Final Examination

Wednesday, March 21, 2018, 3:30 – 6:30 p.m.

Your exam will be scanned for grading, so please write your answers only on the front side of the paper, and only in the area provided for each part. If you must use additional space for a part of a problem, note that in the area provided for that part, and continue writing on one of the blank pages provided at the end.

This exam is open-book and open-note. You may use texts, notes, handouts, problem sets and solutions, tables of integrals, series and transforms. You are not permitted to use any electronic devices. You will receive full credit if you correctly state answers without showing any work. But it is recommended that you show enough work so that your method can be understood. Partial credit will depend on the clarity of your argument. Clearly demarcate your final answers. If you are asked to sketch any graphs, label all key features (vertical and horizontal axes, etc.). You may use any results derived in class or in the homework without re-deriving them. Any new terms or symbols introduced in your solution must be defined.

Please write your name and SUNet ID here.

_____________________________________________________________
Name                                                 SUNet ID
_____________________________________________________________

Please sign here acknowledging that you accept the Honor Code.

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Signature

The total number of points is 150.
1. (30 pts.) CT Fourier transform. Consider the four signals $x(t)$, $x_1(t)$, $x_2(t)$, $x_3(t)$, which have Fourier transforms $X(j\omega)$, $X_1(j\omega)$, $X_2(j\omega)$, $X_3(j\omega)$, respectively.

![Graphs of x(t), x1(t), x2(t), x3(t)]

a. (10 pts.) In the table below, use an “X” to indicate any symmetry property that pertains.

<table>
<thead>
<tr>
<th>Fourier Transform</th>
<th>Purely Real</th>
<th>Purely Imaginary</th>
<th>Even Function of $\omega$</th>
<th>Odd Function of $\omega$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X(j\omega)$</td>
<td></td>
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<td></td>
<td></td>
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<tr>
<td>$X_1(j\omega)$</td>
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<tr>
<td>$X_2(j\omega)$</td>
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<tr>
<td>$X_3(j\omega)$</td>
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</tbody>
</table>
b. (12 pts.) Without computing $X(j\omega)$ explicitly, use Fourier transform properties to express $X_1(j\omega)$, $X_2(j\omega)$ and $X_3(j\omega)$ in terms of $X(j\omega)$. More than one correct answer is possible in some cases.
c. (8 pts.) Give an explicit expression for $X(j\omega)$. *Hint:* start by expressing $x(t)$ as a cosine signal times an appropriately shifted and time-scaled rectangular pulse. Evaluate any convolutions appearing in your answer (neither the symbol $*$, nor the convolution integral, should appear in your answer).
2. (30 pts.) *CT Fourier transform and LTI system analysis.*

a. (5 pts.) Consider a signal \( x(t) = \text{sinc}(t) \cos(2\pi t) \). Sketch the magnitude and phase of its Fourier transform \( X(j\omega) \). You need not specify the phase where the magnitude is zero.
b. (10 pts.) Consider an LTI system having a frequency response

\[ H(j\omega) = -j \text{sgn}(\omega) = \begin{cases} -j & \omega > 0 \\ 0 & \omega = 0 \\ j & \omega < 0 \end{cases} \]

Sketch the magnitude and phase of \( H(j\omega) \). You need not specify the phase where the magnitude is zero.
c. (10 pts.) The signal $x(t) \leftrightarrow X(j\omega)$ from part (a) is input to the LTI system from part (b), yielding an output signal $y(t) \leftrightarrow Y(j\omega)$. Sketch the magnitude and phase of $Y(j\omega)$. You need not specify the phase where the magnitude is zero.
d. (5 pts.) Give an expression for the time-domain output signal $y(t)$. *Hint:* it can be expressed in a simple form similar to the input signal $x(t)$. 
3. (30 pts.) *DT Fourier analysis and LTI system analysis.*

a. (9 pts.) Consider an input signal \( x[n] = 2 + (-1)^n \). Express \( x[n] \) as a sum of two scaled imaginary exponentials, i.e., \( x[n] = a_1 e^{j\Omega_1 n} + a_2 e^{j\Omega_2 n} \) for some constants \( a_1, a_2 \) and some frequencies \( \Omega_1, \Omega_2 \), which may include zero. *Hint:* this can be done by inspection.
b. (7 pts.) The signal $x[n]$ from part (a) is input to an LTI system with impulse response

$$h[n] = \frac{1}{4} \text{sinc}\left(\frac{n}{4}\right) \cdot \cos\left(\frac{\pi n}{2}\right).$$

