Lecture 12 Outline:
Quadrature Modulation
FIR Filter Design

Announcements:
- **Reading**: “4: FIR Discrete-Time Filters” pp. 7-18, Optional reading on QAM Modulation: pp. 16-22 of posted lecture slides on Digital Modulation
- Midterm announcements next week: MT likely to be Thurs. May 5 evening and will cover material through FIR Filter Design

Review of Last Lecture

Quadrature Modulation: MQAM

Introduction to FIR Filter Design

Impulse Response Matching

Frequency Response Matching
Digital communication systems modulate bits onto baseband signal \( m(t) \), then use passband modulation on \( m(t) \)
- May also preprocess bits with compression and coding
- Analog information signals converted to bits via ADC

- **Baseband digital modulation**: ON-OFF \((a_k = A, 0)\), POLAR \((a_k = \pm A)\)
  \[
m(t) = \sum_{k=-\infty}^{\infty} a_k \text{rect}(t - kT_b) = x(t) \ast \text{rect}(t) \quad \text{for} \quad x(t) = \sum_{k=-\infty}^{\infty} a_k \delta(t - kT_b)
\]

- **Passband digital modulation for ASK/PSK is a special case of DSBSC**: 
  \[
s(t) = \sum m(t) \cos(\omega_c t + \varphi)
\]

- **Demodulation** downconverts, integrates, and then uses a decision device to determine if a “1” or “0” was sent
  - Downconversion must be coherent; requires acquisition of phase \( \phi \)
- Noise can cause decision device to output an erroneous bit
Quadrature Digital Modulation: MQAM

- Sends different bit streams on the sine and cosine carriers
- Baseband modulated signals often has \( L > 2 \) levels, with \( L = 2^l \)
  - More levels for the same TX power leads to smaller noise immunity and hence higher error probability
  - Has \( M = L^2 \) possible values for \((m_1(kT_s), m_2(kT_s))\)→log_2M bits per symbol time \( T_s \), Data rate is \( \log_2M / T_s \) bps; called MQAM modulation
- 10 Gbps WiFi has 1024-QAM (10 bits / \( T_s \))

\[
\begin{align*}
  m_i(t) &= 10, 11, 00, 01 \\
  A &= 10, 11, 00, 01 \\
  A/3 &= -A/3, -A \\
  A/3 &= -A/3, -A \\
  T_s &= \text{symbol time} \\
  m_1(t)\cos(\omega_c t) + m_2(t)\sin(\omega_c t) + n(t) \\
  \cos(\omega_c t) \\
  -90^\circ \\
  \sin(\omega_c t) \\
  \text{Data rate: } \log_2M \text{ bits} / T_s \\
  T_s \text{ is called the symbol time}
\end{align*}
\]
Introduction to FIR Filter Design

- Signal processing today done digitally
  - Cheaper, more reliable, more energy-efficient, smaller

- Discrete time filters in practice must have a finite impulse response: $h[n]=0, \ |n| > M/2$
  - Otherwise processing takes infinite time

- FIR filter design typically entails approximating an ideal (IIR) filter with an FIR filter
  - Ideal filters include low-pass, bandpass, high-pass
  - Might also use to approximate continuous-time filter

- We focus on two approximation methods
  - Impulse response and filter response matching
  - Both lead to the same filter design
Impulse Response Matching

- Given a desired (noncausal, IIR) filter response $h_d[n]$
  \[ h_d[n] \leftrightarrow H_d(e^{j\Omega}) \]

- Objective: Find FIR approximation $h_a[n]$: $h_a[n]=0$ for $|n|>M/2$ to minimize error of time impulse response
  \[
  \varepsilon = \sum_{n=-\infty}^{\infty} |h_d[n] - h_a[n]|^2 = \sum_{|n|\leq\frac{M}{2}} |h_d[n] - h_a[n]|^2 + \sum_{|n|>\frac{M}{2}} |h_d[n]|^2, \text{ since } h_a[n] = 0, |n| > M / 2
  \]
  Doesn’t depend on $h_a[n]$

- By inspection, optimal (noncausal) approximation is
  \[
  h_a[n] = \begin{cases} 
  h_d[n] & |n| \leq M / 2 \\
  0 & |n| > M / 2 
  \end{cases}
  \]
  Exhibits Gibbs phenomenon from sharp time-windowing
Frequency Response Matching

- Given a desired frequency response $H_d(e^{j\Omega})$

- Objective: Find FIR approximation $h_a[n]$: $h_a[n]=0$ for $|n|>M/2$ that minimizes error of freq. response

$$\varepsilon = \frac{1}{2\pi} \int_{-\pi}^{\pi} \left| H_d(e^{j\Omega}) - H_a(e^{j\Omega}) \right|^2 d\Omega$$

- Set $x[n] = h_d[n] - h_a[n]$ and $X(e^{j\Omega}) = H_d(e^{j\Omega}) - H_a(e^{j\Omega})$

- By Parseval’s identity: $\sum_{n=-\infty}^{\infty} |x[n]|^2 = \frac{1}{2\pi} \int_{-\pi}^{\pi} |X(e^{j\Omega})|^2 d\Omega$

  - Time-domain error and frequency-domain error equal
  - Optimal filter same as in impulse response matching

$$h_a[n] = \begin{cases} h_d[n] & |n| \leq M/2 \\ 0 & |n| > M/2 \end{cases}$$
Main Points

- MQAM modulation sends independent bit streams on cosine and sine carriers where baseband signals have $L = \sqrt{M}$ levels
  - Leads to data rates of $M/T_s$ bps (can be very high)

- FIR filter design entails approximating an ideal discrete or continuous filter with a discrete filter of finite duration

- Impulse response and frequency response matching minimizes time/frequency domain error; have same noncausal design
  - Optimal filter has $M+1$ of original discrete-time impulse response values
  - Sharp windowing causes “Gibbs” phenomenon (wiggles)

- Will refine design to make it causal via a delay and will use smooth windowing to mitigate Gibbs phenomenon