Welcome to the Cookbook Filter Guide!

Don't have enough time to spice out that perfect filter before Aunt Thelma comes down for dinner? Well this handout is for you! The following pages detail a fast set of steps towards the design and creation of passive filters for practical use in communications and signal processing. Amaze your friends and neighbors as you do twice the amount of work in half the time!

Here's a brief overview of the steps:

- ◆ Specify your filter type
- ◆ Implement a low-pass version of your filter
- ◆ Transform it to what you really want (high-pass, bandpass, bandstop)
- ◆ Simulate and Iterate

◆ Step 1: What Filter Do I Want?

This is where you have to do most of your thinking. It's no good to cook up a nice steak dinner if Auntie Thelma turned vegetarian last year. Here are the specifications that are most often quoted with filters:

Filter Type (Low-Pass, High-Pass, Band-Pass, Band-Stop)
Center frequency (rad/s or Hz)
Bandwidth (rad/s or Hz)
Cut-off/Roll-off rate (dB)
Minimum Attenuation Required in Stopband
Input Impedance (ohms)
Output Impedance (ohms)
Overshoot in Step Response
Ringing in Step Response

We will be creating filters through a method known as the insertion loss method. Insertion Loss, or the Power Loss Ratio P_{LR}.is defined as:

$$P_{LR} = \frac{Power_{Source}}{Power_{Delivered}} = \frac{1}{1 - |\Gamma(w)|^2}$$

Γ is our famliar reflection coefficient. It turns out that this is expressible in the following form of :

$$P_{LR} = 1 + \frac{M(w^2)}{N(w^2)}$$

Where M and N are two real polynomials. Which simply means that we can define an arbitrary filter response and use this formula to match it to real components, thus allowing us to make it physically realizable. There are several standard filter responses, each with their own advanatages and drawbacks

Standard Filter Responses
- • *Butterworth (AKA Maximally Flat or Binomial) Filters*
    Butterworth filters are general purpose filters. Another common name for them is a maximally flat filter, which refers to the relatively flat magnitude response in the pass-band. Attenuation is –3dB at the design cutoff frequency with a –20dB/decade roll off per pole above the cutoff frequency.

- *Chebyshev (AKA equal ripple magnitude) Filter*

    Chebyshev filters have a steeper attenuation above the cutoff frequency, but at the expense of amplitude "ripples" in the pass-band. For a given number of poles, a steeper cutoff can be achieved by allowing more pass-band ripple. The cutoff frequency is defined at the point at which the response falls below the ripple band of the pass-band.

- *Bessel (AKA Linear Phase or Maximally Flat Time Delay) Filters*

    Sometimes a design requires a filter to have a linear phase in order to avoid signal distortion. In general a good phase output (i.e. linear) always comes at the expense of a good magnitude response (i.e. fast attenuation).

- *Elliptic Filters*

    For the previous filters, as the frequency gets progressively further from the center frequency the attenuation increases. Sometimes a design only requires a minimum attenuation in the stop-band. This relaxes some constraints on the response which allow a better cutoff rate. However, this filter has ripples in both the passband and stop-band.

◆ **Step 2: Prototyping a Low-Pass Design**

Having finally specified your filter, it's time to prototype a low-pass version of your filter. Although it seems counterintuitive to spend time on a filter that doesn't even necessarily have the pass-band/stop-band characteristics of your desired filter, it will become apparent that there is relation between the values derived for a low-pass situation and the other filter types.

As an example we will design a low-pass filter for a source impedance of 50 ohm, a cut-off frequency of 1MHz and which requires a minimum attenuation of 40dB at 10MHz.

Determine the type of filter and N, the order of the filter:

First we have to determine which of the filter types we want to use. Do we care about having linear phase? Or is maxmium cut-off attenuation the critical factor? Once that's been done, we can then determine the order of the filter necessary to fit the required attenuation spec. Usually, we refer to a graph like the one below showing the attenuation characteristics for various N versus normalized frequency for a particular filter type.