Give an expression for the output signal $y[n]$. 
c. (7 pts.) The signal $x[n]$ from part (a) is input to an LTI system with impulse response $h[n] = a^n u[n], |a| < 1$. Give an expression for the output signal $y[n]$. 
d. (7 pts.) The signal $x[n]$ from part (a) is input to an LTI system with impulse response $h[n]=\begin{cases} 1 & |n| \leq 2 \\ 0 & |n| > 2 \end{cases}$. Give an expression for the output signal $y[n]$. **Hint:** recall that 

$$
\lim_{\Omega \to 0} \frac{\sin[\Omega(N_1 + \frac{1}{2})]}{\sin(\Omega/2)} = 2N_1 + 1.
$$
4. (30 pts.) DT Fourier series and DT Fourier transform.

a. (10 pts.) Consider a periodic signal \( x[n] = \sum_{m=-\infty}^{\infty} \delta[n - 4m - 1] - \delta[n - 4m + 1] \). Sketch \( x[n] \) vs. \( n \).

b. (10 pts.) Specify the period \( N \) and the fundamental frequency \( \Omega_0 \). Find an expression for the DTFS coefficients \( a_k \).
c. (10 pts.) Consider $z[n] = x[n] \cdot y[n]$, where $y[n] = \frac{1}{4} \sin\left(\frac{n}{4}\right)$. Sketch $Z(e^{j\Omega})$, the DTFT of $z[n]$, for $-\pi \leq \Omega < \pi$. You can sketch either the real and imaginary parts, or the magnitude and phase.
5. (30 pts.) Sampling, DT signal processing, reconstruction. Consider a system that performs sampling, DT signal processing and reconstruction. A CT input signal $x(t)$ has the Fourier transform $X(j\omega)$ shown. The sampling interval is $T = 1$. The DT LTI system has an impulse response $h[n] = \frac{1}{2} \text{sinc} \left( \frac{n}{2} \right)$.

\[
\begin{align*}
\text{CT} & \quad x(t) \overset{\text{FT}}{\leftrightarrow} X(j\omega) \\
\text{DT} & \quad x[n] \overset{\text{DTFT}}{\leftrightarrow} X(e^{j\omega}) \\
\text{DT} & \quad w[n] \overset{\text{DTFT}}{\leftrightarrow} W(e^{j\omega}) \\
\text{DT} & \quad y[n] \overset{\text{DTFT}}{\leftrightarrow} Y(e^{j\omega}) \\
\end{align*}
\]

\[
\begin{align*}
\text{Ideal Bandlimited Interpolation} & \\
\end{align*}
\]

\[
\begin{align*}
\text{CT} & \quad y(t) = \sum_{n=-\infty}^{\infty} y[n] \text{sinc} \left( \frac{t-nT}{T} \right)
\end{align*}
\]

a. (6 pts.) Sketch $X(e^{j\omega})$ (real or imaginary part, whichever is nonzero).
b. (6 pts.) Sketch $W\left( e^{j\omega T} \right)$ (real or imaginary part, whichever is nonzero).

c. (6 pts.) Sketch $H\left( e^{j\omega T} \right)$ (real or imaginary part, whichever is nonzero).
d. (6 pts.) Sketch \( Y(e^{j\omega T}) \) (real or imaginary part, whichever is nonzero).

e. (6 pts.) Sketch \( Y(j\omega) \) (real or imaginary part, whichever is nonzero).