transform, and solve for $E(s)$.

(b) Assume $G(s) = \frac{1}{s}$. Find the error $e(t)$. Does the error go to zero as $t \to \infty$?

(c) Now try $G(s) = \frac{1}{s^2}$

(d) Finally, try $G(s) = \frac{2s+1}{s^2}$