Problem Set #3
Due: Friday April 27, 2018 at 5 PM.

Plot sketches must be done manually (Matlab plots are not sketches so will not be accepted). The reason is that manual plots are meant to help your intuition. So please first attempt plot sketches manually without the help of Matlab. If you are stuck you can use Matlab to understand what the plot will look like before sketching it.

1. Bandpass sampling

(Adapted from OWN, Prob. 7.26) The sampling theorem states that a signal $x(t)$ must be sampled at a rate greater than its bandwidth (or, equivalently, a rate greater than twice its highest frequency). This implies that if $x(t)$ has a spectrum as indicated in the figure then $x(t)$ must be sampled at a rate greater than $2W_2$. Since the signal has most of its energy concentrated in a narrow band, it seems reasonable to expect that a sampling rate lower than twice the highest frequency could be used. A signal whose energy is concentrated in a frequency band is often referred to as a bandpass signal. There are a variety of techniques for sampling such signals, and these techniques are generally referred to as bandpass sampling. In this problem we investigate one such method.

Let $x(t)$ be a CT signal with CTFT $X(j\omega)$ given in the figure. Assume that $x(t)$ is sampled at rate $\omega_s = 2\pi/T_s$ by multiplying it by a delta train

$$x_p(t) = x(t) \sum_{n=-\infty}^{\infty} \delta(t - nT_s).$$

The signal $x_p(t)$ is then passed through a filter with frequency response $H(j\omega)$ given in the figure. Denote the output of the filter by $x_r(t)$.

![Figure 1: Spectrum of $x(t)$](image1)

![Figure 2: Filter frequency response](image2)
(a) Assuming that $W_1 > W_2 - W_1$, find the maximum value of $T_s$, and corresponding values of $A$, $L_1$ and $L_2$ such that $x_r(t) = x(t)$.

(b) Sketch a plot of $X_p(j\omega)$ for your given minimum $T_s$, with $W_1 = 3$ and $W_2 = 3.8$

2. **Discrete Time Quadrature Multiplexing**

(Adapted from OWN 8.41) In class we introduced the concept of quadrature multiplexing, where two signals are modulated by the same carrier frequency and sent at the same time along the sine and cosine waves (at a phase difference of $90^\circ$). This problem demonstrates the same process in discrete time. Figure 1a and 1b below show a discrete time multiplexer and demultiplexer. The signals $x_1[n]$ and $x_2[n]$ are both assumed to be band limited to $\Omega_m$, so that:

$$X_1(e^{j\Omega}) = X_2(e^{j\Omega}) = 0, \Omega_m < \Omega < 2\pi - \Omega_m$$

a) Determine the range of values for $\omega_c$ for which $x_1[n]$ and $x_2[n]$ can be recovered from $r[n]$. (8 pts)

b) With $\omega_c$ satisfying the conditions in part (a), determine and draw $H(e^{j\Omega})$ that guarantees that $y_1[n] = x_1[n]$ and $y_2[n] = x_2[n]$. (7 pts)
3. Quadrature Modulation and Complex Baseband Representations (20 pts)

Consider a quadrature modulated signal \( s(t) = m_1(t) \cos(\omega_c t) + m_2(t) \sin(\omega_c t) \) for both analog communications \((m_1(t) \text{ and } m_2(t) \text{ analog signals})\) and digital communications \((m_1(t) \text{ and } m_2(t) \text{ baseband digital signals})\). The receiver for the quadrature system in analog case is as shown in the figures below. Note that for this problem you may find the following trig identities useful: \( \cos(a) \cos(b) = \frac{1}{2}(\cos(a+b) + \cos(a-b)) \) and \( \cos(a) \sin(b) = \frac{1}{2}(\sin(a+b) - \sin(a-b)) \)