In this case, we decide that a flat magnitude response in the most appropriate. So then we look at the graph.The definition of normalized frequency is:

$$f_{norm} = \left| \frac{w}{w_c} \right| - 1$$

we find that w_c=1MHz w_o=10MHz which means the normalized frequency is 9. Looking on the graph we see that N=2 will easily satisfy our requirements for 40dB of attenuation.
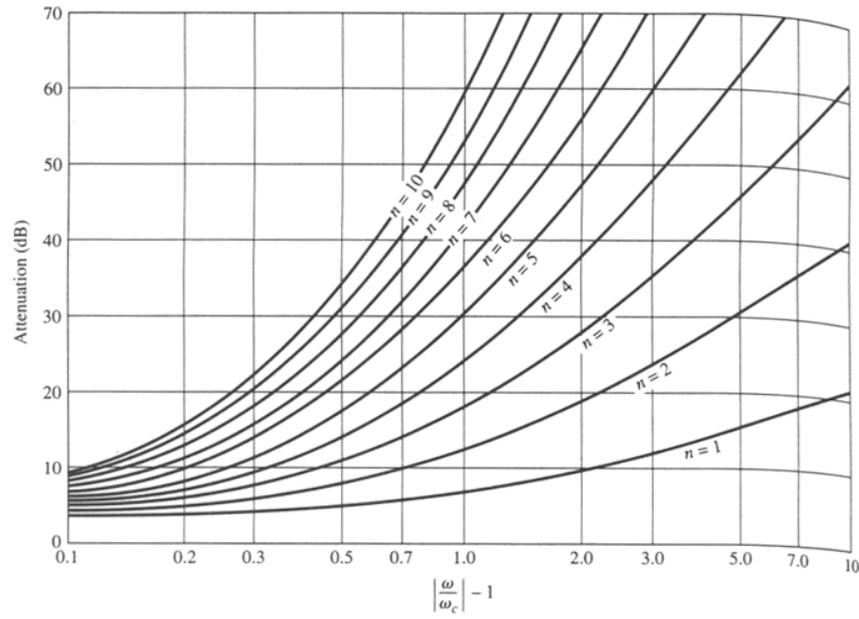
Attenuation versus normalized frequency for maximally flat filter prototypes. Adapted from G. L. Matthaei, L. Young, and E. M. T. Jones, *Microwave Filters, Impedance-Matching Networks, and Coupling Structures* (Dedham, Mass.: Artech House, 1980) with permission.

Design a Normalized Low-Pass Filter using a Table:

Once that is done, we can now design a second order prototype filter for a source impedance of 1 ohm, a cut-off frequency of 1 rad/sec. As shown in the figure below, we use one of two equivalent ladder circuits. Note the way the element values are numbered, with g_0 at the generator to g_{N+1} at the load.
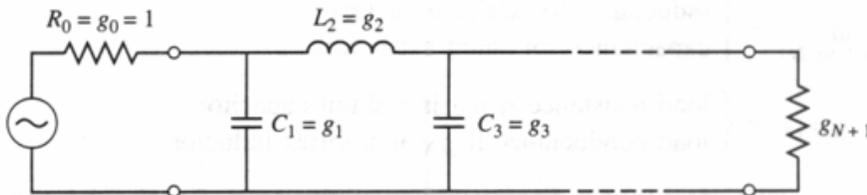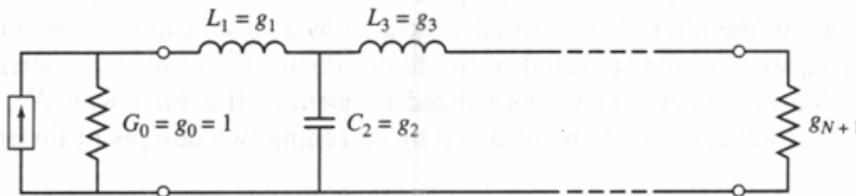
Ladder circuits for low-pass filter prototypes and their element definitions. (a) Prototype beginning with a shunt element. (b) Prototype beginning with a series element.

How to read this chart:

g_0 = generator resistance
  or a
  generator conductance

g_k =
 inductance for series inductors
  or a
 capacitance for shunt capacitors

g_{N+1} =
 load resistance if g_N is a shunt C
  or a
 load conductance if g_N is a series L

A key point is that the components alternate between shunt and series. Note that during out prototyping, inductors are always series, capacitors are always shunt. The only difference is whether or not the *first* element is series or shunt.

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To design a filter of a particular response (i.e. Butterworth, or Bessel) there is a unique ratio of components to be used. These ratios are usually kept is handy tables like the one below.