a) Quadrature-modulated signals are often represented in terms of their in-phase and quadrature signal components. In this representation we write \( s(t) = s_I(t) \cos(\omega_c t) - s_Q(t) \sin(\omega_c t) \), where \( s_I(t) \) is called the in-phase component of \( s(t) \) and \( s_Q(t) \) is called its quadrature component. We can alternatively write \( s(t) = \Re \{ v(t) e^{j\omega_c t} \} \) where \( v(t) \) is called the complex baseband representation of \( s(t) \). Find \( s_I(t) \), \( s_Q(t) \), and \( v(t) \) in terms of \( m_1(t) \) and \( m_2(t) \) for \( s(t) = m_1(t) \cos(\omega_c t) + m_2(t) \sin(\omega_c t) \)

b) For the analog system with your demodulator shown below, determine the branch outputs when the receiver phase \( \phi \neq 0 \) as a function of \( \phi \). You will see that the output of each branch depends on both signal components \( s_I(t) \) and \( s_Q(t) \), i.e. the quadrature component is an interfering signal to the in-phase branch, and the in-phase component is an interfering signal to the quadrature branch. We define the signal-to-interference power ratio (SIR) as the ratio of the power in the desired signal over the power in the interference, i.e. for the in-phase branch \( \text{SIR} = P_{s_I}/P_{s_Q} \), where \( P_{s_I} \) is the power in the in-phase signal component and \( P_{s_Q} \) the power in the interfering quadrature signal, and similarly for the quadrature branch. Find an expression for the SIR in the in-phase and quadrature branch with a receiver phase offset \( \phi \) as a function of \( \phi \), and evaluate its dB value assuming \( \phi = \pi/8 \) and the power in \( s_I(t) \) and in \( s_Q(t) \) is 100 mW.
4. *Frequency synchronization in amplitude demodulation*  

*(Taken from OWN, Prob. 8.23)* Consider the amplitude modulation and demodulation systems given by

\[
\begin{align*}
y(t) &= x(t) \cos(\omega_c t) \\
w(t) &= y(t) \cos(\omega_d t) \\
\hat{x}(t) &= w(t) \ast h(t).
\end{align*}
\]

Let us denote the difference in frequency between the modulator and demodulator as \(\Delta \omega\) (i.e., \(\omega_d - \omega_c = \Delta \omega\)). Also, assume that \(x(t)\) is band limited with \(X(j\omega) = 0\) for \(|\omega| \geq \omega_M\), and assume that the cutoff frequency \(w_{co}\) of the lowpass filter (with impulse response \(h(t)\) whose transform is indicated in the figure below) in the demodulator satisfies the inequality

\[
\omega_M + \Delta \omega < w_{co} < 2\omega_c + \Delta \omega - \omega_M.
\]

(a) Show that the output \(\hat{x}(t)\) of the lowpass filter in the demodulator is proportional to \(x(t) \cos(\Delta \omega t)\).  

(b) If the spectrum of \(x(t)\) is as shown below, sketch the spectrum of the output \(\hat{x}(t)\) of the demodulator.
5. **Extra credit: Single sideband modulation**

*(Adapted from OWN, Prob. 8.28)* Figure 5 shows a system required to retain the upper sidebands for performing single sideband sinusoidal amplitude modulation in which the upper sidebands are retained.

With the $X(j\omega)$ illustrated in figure 6, sketch $Y_1(j\omega)$, $Y_2(j\omega)$ and $Y(j\omega)$ for the system in the figure and demonstrate that only the upper sidebands are retained.

![SSB Modulator Diagram](image)

Figure 5: SSB modulator

![X(j\omega) Spectrum](image)

Figure 6: $X(j\omega)$
MATLAB Assignment

General Instructions
Answer all questions asked. Your submission should include all m-file listings and plots requested. All plots should have a title and x- and y-axes properly labeled.

Amplitude Demodulation (25 pts)

(Taken from BDS) This project explores some of the issues involved in the sinusoidal amplitude modulation and demodulation of speech signals, including the synchronization between the transmitter and receiver. The general form for a sinusoidal AM signal is given by

\[ y(t) = x(t) \cos(\omega_c t), \]

where \(x(t)\) is called the message signal and \(\omega_c\) is the carrier frequency of the AM signal. At the receiver, the message signal \(x(t)\) can be recovered through a technique called synchronous AM demodulation via

\[ w(t) = y(t) \cos(\omega_c t), \]
\[ = x(t) \cos^2(\omega_c t), \]
\[ = 0.5x(t)(1 + \cos(2\omega_c t)). \]

To recover \(x(t)\), the signal \(w(t)\) can be lowpass filtered to eliminate the component of the spectrum centered about \(2\omega_c\). A potential difficulty in AM systems of this form is that the receiver must have a local oscillator that is synchronized with the transmitter. If there is a phase difference between the transmitter and receiver oscillators, then some demodulation degradation can occur. For example, if the carrier signal is \(\cos(\omega_c t)\) and the demodulator signal is \(\cos(\omega_c t + \phi)\), then \(w(t)\) will be

\[ w(t) = x(t) \cos(\omega_c t) \cos(\omega_c t + \phi), \]
\[ = x(t)(\cos(\phi) + \cos(2\omega_c t + \phi)). \]

Preparation: The problems use origbl.mat, which is contained in the Computer Explorations Toolbox: [http://bit.ly/1Q2tg6T](http://bit.ly/1Q2tg6T) or [http://ee102b.stanford.edu/contents/buck.zip](http://ee102b.stanford.edu/contents/buck.zip). This file can be loaded by typing load origbl. Since the speech is sampled at \(f_s = 8192\) Hz, you will use a set of time samples at this rate to simulate continuous-time signals, i.e., \(t = [0:1/f_s:(N-1)/f_s]\), where \(N = \text{length}(x)\). Verify that you have loaded the speech signal correctly by executing the command \(\text{sound}(x, 8192)\). You should hear a slightly muffled phrase “line up”. The speech may sound muffled to you since it has been bandlimited to 1000 Hz.

Questions:

(a) Use \texttt{fft} to compute 8192 samples of the CTFT of \(x\) and store your results in \(X\). Make an appropriately labeled plot of the magnitude of the CTFT of \(x\) to verify that it is indeed bandlimited to about 1000 Hz.

(6 pts)
(b) Assuming that \( A = 0 \), amplitude modulate the speech signal in \( x \) at a carrier frequency of \( f_c = 1500 \) Hz and store the modulated speech in the vector \( y \). Make an appropriately labeled plot of the magnitude of the CTFT of \( y \).

(6 pts)

(c) Assuming that the demodulator has a perfectly synchronized carrier signal at 1500 Hz, create the demodulated signal \( w(t) = y(t) \cos(\omega_c t) \) and store the result in the vector \( w \). Use \( \text{fft} \) to compute 8192 samples of the CTFT of \( w \) and make an appropriately labeled plot of its CTFT magnitude.

(6 pts)

(d) Analytically determine the impulse response \( h(t) \) of an ideal lowpass filter with cut-off frequency 1500 Hz. Store in a row vector \( h \) the first 41 samples of \( h(t - T) \), for \( T = 40 / f_s / 2 \). Multiply your samples by a hamming window of length 41 by executing \( h = h .* \text{hamming}(41) \)’. Plot the impulse response \( h \) versus \( t(1:41) \). Make an appropriately labeled plot of the CTFT magnitude of \( h \) by executing

\[
\begin{align*}
>> H &= \text{fftshift}(\text{fft}(h, 8192)); \\
>> df &= f_s / (8192); \\
>> \text{plot}([-f_s/2:df:f_s/2-df], \text{abs}(H))
\end{align*}
\]

(7 pts)

(e) **Extra credit:** Filter the signal \( w \) with the impulse response stored in \( h \) using \( \text{filter} \) and store the result in the vector \( z \). Use \( \text{fft} \) to compute 8192 samples of the CTFT and make an appropriately labeled plot of the CTFT magnitude of \( z \). Verify that you have properly demodulated the speech signal by playing it using \( \text{sound} \).

(10 pts)