Element Values for Butterworth (Maximally Flat) Low-Pass Filter Prototypes (g_o=1,w_c=1,N=1 to 10)

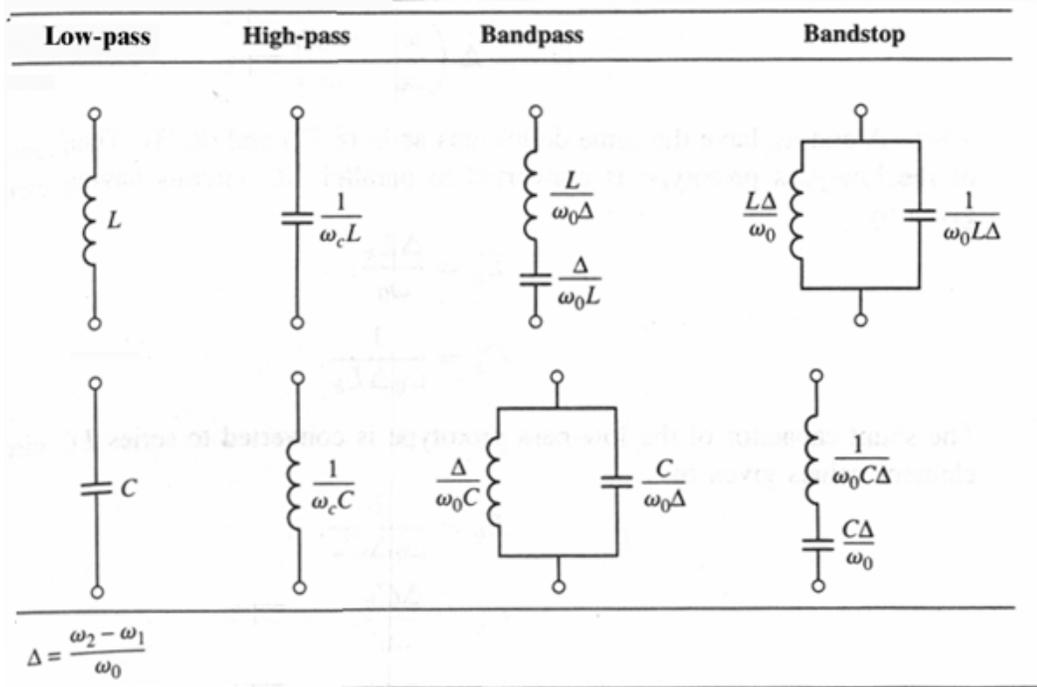| N	$g1$	$g2$	$g3$	$g4$	$g5$	$g6$	$g7$	$g8$	$g9$	$g10$	$g11$
1	2.0000	1.0000									
2	1.4142	1.4142	1.0000								
3	1.0000	2.0000	1.0000	1.0000							
4	0.7654	1.8478	1.8478	0.7654	1.0000						
5	0.6180	1.6180	2.0000	1.6180	0.6180	1.0000					
6	0.5176	1.4142	1.9318	1.9318	1.4142	0.5176	1.0000				
7	0.4450	1.2470	1.8019	2.0000	1.8019	1.2470	0.4450	1.0000			
8	0.3902	1.1111	1.6629	1.9615	1.9615	1.6629	1.1111	0.3902	1.0000		
9	0.3473	1.0000	1.5321	1.8794	2.0000	1.8794	1.5321	1.0000	0.3473	1.0000	
10	0.3129	0.9080	1.4142	1.7820	1.9754	1.9754	1.7820	1.4142	0.9080	0.3129	1.0000

For a second order, we see that g_1 and g_2 must both equal 1.4142 and g_3, the load, must equal the g_o, the source resistance. So the following circuit will get us a low pass filter centered about 1 rad/sec.

◆ Step 3: Transforming the Low-Pass Prototype to the Desired Filter

All we have to do now is transform the prototype into something we can really use. The tables assume a set impedance R_s=R_l=1 ohm (except for Chebyshev filters, where the source and load resistance differ by a set amount). In addition, the filter's cutoff frequency is 1 rad/sec, a fairly useless value in general. So we need to scale for impedance and frequency.

Let's start with impedance scaling. To take this into account, we need to multiply the impedances of the prototype by the desired R_s. Since the prototype input impedance is always 1 ohm, we can simply multiply by the ratio. R_l naturally needs to scale proportionally, while all capacitors need to be divided by R_s and all inductors need to be multiplied by R_s.

Now that we have these original values, we need to swap components in order to obtain the correct filter type. Below are the substitutions.

Low-pass	High-pass	Bandpass	Bandstop
L	$\dfrac{1}{\omega_c L}$	$\dfrac{L}{\omega_0 \Delta}$, $\dfrac{\Delta}{\omega_0 L}$	$\dfrac{L\Delta}{\omega_0}$, $\dfrac{1}{\omega_0 L\Delta}$
C	$\dfrac{1}{\omega_c C}$	$\dfrac{\Delta}{\omega_0 C}$, $\dfrac{C}{\omega_0 \Delta}$	$\dfrac{1}{\omega_0 C\Delta}$, $\dfrac{C\Delta}{\omega_0}$

$$\Delta = \frac{\omega_2 - \omega_1}{\omega_0}$$

$$\Delta = \frac{w_1 - w_2}{w_0} \quad \text{is your friend}$$

Δ is a term to describe the sharpness of the filter. If you look at it really hard, you'll see it's actually the inverse of Q. In filter design the center frequency is not chosen as the arithmetic mean, but the geometric mean. In other words:

$$w_o \neq \frac{w_1 + w_2}{2}$$

$$w_o = \sqrt{w_1 w_2}$$

This allows for the math to work out such that straight substitutions are allowable. What this means is that often times, you'll pick your bandwidth and just go for a center frequency in that area. Of course, you could work out the math to get exactly the center frequency you want, but sometimes, you really don't want to think that hard about a minor filter.

With that all said here are the transformations. The formulas on the next page were derived by comparing the frequency dependent impedances and mapping the cutoff frequency to different situations. In each one, the L_k or C_k from your prototype is coverted into a new L_k' or C_k'. A derivation of these formulas can by found in Posner's *Microwave Transmission*. NOTE: These formulas ALREADY take into account the scaling for impedances.

TRANSFORMATION TABLE

Low-Pass

Simple impedance and frequency scaling.

$$L'_k = \frac{1}{w_c} R_s L_k$$

$$C'_k = \frac{1}{w_c} \frac{C_k}{R_s}$$

High-Pass

Note the swapping of capacitors and inductors:

$$C'_k = \frac{1}{w_c} \frac{1}{R_s L_k}$$

$$L'_k = \frac{1}{w_c} \frac{R_s}{C_k}$$

Bandpass

Series inductor L_k transforms to a series LC circuit defined by:

$$L'_{Lk} = \frac{1}{w_o} \frac{R_s L_k}{\Delta}$$

$$C'_{Lk} = \frac{1}{w_o} \frac{\Delta}{R_s L_k}$$

Shunt capacitor C_k transforms to a parallel LC circuit with element values:

$$L'_{Ck} = \frac{1}{w_o} \frac{R_s \Delta}{C_k}$$

$$C'_{Ck} = \frac{1}{w_o} \frac{C_k}{R_s \Delta}$$

Bandstop

Series inductor L_k transforms to a parallel LC circuit defined by:

$$L'_{Lk} = \frac{1}{w_o} \Delta R_s L_k$$

$$C'_{Lk} = \frac{1}{w_o} \frac{1}{\Delta R_s L_k}$$

Shunt capacitor C_k transforms to a series LC circuit with element values:

$$L'_{Ck} = \frac{1}{w_o} \frac{R_s}{\Delta C_k}$$

$$C'_{Ck} = \frac{1}{w_o} \frac{\Delta C_k}{R_s}$$

In our case, we can see that we need to multiply our L value by R_s/w_c and our C value by $1/w_c R_s$. L=11.25uH and C=4.50nF. Now we test this in HSPICE or Matlab and see if our response is good enough for our purposes. If not, we iterate, making educated tweaks. Note that we've mainly designed for the frequency domain in this methodology, which is fairly straightforward. Time-domain characterstics, such as ringing, or overshoot, are often equally important but the methods for dealing with them are completely different. But that's for another handout to cover…

Element Values for Butterworth (Maximally Flat) Low-Pass Filter Prototypes
(g_o=1,w_c=1,N=1 to 10)

N	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
1	2.0000	1.0000									
2	1.4142	1.4142	1.0000								
3	1.0000	2.0000	1.0000	1.0000							
4	0.7654	1.8478	1.8478	0.7654	1.0000						
5	0.6180	1.6180	2.0000	1.6180	0.6180	1.0000					
6	0.5176	1.4142	1.9318	1.9318	1.4142	0.5176	1.0000				
7	0.4450	1.2470	1.8019	2.0000	1.8019	1.2470	0.4450	1.0000			
8	0.3902	1.1111	1.6629	1.9615	1.9615	1.6629	1.1111	0.3902	1.0000		
9	0.3473	1.0000	1.5321	1.8794	2.0000	1.8794	1.5321	1.0000	0.3473	1.0000	
10	0.3129	0.9080	1.4142	1.7820	1.9754	1.9754	1.7820	1.4142	0.9080	0.3129	1.0000

Element Values for Bessel (Maximally Flat Time Delay) Low-Pass Filter Prototypes (g_o=1,w_c=1,N=1 to 10)

N	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
1	2.0000	1.0000									
2	1.5774	0.4226	1.0000								
3	1.2550	0.5528	0.1922	1.0000							
4	1.0598	0.5116	0.3181	0.1104	1.0000						
5	0.9303	0.4577	0.3312	0.2090	0.0718	1.0000					
6	0.8377	0.4116	0.3158	0.2364	0.1480	0.0505	1.0000				
7	0.7677	0.3744	0.2944	0.2378	0.1778	0.1104	0.0375	1.0000			
8	0.7125	0.3446	0.2735	0.2297	0.1867	0.1387	0.0855	0.0289	1.0000		
9	0.6678	0.3203	0.2547	0.2184	0.1859	0.1506	0.1111	0.0682	0.0230	1.0000	
10	0.6305	0.3002	0.2384	0.2066	0.1808	0.1539	0.1240	0.0911	0.0557	0.0187	1.0000

Element Values for Chebyshev (Equal-Ripple) Low-Pass Filter Prototypes (g_o=1,w_c=1,N=1 to 10)
0.5dB Ripple Band

N	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
1	0.6986	1.0000									
2	1.4029	0.7071	1.9841								
3	1.5963	1.0967	1.5963	1.0000							
4	1.6703	1.1926	2.3661	0.8419	1.9841						
5	1.7058	1.2296	2.5408	1.2296	1.7058	1.0000					
6	1.7254	1.2479	2.6064	1.3137	2.4758	0.8696	1.9841				
7	1.7372	1.2583	2.6381	1.3444	2.6381	1.2583	1.7372	1.0000			
8	1.7451	1.2647	2.6564	1.3590	2.6964	1.3389	2.5093	0.8796	1.9841		
9	1.7504	1.2690	2.6678	1.3673	2.7239	1.3673	2.6678	1.2690	1.7504	1.0000	
10	1.7543	1.2721	2.6754	1.3725	2.7392	1.3806	2.7231	1.3485	2.5239	0.8842	1.9841

3.0dB Ripple Band

N	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
1	1.9953	1.0000									
2	3.1013	0.5339	5.8095								
3	3.3487	0.7117	3.3487	1.0000							
4	3.4289	0.7483	4.3471	0.5920	5.8095						
5	3.4817	0.7618	4.5381	0.7618	3.4817	1.0000					
6	3.5045	0.7685	4.6061	0.7929	4.4641	0.6033	5.8095				
7	3.5182	0.7723	4.6386	0.8039	4.6386	0.7723	3.5182	1.0000			
8	3.5277	0.7745	4.6575	0.8089	4.6990	0.8018	4.4990	0.6073	5.8095		
9	3.5340	0.7760	4.6692	0.8118	4.7272	0.8118	4.6692	0.7760	3.5340	1.0000	
10	3.5384	0.7771	4.6768	0.8136	4.7425	0.8164	4.7260	0.8051	4.5142	0.6091	5.8095

Please note that these tables assume the following:

$$w_c = \frac{2}{\sqrt{LC}}$$

Which means that your filters will have a smaller bandwidth than you usually expect if you define 3dB as your cutoff frequency. You may easy compensate this by just roughly doubling w_c